by introducing a cut-off angle θ_{max}. The higher moments in this case, of course, are greater than in case A: ($\langle\theta^4\rangle_{Av}$ by 18 percent and $\langle\theta^6\rangle_{Av}$ by 45 percent, according to Green and Messel). But it is obvious that the higher moments in this case are very sensitive to the choice of the cut-off θ_{max}. On the other hand, E. J. Williams' expression for θ_{max} merely represents an estimation of the order of magnitude and the cut-off procedure as a whole, of course, is somewhat rough. (By a more rigorous treatment the largest scattering angles would not be entirely suppressed but only reduced in frequency by a factor Z^{-1} since in this region the protons of the nucleus scatter individually.) The exact numerical values of the higher moments in this case also are therefore physically meaningless. Moreover, the influence of the finite size of the nucleus is practically negligible in shower theory (see Sec. C). Finally, it may be stated that the method of calculating the distribution functions by means of the moments, as proposed by Green and Messel, seems not very suitable because a very large number of moments (strictly speaking, all of them), would be needed if one wished to obtain reasonable accuracy in the interesting region of small and medium arguments.[4]

C. *Single scattering is taken into account neglecting the influence of the finite size of the nuclei:*—in shower theory single scattering is altogether of little importance. This has been pointed out by Nishimura and Kamata[5] who have shown that the main contribution to strongly deflected electrons (i.e., those found at large values of θ and r), is due to multiple scattering of electrons slowed down by ionization loss rather than to single scattering. Large single scattering angles, in general, are rare events. The modification of the frequency of very large single scattering angles by the finite size of the nucleus, therefore, is of still less importance. [As an illustration, it may be estimated that the influence of the finite size of the nucleus on the distribution function $f(E,\theta)$ is not appreciable up to angles of about seven times the root-mean-square angle of multiple scattering.] It may be noted, further, that in case C and also in the above-mentioned rigorous treatment of case B, all the moments turn out to be infinite in the usual small-angle approximation. This illustrates the irrelevance of the moments proposed by Green and Messel which depend entirely on the cutoff.

Case C was used in the present author's theory,[3] the procedure being the following. Starting with case A, the Fourier-transforms of the distribution functions were calculated numerically with great accuracy. The moments valid in case A play the rôle of the coefficients of the power series of the Fourier-transforms and were duly used in this calculation. As a next step, for convenience in performing the Fourier-transformation, the exact Fourier-transforms were approximated by analytical expressions in such a way that stress was laid upon a good representation at large and intermediate arguments of the Fourier-transforms. In this way great accuracy was reached at small and intermediate arguments θ and r of the distribution functions resulting from the Fourier-transformation, whereas errors were admitted in the region of large θ and r where the distribution functions due to multiple scattering have fallen off practically to zero, these errors also influencing the higher moments. The criticism expressed in the literature in this connection (Blatt,[6] Eyges[7]) is not significant because in this domain of large arguments the distribution function is due solely to single scattering. The influence of the latter was determined in a final step of the theory, as follows. The resulting distribution functions of case A were decomposed into Gaussian functions ("Gauss transformation"), each Gaussian function associated with electrons of a certain "energetic history," characterized by a certain parameter. These Gaussian functions, then, were replaced by the more exact functions which contain the "single-scattering tail." Finally, by the inverse Gauss transformation, the distribution functions of shower theory duly containing the effect of single scattering were obtained.

(2) Another point of Green and Messel's criticism concerns the neglect of the variation of the atmospheric density in all previous theories. The influence of this variation of density is closely connected with the path length which is needed for equilibrium in the distribution in θ and r. As to this question, Green and Messel have given an example. They have calculated $\langle r^2\rangle_{Av}$ as a function of depth in a homogeneous layer of matter in the case of a primary (integral) power spectrum of exponent 1.5. From the figures given by them the conclusion may be drawn that in the case of a power 1.5 a path length of about 12 to 14 radiation lengths is needed for equilibrium in the radial distribution. Roughly, this means that the radial deviations of the electrons in the depth of observation have their origin mainly at a depth smaller by 6 to 7 radiation lengths. For showers observed at sea level, therefore, the inhomogeneity of the atmosphere can be roughly accounted for by using a value of the radiation length which is greater by 25 to 30 percent. For a power law with exponent 1, i.e., for air showers near the maximum, the situation is still better. It may be estimated that in this case 6 to 8 radiation lengths are sufficient to reach equilibrium, which means that a layer higher by 3 to 4 radiation lengths is responsible for the radial distribution. The radiation length, therefore, has to be increased by 15 to 20 percent for observation at sea level and by 30 to 40 percent for 5000 m.

A more exact computation of this correction, *viz.*, an improved theory taking into account the variation of density, would be useful. But the assertion of Green and Messel that the neglect of the effect of density variation would introduce errors as large as 5000 percent (!) cannot be understood. Presumably this large error would concern the higher moments and so is of no interest.

(3) In a further point in their paper, Green and Messel make the criticism that previous authors either consider only the maximum of showers or integrate over all depths. However, both the maximum and the integration over all depths are suitable starting points. Besides, Green and Messel seem to overlook the fact that in the meantime, Nishimura and Kamata[5] extended the theory to a shower age of 1.5.

The points 4 and 5 enumerated by Green and Messel need no commentary. The further criticism expressed by Green and Messel in the text of their paper, claiming that ionization loss has not been duly treated by previous authors, is also made obsolete by the work of Nishimura and Kamata, which seems to be unknown to them.

Finally it may be noted that the lateral spread of the electronic component of large air showers is only slightly modified by the contribution of the nucleon-meson component. This contribution is practically restricted to small distances from the center and consists in the formation of plural cores at separations of the order of some tens of centimeters from each other.[8]

[1] H. S. Green and H. Messel, Phys. Rev. **88**, 331 (1952). We are thankful to Professor W. Heisenberg for sending us a prepublication print of the paper.
[2] L. Eyges and S. Fernbach, Phys. Rev. **82**, 23 (1951).
[3] G. Molière, in *Cosmic Radiation*, edited by W. Heisenberg (Dover Publications, New York, 1946). The theory of the spread of large air showers will be presented in detail in the second edition of this book (Springer, Berlin, 1953).
[4] In their application of this method to the nucleonic cascade, Green and Messel [Phys. Rev. **87**, 738 (1952)] used an arbitrary extrapolation for the asymptotic behavior of the higher moments. As stated above, just this asymptotic behavior determines the shape of the distribution function.
[5] J. Nishimura and K. Kamata, Progr. Theoret. Phys. **6**, 262, 628 (1951); **7**, 185 (1952).
[6] J. M. Blatt, Phys. Rev. **75**, 1584 (1949).
[7] L. Eyges, quoted by Blatt, reference 6. See also Nordheim, Osborne, and Blatt, Echo-Lake Report, 1949 (unpublished).
[8] See H. Messel and H. S. Green, Phys. Rev. **87**, 738 (1952).

Total Interaction Cross Section of Pions with Protons and Deuterons at 1.0 Bev*

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THE total cross sections of negative pions with hydrogen and the deuterium-hydrogen difference have been measured at an average pion kinetic energy of 1.0 Bev. The apparatus described

TABLE I. The total interaction cross section of 1.0-Bev negative pions with H and (D–H). Measurements are listed for absorbers of various hydrogen or deuterium content and for several rms values θ_{rms} of the half-angle subtended by counter No. 5 at the absorber.

θ_{rms}	Absorber g cm^{-2} of H	$\sigma(\pi^-,p)$ mb	Absorber g cm^{-2} of D	$\sigma(\pi^-,d-p)$ mb
7.2°	8.27	39 ±3	13.34	17 ±3
5.4°	3.90	42 ±4.5	···	···
3.2°	12.17	49 ±3	···	···
2.7°	···	···	13.34	21.2±2.6
2.5°	3.90	47.5±5	···	···
2.2°	12.17	46 ±3.5	···	···

in a previous letter[1] was placed in the 1.1-Bev negative pion beam which, like the 1.55-Bev beam, is deflected by the Cosmotron magnet. The method for measuring the cross sections was the same as that described before.

In this beam the muon contamination was larger than at 1.55 Bev. It was determined by two arrangements; both make use of the strong-pion and weak-muon nuclear interaction so that, after passing through thick absorbers, the beam contains most of the initial muons and comparatively few pions. The counting rate beyond the absorber has to be corrected for the pions which remain and for the loss of muons by multiple Coulomb scattering in the absorber. In the first arrangement, an absorption curve in Al was obtained up to 307 g cm^{-2}, by placing the Al between counters 4 and 5. The correction for the loss of muons by scattering was calculated in this case to be 17 percent. In the second arrangement, counters 4 and 5 were arranged as for the CH_2-C measurements with the 73 g cm^{-2} of C in place. In addition, 372 g cm^{-2} of Fe was placed behind counter 5. A single large counter was placed behind the Fe and was used in several successive positions to integrate the remaining beam over an effective diameter of 14 in. In this arrangement, the loss of muons by scattering was negligible, but a somewhat greater uncertainty than that of the first method was involved in placing safe limits on the remaining pions. The result of the first method is $I_\mu/I_0 = 0.110\pm0.025$ while the second gives $I_\mu/I_0 = 0.117\pm0.012$.

Two thicknesses of absorber have been used in the CH_2-C measurements (12.17 and 3.9 g cm^{-2} of H). The thick absorber introduces less statistical uncertainty and an effectively larger muon correction than the thin absorber. The muon contamination produces a correction in $\sigma(\pi^-,p)$ of 8 mb for the thin absorber and 12 mb for the thick. The results are in agreement, and quoted errors include the uncertainty in evaluating the muon contamination. The electron contamination has been measured as at 1.5 Bev, and found to be less than 1 percent.

Several geometries have been used to check that the cross-section values obtained are equal to the total cross section; that is, that the fraction of events in which secondaries enter our last counter is, indeed, very small with the geometry most extensively used. Taking account of the differences in solid angle, the data reported in Table I show that $\sigma(\pi^-,p)=48\pm4$ mb at 1.0 Bev, which is distinctly larger than the value of 27.5±6 mb obtained by Lindenbaum and Yuan[2] at 450 Mev and, with reasonable certainty, exceeds our value of 34±3.5 at 1.5 Bev.[1] The value of Shapiro, Leavitt, and Chen[3] of $\sigma(\pi^-,p)=47\pm5$ mb at 850 Mev fits quite smoothly on a curve showing a broad maximum at roughly 1 Bev.

Another interesting feature shown by our data is the ratio between $\sigma(\pi^-,p)$ and $\sigma(\pi^-,d-p)$. $\sigma(\pi^-,d-p)=21\pm3$ mb and should be equal to $\sigma(\pi^+,p)$ if the principle of charge symmetry and the additivity of cross sections in deuterium were rigorously correct. While actually some difference can be expected, principally because of the second assumption, the qualitative fact that $\sigma(\pi^-,p)$ is considerably larger than $\sigma(\pi^+,p)$ at 1 Bev can hardly be doubted. Preliminary measurements with positive pions at 800 Mev support our present conclusion.

From $\sigma(\pi^-,p)$ and $\sigma(\pi^+,p)$ one can derive the value of the cross section for the pure state having total isotopic spin of one-half: $\sigma(\pi^-,p)_{I=\frac{1}{2}}=\frac{3}{2}\sigma(\pi^-,p)-\frac{1}{2}\sigma(\pi^+,p)$. It is 62±7 mb at 1 Bev and about one-half of this value at both 0.45 Bev and 1.5 Bev. This state seems, therefore, to have a very marked maximum at about 1 Bev.

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Transient Nuclear Induction Signals Associated with Pure Quadrupole Interactions*

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WE have observed transient "pure quadrupole" induction signals corresponding to the "Bloch decays" and echoes found in pulsed nuclear magnetic resonance experiments.[1] The induction signals arise from oscillating components of magnetization established along the axis of the applied rf magnetic field H_1. A quantum-mechanical calculation predicted that these transient components should occur even though, at equilibrium, the electric interaction $Q\cdot\nabla E$ produces no macroscopic magnetization. The presence of the induction decays had been suggested by Dean[2] to explain the anomalous signal to noise behavior of his quenched oscillator in an earlier investigation of pure quadrupole spectra. We have found the predicted induction signals in $NaClO_3$ single crystals and powders, following the application of rf pulses at the Cl^{35} pure quadrupole resonance frequency (29.920 Mc/sec at room temperature). We observe the effects of nuclear Zeeman splittings upon the induction signals by orienting the single crystals within a small magnetic field.

To develop a theoretical expression for the induction signals, we consider a Hamiltonian of the form $\mathcal{H}=Q\cdot\nabla E+\mu\cdot(H_0+H_1\cos\omega t)$, where H_0 and H_1 are the amplitudes respectively of the applied dc and rf magnetic fields. We expand the wave function of the Cl nuclei (spin $\frac{3}{2}$) in terms of the eigenfunctions of the electric quadrupole term, $\psi=\Sigma C_m \exp(-iE_m t/\hbar)\psi_m$. The induction signals observed arise from the components along H_1 of the bulk magnetization $M_{x(y)}$, which is created transverse to the symmetry axis of $\nabla E\cdot M_{x(y)}\propto \bar{I}_{x(y)}=(\psi^*/I_{x(y)}/\psi)$, where the $I_{x(y)}$ are the transverse components of the nuclear angular momentum operator.

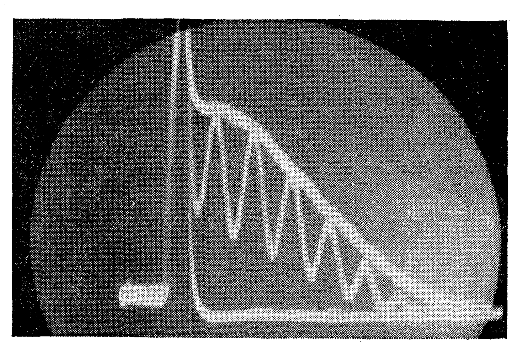

FIG. 1. Induction decay in a single crystal of $NaClO_3$. The sweep is 800 μsec long. The beat structure appears in the presence of a Zeeman field of 12 gauss applied, parallel to H_1, along (0,0,1).