The Zenith Angle Dependence of Cosmic-Ray Protons*

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The proton spectrum for particles in the momentum band $0.4-1.0$ Bev/c and the meson spectrum for particles in the momentum band $0.09-0.34$ Bev/c at zenith angles 45° east, vertical, and 45° west have been obtained at 3.3-km elevation and magnetic latitude 48° north. The zenith angle dependence of protons, along with a simplified analysis, is presented as data on the nucleonic cascade. It is found that the gross zenith angle dependence will fit a $\cos^{3.2}\theta$ distribution at 45 degrees east and west, there being no statistically significant east-west asymmetry. The simplified calculation assumes that a unidirectional component of the primary radiation will (1) suffer exponential absorption along its original path with a path length for removal of 120 g/cm^2 of air and (2) acquire an ultimate distribution at 3.3 km due to scattering fitting a $\cos^n\omega$ law, n being an adjustable parameter.

It is found that a value of n of 6.5 ± 0.7 yields a unidirectional distribution which, when integrated over all angles, results in a gross zenith angle dependence agreeing with $\cos^{3.2}\theta$. The $\cos^n\omega$ distribution with n equal to 6.5 is quite sharp and leads to the conclusion that the average projected scattering angle per collision is only about 6 degrees.

INTRODUCTION

A GREAT deal of effort in cosmic-ray research has been expended toward determining the behavior of a generalized nuclear component in its traversal of the atmosphere. It is known that the development of this nuclear component involves a cascade process qualitatively similar to that of the soft component. This has been discussed by Janossy, Heitler, Messel, and others.[1-3] The validity of their attempts to provide an adequate theoretical description of the cascade suffers from an almost complete lack of experimental knowledge of the cross sections of the many processes involved. Recent experiments on penetrating showers by Branch[4] and others have provided evidence to indicate that the differential cross sections in inelastic nucleon-nucleus collisions must be sharply peaked in the forward and backward directions.

In successively refined experiments[5] conducted by this Laboratory, a technique of proton-meson analysis of the cosmic radiation has been developed using a magnetic cloud chamber. This has been applied to the present study of the zenith angle dependence of the nonelectronic component of the ionizing radiation of range less than 15 cm of lead present at an altitude of 3.3 km. This radiation consists almost entirely of mu-mesons and protons. The ability of this apparatus to accept these particles in distinct momentum bands makes it possible to study the zenith angle dependence of protons alone.

A simple phenomenological analysis of this dependence of proton radiation on zenith angle has been made in an attempt to separate effects of nuclear scattering from the geometrical factors arising from the fact that contributions to the total radiation come from all directions of primary incidence. If the effect of exponential absorption is also taken into account, it is possible to arrive at a closer approximation to the angular distribution of secondaries due to nuclear collisions. From this distribution, it is found that the mean projected angle of scatter is in agreement with the sharply peaked differential cross section mentioned above.

THE EXPERIMENT

A. Apparatus

A sketch of the apparatus is shown in Fig. 1. Construction and operating details have been given in previous papers.[5] The apparatus records the intensity of those particles capable of traversing, in turn, the 2.5-cm lead absorber over the telescope, the Geiger tube $C1$, the cloud chamber, the 1-cm lead absorber over $C2$, $C2$ itself, but stopping in the 15-cm lead absorber at the bottom. Coincidence events of the type $C1+C2-A1-A2-A3$ are selected by suitable electronic circuits and initiate chamber expansion. Anticoincidence counters $A1$, lying outside the acceptance beam defined by $C1$ and $C2$, serve to limit the number of cases in which shower events are recorded, while the tubes $A2$ and $A3$, also in anticoincidence, establish the maximum range of particles accepted. As discussed before,[5] such range definition serves to resolve protons and mesons into distinct momentum bands. To the extent that scattering and nuclear interaction in the absorbers are ignored, the apparatus accepts protons in the momentum band $0.4-1.0$ Bev/c.

Particles are deflected in the chamber by a magnetic field of 8200 gauss, and momentum measurements are obtained by photographing the chamber and comparing

* This work supported by U. S. Office of Naval Research.
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1 W. Heitler and L. Janossy, Proc. Phys. Soc. (London) **A62**, 374 (1949).
2 L. Janossy and H. Messel, Proc. Roy. Irish Acad. **A54**, 217, 245 (1949).
3 H. Messel, Phys. Rev. **83**, 21, 26 (1951).
4 G. M. Branch, Phys. Rev. **84**, 147 (1951).
5 Miller, Henderson *et al.*, Phys. Rev. **79**, 459 (1950); **84**, 981 (1951).

the curvature of the projected image of the track with that of standard arc.

As in previous work, the 2.5-cm absorber above $A1$ serves to insure the recognition of electron showers. Such showers either actuate one of the counters $A1$, in which the case the event is not recorded, or else are recognized by the appearance of multiple tracks in the chamber.

The entire assembly sketched in Fig. 1 was so mounted that it could be rotated about an axis perpendicular to the plane of the page, so that the beam at any zenith angle up to 50° could be accepted.

B. Experiments Performed

Three sets of measurements were taken with this apparatus located at Climax, Colorado, elevation 3.3-km, magnetic latitude 48° north during the months of August, September, and October, 1951. Initial conjectures as to the results of the measurements led to the conclusion that appreciable differences in intensity would require rather large angles of observation. For this reason, an angle of 45° was chosen. The three sets of measurements differed only in the zenith angle to which the apparatus was tilted, a measurement being made at vertical incidence, at 45° to the east of the zenith and at 45° to the west. Accumulation of statistically meaningful data was too slow to allow measurements at other angles.

Nearly 12 000 photographs were taken, about equally divided between the three cases. The method of data analysis, including the determination of absolute rates quoted here, is described fully in the previous papers.

RESULTS

Figure 2 gives the results of the three sets of measurements. The dark bars on the horizontal axis indicate the momentum bands determined by the range limits imposed by the apparatus for protons and mesons as labeled. The negative distributions show a single peak, since they contain mu mesons only, while the positive distributions show a second maximum at the momentum expected for protons. The rapid cutoff of the negative distributions at high momentum insures that the positive curve contains very few mesons above 0.5 Bev/c. The high momentum tail of the positive curve indicates the strong nuclear interaction in the lead absorber characteristic of protons. From these graphs, it is apparent that the 45° east and 45° west spectra are identical within statistics for both protons and mesons, and that the vertical spectrum differs from the 45° spectra only by a scale factor of about 3.2. These results are better shown in Fig. 3, in which the solid curve is the vertical spectrum, while the plotted points are those of the 45° data averaged over east and west and multiplied by the scale factor of 3.2. This scale factor corresponds to an angular distribution described by $\cos^n\theta$, with $n=3.2\pm0.3$.

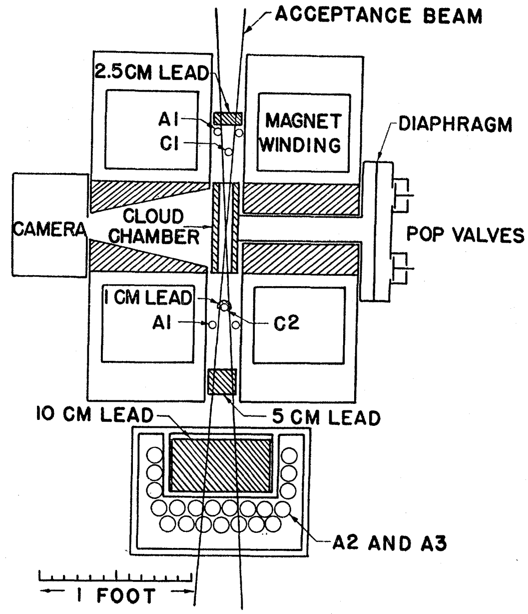

FIG. 1. Sketch of experimental apparatus.

Figure 3 indicates that, within the accuracy of the experiment and at this altitude, protons of momentum near 0.75 Bev/c show the same zenith angle dependence as mesons of momentum near 0.25 Bev/c. It is not to be expected that protons and mesons should show the same zenith angle dependence, since these particles are not closely related genetically, and the factors determining these zenith angle dependences are not at all similar. The present result must thus be considered fortuitous for the particular momentum and altitude investigated. Indeed, the results of another experiment[6] to be published soon show that at higher momentum (2 Bev/c), meson intensities in the vertical and 45° directions differ by a factor of only 2.

DISCUSSION

As compared to the meson radiation, the atmospheric development of the proton radiation (and the nucleonic radiation in general) is greatly complicated by nuclear interaction which here becomes the most important single process. The cascade is propagated through the atmosphere principally by protons and neutrons produced in successive penetrating showers. Pi mesons originating in these showers decay too quickly to have high probability of undergoing further nuclear interaction, while the mu mesons secondary to them do not undergo appreciable nuclear interaction at all and so take almost no part in further propagation of the cascade.

Absorption of the radiation which produces penetrating showers has been studied by Van Allen, Ber-

[6] F. M. Charbonnier *et al.* (to be published).

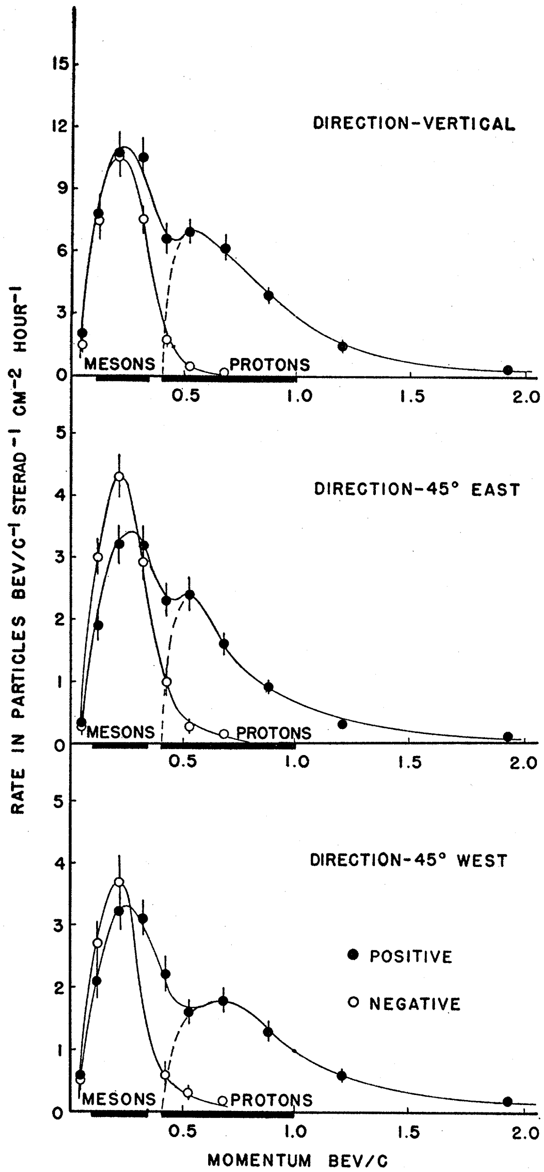

FIG. 2. Proton and meson momentum spectra for three zenith angles.

nardini, Rossi, and others[7–10] who find an exponential absorption with a coefficient of about 120 g/cm^2 of air. Studies of the absorption of radiation producing photographic stars as well as direct measurements by this Laboratory of the relative proton intensity between 3.3 km and sea level give similar absorption path lengths. It should be pointed out that an exponential absorption with this path length cannot hold to the top of the atmosphere, as in this case, the observed proton intensity at 3.3 km would be much too large, as compared with the primary intensity at the top of the atmosphere. In the following treatment of the angular spread of the proton radiation, a constant removal path length of 120 g/cm^2 is used. In making comparisons with observations made at 3.3 km, it is not necessary that this assumption hold to the top of the atmosphere, but rather that it be good from heights of 3.3 km to sea level. Protons observed in this investigation lie principally between 0.5 Bev/c and 1.0 Bev/c. On the basis of ionization loss alone, these protons must have had momenta greater by 2 Bev/c at the top of the atmosphere. As they will have undergone, on the average, several nuclear collisions, the primary energy must have been several times greater than this amount. For protons of momenta above 7.5 Bev/c the primary distribution is isotropic above the horizon, as shown by

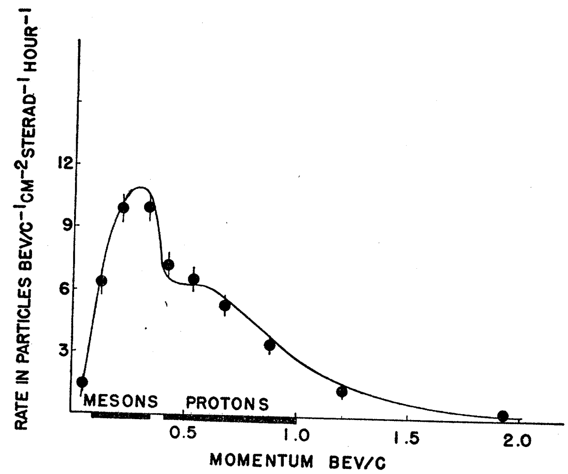

FIG. 3. Comparison of momentum spectra. Points plotted are for the 45° positive spectra averaged over east and west and multiplied by a scale factor of 3.2. The curve is the vertical positive spectrum.

the Lemaitre-Vallarta theory of the geomagnetic effect. It can be expected then that the protons observed at 3.3 km are derived mainly from an initially isotropic distribution. The radiation at 3.3 km then can be considered as the superposition of contributions at this altitude from equal primary intensities for all directions at the top of the atmosphere. The observed angular spread at 3.3 km depends in part on geometrical factors resulting from this superposition of contributions from many primary directions. It is clear, for example, that a primary radiation all of vertical incidence would not give rise to so great an angular spread at 3.3 km as that observed. Thus, the observed angular spread represents an upper limit to that due to nuclear scattering alone in traversing the atmosphere. We attempt to obtain a closer approximation to the angular spread due solely to the nuclear scattering through the following assumptions:

[7] H. E. Tatel and J. A. Van Allen, Phys. Rev. 73, 87 (1948).
[8] Bernardini, Cortini, and Manfredini, Phys. Rev. 74, 1878 (1948).
[9] B. Rossi, Revs. Modern Phys. 20, 537 (1948).
[10] J. Tinlot, Phys. Rev. 73, 1476 (1948).

(1) The primary proton intensity is the same at all angles of incidence.

(2) The intensity of secondaries at depth x due to primaries of zenith angle θ (independent of azimuth) depends only on the diagonal distance d in g/cm^2 as measured along this zenith direction.

(3) This intensity decreases exponentially with a path length of 120 g/cm^2.

(4) The angular spread of secondaries due to the primary radiation in a given direction is represented by a $\cos^n\omega$ law, centered about this direction, with n independent of depth.

The first assumption is completely justified for primary momenta above 7.5 Bev/c by the geomagnetic theory. As a result of the following computation the assumption that the primary radiation is isotropic for angles greater than 45° is found unnecessary. For this smaller angle, geomagnetic theory insures isotropy for momenta as low as 3.3 Bev/c.

The second assumption would be strictly true if there were no scattering. With scattering, the geometrical situation is not strictly the same for all zenith angles. A primary traversing a given path near the top of the atmosphere will produce secondaries at lower altitudes with lateral displacements from its original path. As compared to vertical incidence, minimum path lengths for these secondaries (from the point of observation to the top of the atmosphere) will be increased for 180° of azimuth and correspondingly decreased for the other 180°. Except for large zenith angles, since average path lengths are the same, the situation will not differ much from that for vertical incidence.

Assumption three conforms to the large body of experimental determinations of this attenuation. While, as has been pointed out, this cannot hold to the top of the atmosphere, for the present purpose it need only be a close approximation for atmospheric depths from about 700 to 1000 g/cm^2. Experimental evidence indicated its validity below 60 000 feet.[11]

With regard to assumption four, a $\cos^n\omega$ scattering law has no theoretical foundation. Its use conforms to usual practice, n being adjusted to give the experimentally determined rate of decrease. The assumption that n is a constant cannot be true over large ranges of distance. Again this need only be approximately true over a range of 700 to 1000 g/cm^2. As n will be a function of the proton energies involved, similarity of the proton spectra at 700 and 1000 g/cm^2 can be taken as a measure of the constancy of n. This evidence has been provided by Wilson.[12]

The various quantities used in the calculation are defined as follows:

$I_0(\theta_0,\varphi_0)$ denotes proton intensity at the top of the atmosphere in a direction θ_0, φ_0 (see Fig. 4). This is assumed to be constant.

x denotes the depth of the point of observation measured in path lengths of 120 g/cm^2.

$I_x(\theta_0,\varphi_0,\theta_x,\varphi_x)$ is the portion of the observed proton intensity at depth x at angles θ_x, φ_x contributed by the primary radiation incident at angles θ_0, φ_0 at the top of the atmosphere.

$I_x(\theta_x,\varphi_x)$ denotes the proton intensity at depth x observed at angles θ_x, φ_x.

ω is the angle between $d\Omega_0$ and $d\Omega_x$.

The counting rate of a Geiger counter telescope at the top of the atmosphere will be proportional to $I_0(\theta_0,\varphi_0)d\Omega_0$, where $d\Omega_0$ is the aperture of the telescope. The telescope at depth x will record $I_x(\theta_x,\varphi_x)d\Omega_x$. We wish to account for the fact that this latter rate falls by a factor of 3.2 from the vertical to 45° zenith angle. For this purpose, we assume that

$$I_x(\theta_0\varphi_0,\theta_x,\varphi_x)=kI_0(\theta_0,\varphi_0)[\exp(-x/\cos\theta_0)]\cos^n\omega.$$

We quickly find from Fig. 4 that

$$\cos\omega=\sin\theta_0\sin\theta_x\cos(\varphi_0-\varphi_x)+\cos\theta_0\cos\theta_x,$$

so that

$$I_x(\theta_x,\varphi_x)d\Omega_x=kI_0(\theta_0,\varphi_0)\int_{\Omega_0}[\cos\theta_0\cos\theta_x$$
$$+\sin\theta_0\sin\theta_x\cos(\varphi_0-\varphi_x)]^n\exp(-x/\cos\theta_0)d\Omega_0 d\Omega_x,$$

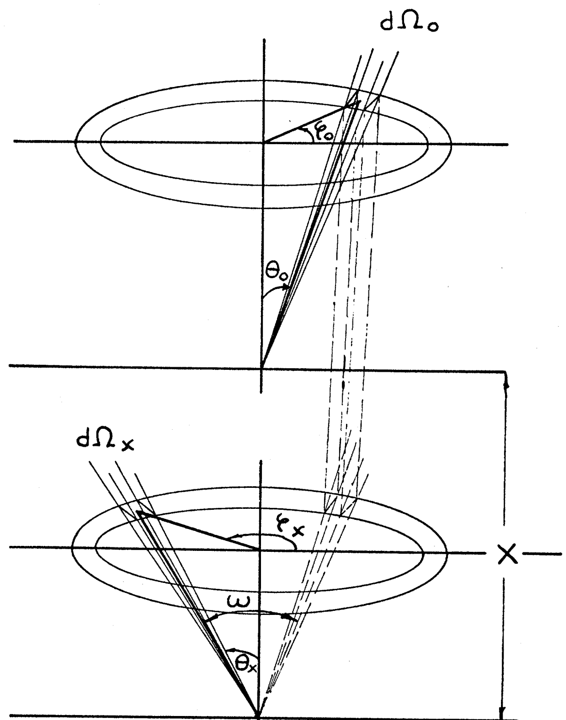

FIG. 4. Sketch of coordinate system used.

[11] M. Blau and H. Wambacher, Akad. Wiss. Wien **IIa**, **146**, 623 (1937).

[12] M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 417 (1951).

where k is a normalization factor, and Ω_0 is the upper hemisphere. The determination of k can be made immediately by noticing that

$$\int_{\Omega_x} I_x(\theta_0,\varphi_0,\theta_x,\varphi_x)d\Omega_x = kI_0(\theta_0,\varphi_0)\exp(-x/\cos\theta_0)\int_{\Omega_x}\cos^n\omega d\Omega_x d\Omega_0$$

must reduce to $I_0(\theta_0,\varphi_0)d\Omega_0$ for the case of no absorption, i.e., infinite removal path length. This means that x is zero and the exponential factor is unity. Thus, from

$$k\int_{\Omega_x}[\cos\theta_0\cos\theta_x+\sin\theta_0\sin\theta_x\cos(\varphi_0-\varphi_x)]^n d\Omega_x=1,$$

where Ω_x is again the upper hemisphere, k can presumably be found. The result, however, evidently depends on θ_0. This is to be expected, since for larger values of θ_0, the $\cos^n\omega$ distribution places particles outside the limits of integration. However, it will be shown immediately below that the exponent n is large enough so that the contributions to the above integral for values of θ_x greater than 90° are negligibly small for θ_0 less than 45°, which is the region of interest. Thus, it is legitimate to set $\theta_0=0$ with the result

$$k\int_0^{\pi/2}\int_0^{2\pi}\cos^n\theta_x\sin\theta_x d\varphi_x d\theta_x=1$$

or

$$k=(n+1)/2\pi.$$

The determination of n is independent of this result.

Involved in the choice of the $\cos^n\omega$ distribution for an assumed scattering law is the fact that forward scattering predominates and that scattering in excess of 90° is negligibly small. Since $\cos^n\omega$ vanishes at 90°, the behavior of the forward scattering is fairly well approximated. However, $\cos^n\omega$ is not zero beyond 90°, and contributions to the integral will occur unless appropriate limits for the integration are defined. Again on account of the size of n, this difficulty is obviated.

The physical situation demands that we have axial symmetry about the vertical (no deflection by the earth's magnetic field is assumed), and hence, the result must be independent of φ_x. This allows the choice $\varphi_x=0$. The integral can then be transformed to the expression:

$$I_x(\theta_x,\varphi_x)d\Omega_x=kI_0(\theta_0,\varphi_0)\int_0^{\pi/2+\alpha}\cos^n(\xi-\alpha) \times\int_0^{\pi}[\exp(-x/\sin\lambda\sin\xi)\sin^{n+1}\lambda d\lambda d\xi d\Omega_x,$$

where $\alpha=\pi/2-\theta_x$, which is in a better form for the necessarily graphical integration. We have data on this quantity for 2 directions of observation, and a determination of n can be had by computing the ratio

$$\rho=I_x(0,\varphi_x)/I_x(\pi/4,\varphi_x)$$

as a function of n and comparing with the experimental value of 3.2. Table I presents the results of this calculation. It is seen that ρ is a fairly sensitive function of n, and that n can be assigned the value 6.5. This is a sharp distribution, falling by a factor of nine from zero to 45° zenith angle, and is ample justification for the various approximations made in evaluating the integral, since contributions from primary radiation incident at zenith angles greater than 45° are less than 5 percent for n greater than 5.

The value of n so determined is dependent on the choice of path length. The sensitivity of the dependence is not great, however. The figure for the path length as determined by this laboratory is 125 ± 8 g/cm². Values from nuclear plate data[8] are somewhat higher (about 135 g/cm²) but refer to particles in much broader momentum bands. Within the region $L=100-140$ g/cm², n varies from 6.0 to 8.5. Within the region $L=125\pm8$ g/cm² and within the statistical accuracy of the experimental value of ρ, n has the value 6.5 ± 0.7.

TABLE I. Sharpness of assumed scattering distribution *vs* zenith angle dependence.

n	ρ
5	2.66
6	2.93
7	3.51
Inf.	9.70

The case of no scattering at all (n infinite) gives a value of ρ of 9.7, while the introduction of a distribution as sharp as $\cos^{6.5}\omega$ brings this ratio down by a factor of three. One might wonder whether attenuation alone might account for the ratio $\rho=3.2$. A short calculation shows that L would have to exceed 300 g/cm² of air, indicating that the effect of scattering cannot be neglected.

An approximation to the average projected angle of scatter may be had by a simple analysis. Protons in the atmosphere undergo nuclear collisions with a path length of about 60 g/cm²,[1] corresponding to the geometrical cross section. The distance from 3.3 km to the top of the atmosphere is then about 10 such path lengths. We can thus assume that observed protons are, on the average, in the tenth generation in the sense that the proton is separated by a total of ten collisions from its primary parent. The root-mean-square total angular deviation of a proton undergoing N collisions with an average scattering angle α, regarding this as a simple diffusion process, is given by

$$\bar{A}=\langle A^2\rangle^{\frac{1}{2}}=\alpha\sqrt{N}.$$

$\bar{A}$ for the $\cos^{6.5}\omega$ distribution is about 20°. With $N=10$, one finds $\alpha=6.3°$. This number is far removed from the value 15–20 degrees predicted by Messel in early theoretical treatment of the cascade.

Branch[4] has determined the angular distribution about the shower axis of penetrating particles in extensive air showers. Such showers, of course, include, in addition to nucleons, many pi and mu mesons. A conclusion therefrom, relevant to this problem, is nevertheless possible, since Branch finds a root-mean-square angular deviation of emergent particles of about three degrees. This figure agrees fairly well with the figure 6.3 degrees presented here. Branch's result, which this experiment independently corroborates, has caused drastic changes in the nature of the cross section assumed in the theoretical papers by Green, Messel, and Chartres,[13,14] with the adoption of a distribution sharply peaked in the forward and backward directions.

Other published data concerned with the problem treated here is the work of Walker.[15] He performed a counter-absorber type of experiment at 3.26-km elevation and at sea level in which recorded events are those due to charged particles producing penetrating showers in blocks of lead absorber of varying thicknesses. After elaborate precautions to remove the soft and mesonic components, he is able to call such particles protons, but the method of selection only vaguely defined the accepted momentum. Walker's requirements for the lowest energy case are that a charged particle traverse two trays of counters separated by 12 inches, enter an eight-inch block of lead and produce a shower there, whose presence shall be indicated by the discharge of three or more counters in a tray located below the absorber. Knowledge of which counters in the upper two trays were discharged yields the zenith angle dependence to 25° in six steps. The only limit that can be placed on the extent of the accepted momentum band is a lower one, which, because of the requirement for shower production, is certainly very much higher than the 1-Bev/c maximum cutoff for this experiment. This fact is further substantiated by Walker's ratio of the intensity at elevation to that at sea level. This ratio is about four, which is far removed from the well-established ratio of about twenty as obtained by this laboratory, by Wilson[12] and by Whittemore and Shutt[16] for low momentum (less than 1 Bev/c) protons.

Walker finds that his data for 3.26 km can best be represented by a $\cos^n\theta$ distribution with n equal to 5.5 ± 0.7, while at sea level, the exponent is 3.4 ± 1.0.

The very high-energy radiation considered by Walker is certainly isotropic at the top of the atmosphere. Only absorption can lead to a sharper zenith angle dependence with increasing depth. Walker's zenith angle dependence at 3.26 km can be accounted for by an absorption path length of 130 g/cm^2 in the absence of scattering. With scattering, an even shorter path length is required. This is inconsistent with Walker's value of 4 for the intensity ratio between 3.26 km and sea level. Continued absorption with this path length (130 g/cm^2) without scattering would give a zenith angle dependence of $\cos^{8.5}\theta$ at sea level. The experimental value quoted by Walker ($n=3.4\pm1.0$) would indicate sharply increased scattering below 3.26 km which seems unlikely at these energies.

This experiment was performed at a site provided by the Climax Molybdenum Company at Fremont Pass in Colorado. The authors wish to acknowledge the generous cooperation of C. J. Abrams, General Manager, and other personnel of the company. Don Eng and Elmer Wright of our group are also responsible for many contributions to this research.

[13] Green, Messel, and Chartres, Phys. Rev. **88**, 1277 (1952).
[14] H. Messel and H. S. Green, Phys. Rev. **87**, 378, 738 (1952).
[15] W. D. Walker, Phys. Rev. **77**, 686 (1950).
[16] W. L. Whittemore and R. P. Shutt, Phys. Rev. **86**, 940 (1952).