except that it is less steep within one meter or so from the axis. The energy spectrum of the electron and photon component is "softer" than that of a pure electronic cascade, that is, there are fewer high-energy rays than one would expect.

However, a one step multiple-production model of the shower development does not seem adequate since the observed angular distribution of the showers indicates too great a penetration into the atmosphere. A simplified nucleonic cascade model is only partially successful in reconciling these data. At least one required modification is that an appreciable fraction of the energy in a high-energy collision is transferred to nucleons so that the penetration of the cascade is not limited by the decay of pi mesons. This may occur because of a lack of complete inelasticity or because of increasing importance of nucleon-pair formation at high energies.

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The Mass Difference of Neutral and Negative π Mesons*

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The angular correlation of the decay γ rays from neutral π mesons produced in the reaction $\pi^- + p \rightarrow n + \pi^0$ has been measured, using counter techniques. Analysis of the observed correlation function yields a value of the $\pi^- - \pi^0$ mass difference $m_{\pi^-} - m_{\pi^0} = 8.8 \pm 0.6 m_e$.

I. INTRODUCTION

THE $\pi^- - \pi^0$ mass difference was first measured by Panofsky, Aamodt, and Hadley[1] in a study of the reactions of π^- mesons stopped in hydrogen. In this experiment hydrogen gas, inside the cyclotron chamber and under 3000-lb/in.[2] pressure, was exposed to the secondary radiation from a target bombarded by 330-Mev protons. Measurement of the energy spectrum of the γ rays produced in the hydrogen disclosed two groups, one of them rather broad near 70 Mev. This was presumed due to the reaction $\pi^- + p \rightarrow n + \pi^0$, $\pi^0 \rightarrow 2\gamma$. The width of the distribution is a measure of the velocity of the neutral meson. Assuming the π^- at rest, the mass difference was then calculated, $\delta = m_{\pi^-} - m_{\pi^0} = 10.6 \pm 2.0 m_e$.

Sachs and Steinberger[2] have verified the assumption that the low-energy group is due to the decay of neutral mesons produced by stopped negative mesons. Hydrogen was bombarded by a monochromatic external meson beam and the two γ rays from the neutral meson decay observed in coincidence.

In a similar experiment, but using the liquid hydrogen target previously described,[3] we have investigated the angular correlation of the two γ rays. This provides a measure of the π^0 velocity, as did the energy distribution of Panofsky *et al.*,[1] and therefore a new measurement of the $\pi^- - \pi^0$ mass difference.

II. KINEMATICS

A. Reaction Kinematics

Mesons coming to rest in liquid hydrogen become bound in the lowest Bohr orbit of a meson-proton atomic system in a time short compared to the mean life of $\pi - \mu$ decay.[4,5] The velocities in the bound state are negligible compared to the sensitivity of this experiment. The meson and proton are therefore considered to interact at rest. The process $\pi^- + p \rightarrow n + \pi^0$ gives a neutron and neutral meson, the sum of whose kinetic energies is then equal to the $\pi^- - \pi^0$ mass difference, minus the neutron-proton mass difference. From momentum and energy conservation,

$$\delta = m_{\pi^-} - m_{\pi^0} = [(m_{\pi^-} + m_p)^2 \beta^2 / (1-\beta^2) + m_n^2]^{\frac{1}{2}} - m_p/(1-\beta^2)^{\frac{1}{2}} - m_{\pi^-}[1/(1-\beta^2)^{\frac{1}{2}} - 1],$$

where βc is the velocity of the neutral meson. For small β,

$$\delta = m_n - m_p + \tfrac{1}{2}\beta^2 [m_{\pi^-} + m_{\pi^-}{}^2 / m_n].$$

B. $\gamma - \gamma$ Correlation

In the π^0 rest system the two γ rays are emitted in opposite directions. Let ξ be the angle between the direction of motion of one of the γ rays in the rest system and the π^0 velocity in the laboratory system.

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[1] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

[2] A. Sachs and J. Steinberger, Phys. Rev. **82**, 973 (1951).

[3] Lindenfeld, Sachs, and Steinberger, Phys. Rev. **89**, 531 (1953).

[4] A. S. Wightman, Phys. Rev. **77**, 521 (1950).

[5] Sargent, Rinehart, and Lederman (private communication) have observed, in hydrogen gas at 18-atmos pressure, in a cloud chamber experiment, a ratio of $\pi - \mu$ decays to stoppings <15 percent. In liquid hydrogen the ratio is then $<\frac{1}{2}$ percent.

For the angle ϕ between the two γ rays in the laboratory system, we have

$$\sin(\phi/2)=(1-\beta^2)^{\frac{1}{2}}/[(1-\beta^2)\cos^2\xi+\sin^2\xi]^{\frac{1}{2}}.$$

Considering the isotropy of the γ-ray distribution in the rest system, the probability of observing a pair with correlation angle between ϕ and $\phi+d\phi$ is

$$P(\phi)d\phi=\frac{(1-\beta^2)\sin\phi d\phi}{(1-\cos\phi)^{\frac{3}{2}}\beta[(1-\cos\phi)/(1-\beta^2)-2]^{\frac{1}{2}}}.$$

This correlation function is shown in Fig. 1 for $\beta=0.2$. The smallest correlation angle at which γ-ray coincidences should be observed is

$$\phi_c=\cos^{-1}(2\beta^2-1).$$

For small velocities this cut-off angle is a linear function of β, with $\pi-\phi_c\cong 2\beta$. The error in β is therefore half the error ϵ_ϕ in the experimentally determined cut-off angle, and the error in the mass difference will be approximately $\frac{1}{2}m_{\pi^-}\beta\epsilon_\phi$.

III. EXPERIMENTAL PROCEDURE

The negative mesons produced at the internal target of the Columbia University cyclotron at Nevis are collimated in a channel of the 8-ft iron shielding wall and further sorted in momentum by a double-focusing magnet and the beam defining counters No. 1 and No. 2. See Fig. 2. Counter No. 1 is a liquid scintillator $4\frac{1}{2}$ in. in diameter by $\frac{5}{8}$ in. thick. Counter No. 2 is a stilbene crystal $2\frac{1}{4}$ in. horizontally by $2\frac{3}{4}$ in. vertically by $\frac{1}{8}$ in. in thickness. Between the two counters 5 g/cm^2 of carbon and 5 g/cm^2 of LiH are inserted to maximize the number of mesons stopped in the hydrogen. The liquid hydrogen container[3] is a vertical cylinder 3.1 in. in diameter and $4\frac{1}{2}$ in. high. However, only $3\frac{3}{4}$ in. vertically are exposed by the thin part (0.006-in. Al foil) of the vacuum chamber. The γ-ray detectors, counters No. 3 and No. 4, are liquid scintillators 4 in. horizontally by 8 in. vertically by 1 in. in thickness. The centers of the γ-ray detection counters are each $27\frac{1}{2}$ in. from the center of the hydrogen well and move in the horizontal plane through the target center.

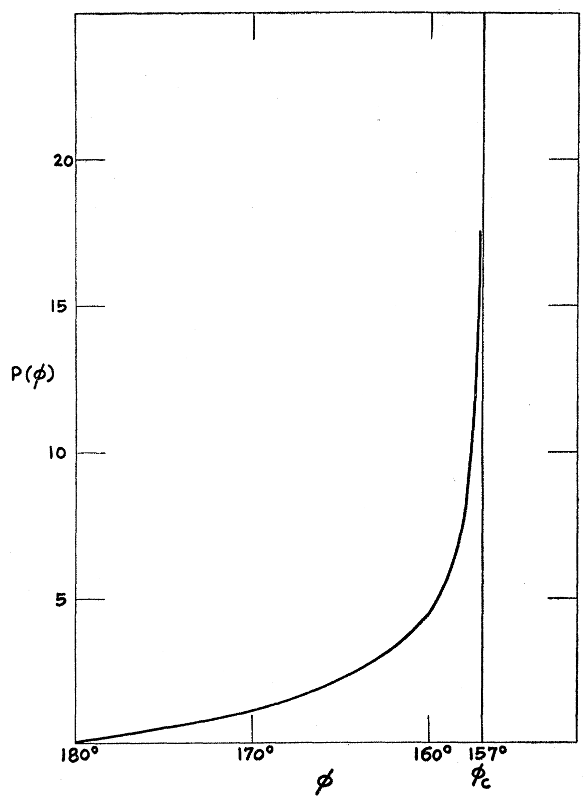

FIG. 1. γ-γ angular correlation function $P(\phi)$ *vs* ϕ for $\beta=0.2$.

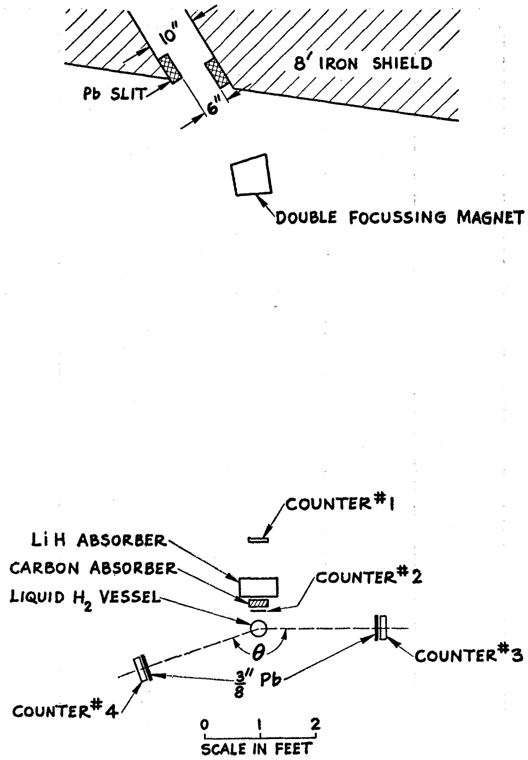

FIG. 2. Experimental geometry, top view.

To obtain the $\gamma-\gamma$ coincidence rates we measure the four counting rates:

(a) Pb converter in place, H_2 in well;
(b) Pb converter in place, no H_2 in well;
(c) Pb converter removed, H_2 in well;
(d) Pb converter removed, no H_2 in well.

The coincidence rate is then taken to be $(a-b)-(c-d)$. The rates without converter were however so small that they were usually omitted.

IV. EXPERIMENTAL RESULTS

Preliminary measurements were made with counters No. 3 and No. 4 at 10 in. from the hydrogen well to

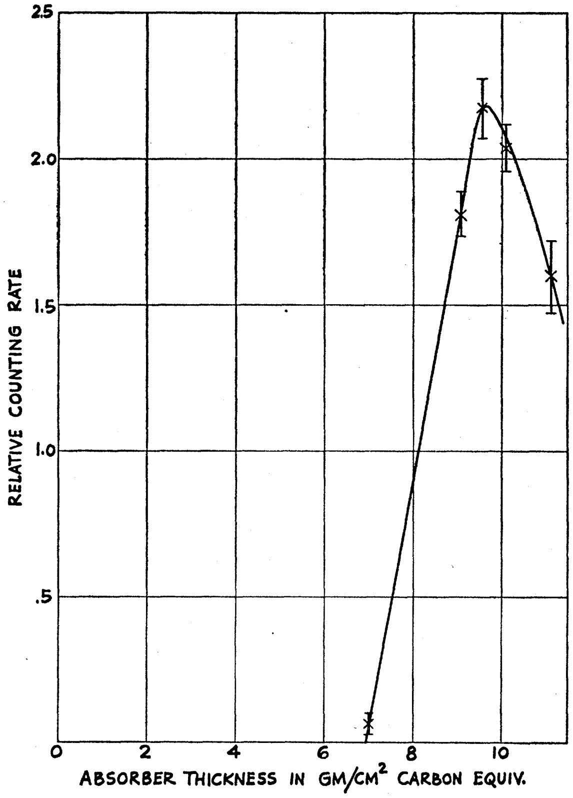

FIG. 3. Fourfold coincidence counting rates *vs* absorber thickness.

determine the most advantageous absorber thickness between counters No. 1 and No. 2. The results are shown in Fig. 3. The measurements were not extended to larger and smaller absorber thicknesses since it has already been shown that such coincidences are due to stopped mesons.[2]

Table I shows fourfold coincidences as a function of the angle θ between the γ-ray detectors, each counter at a fixed distance of $27\frac{1}{2}$ in. from the hydrogen cup. Incident beam intensity in the 1, 2 telescope was of the order of 2×10^5 counts per minute. Fourfold rates with hydrogen in the cup varied from about 2 counts/min at 165° to 0.1 count/min at 145°. Background rates varied from approximately 1 percent of hydrogen events to 25 percent at small angles. The data are presented in Fig. 3 in graphical form together with computed curves to be discussed below.

TABLE I. Experimental results. Variation of fourfold coincidence rates with θ, the angle between γ-ray detectors. Counts are per 2.048×10^6 monitor counts.

	Number counts hydrogen in cup	Number counts without hydrogen	Net due to hydrogen
180°	22.2±1.5	1.3±0.5	20.9±1.6
170°	21.6±1.5	1.3±0.7	20.3±1.7
165°	24.6±1.9	1.0±1.0	23.6±2.1
160°	22.4±1.4	0.9±0.9	21.3±1.7
155°	12.8±1.2	0.7±0.5	12.1±1.3
150°	6.5±0.8	0.8±0.6	5.7±1.0
145°	2.3±0.5	1.4±0.5	0.9±0.7

V. RESOLUTION OF APPARATUS

Quantitative evaluation of the cut-off angle ϕ_c requires a knowledge of the resolution of the detection system. To this end, consider counter No. 3 as fixed and the H_2 cylinder collapsed into a plane parallel to that of counter No. 3. Let $N(\phi)\sin\phi d\phi$ be the rate, per unit volume of well, of emission of γ rays with correlation angle between ϕ and $\phi+d\phi$, and per unit solid angle of one γ ray. The corresponding rate on the collapsed surface is then

$$n(x,y,\phi)dxdyd\phi=N(\phi)\sin\phi d\phi(r^2-x^2)^{\frac{1}{2}}dxdy.$$

Here r is the well radius, x and y, respectively, the horizontal and vertical displacements from the well center. Let x', y' be the coordinates of a point on counter No. 3, and let a pair of γ rays, γ_1 and γ_2, be emitted in $dxdy$, γ_1 striking counter No. 3 in $dx'dy'$. The rate for this process is $N(\phi)\sin\phi d\phi(r^2-x^2)^{\frac{1}{2}}dxdydx'dy'/R^2$. R is the radius of the right circular cylinder C, with axis coincident with the well axis, on which counters No. 3 and No. 4 move.

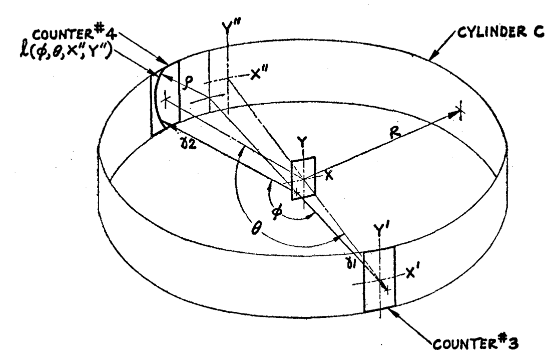

FIG. 4. Details of geometry used for calculation of resolution of apparatus.

Figure 4 shows the geometrical details. The trajectories of γ_2 at the angle ϕ with respect to γ_1 generate a cone of half-angle $\psi=\pi-\phi$ and axis along γ_1 projected back. Let x'', y'' be the point of intersection of γ_1 on C opposite counter No. 3. Then for small angles ψ the intersection of the cone and counter No. 4 may be approximated by the arc of a circle of radius $\rho=R\psi$ with center at x'', y''. We note that $x'=2x-x''$, $y'=2y-y''$. Further, let $l(\phi,\theta,x'',y'')$ be the length of arc of the intersection of the γ_2 cone with counter No. 4, where θ is the angle between counters No. 3 and No. 4. The coincidence counting rate is

$$C(\theta)=\int N(\phi)d\phi\times L(\theta,\phi),$$

where

$$L(\theta,\phi)=\int\int\frac{dx''dy''}{R^2}\frac{l(\theta,\phi,x'',y'')}{2\pi R}\int\int dxdy(r^2-x^2)^{\frac{1}{2}}$$

is the resolution function of the apparatus. The x, y integral is elementary. The x'', y'' integration was done numerically for a number of values of ϕ and θ by constructing regions in x'', y'' space with equal probability of being bombarded by γ_1 (given by the x, y integral$\times[\Delta x''\Delta y'']$) and summing the arc lengths l on a map measure fixed to a compass, the centers of the circles being placed in the various regions of x'', y'' space. The results, giving the resolution function $L(\theta,\phi)$ for several values of θ are shown in Fig. 5.

VI. THE $\pi^- - \pi^0$ MASS DIFFERENCE

The curves of counting rate *vs* θ shown in Fig. 6 were computed by performing, numerically, the integral

$$C(\theta)=\int_{\phi_1(\theta)}^{\phi_2(\theta)}L(\theta,\phi)\frac{(1-\beta^2)d\phi}{(1-\cos\phi)^{\frac{3}{2}}\beta[(1-\cos\phi)(1-\beta^2)^{-1}-2]^{\frac{1}{2}}}$$

for different values of β^2. All curves were normalized to the same area as that of the experimentally determined curve. The experimental resolution is seen to be somewhat poorer than the calculated resolution. This may be at least in part due to scattering of the conversion electrons in the edges of the lead foil. The best fit is obtained with $\beta^2=0.040$, and we estimate the limits on β^2 to be 0.0365 and 0.0435. The $\pi^- - \pi^0$ mass difference is then

$$m_{\pi^-}-m_{\pi^0}=8.8\pm0.6m_e.$$

This is in good agreement with the earlier result of Panofsky, Aamodt, and Hadley, $m_{\pi^-}-m_{\pi^0}=10.6\pm2m_e$.

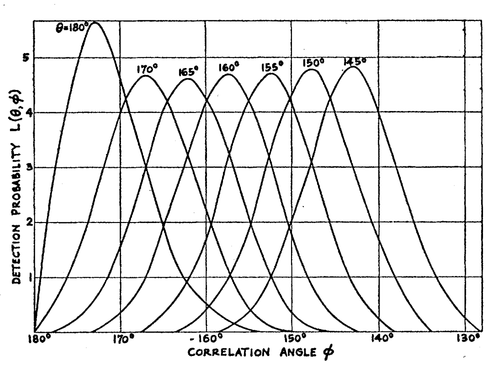

FIG. 5. Resolution function $L(\theta,\phi)$. Values of θ are indicated on the curves.

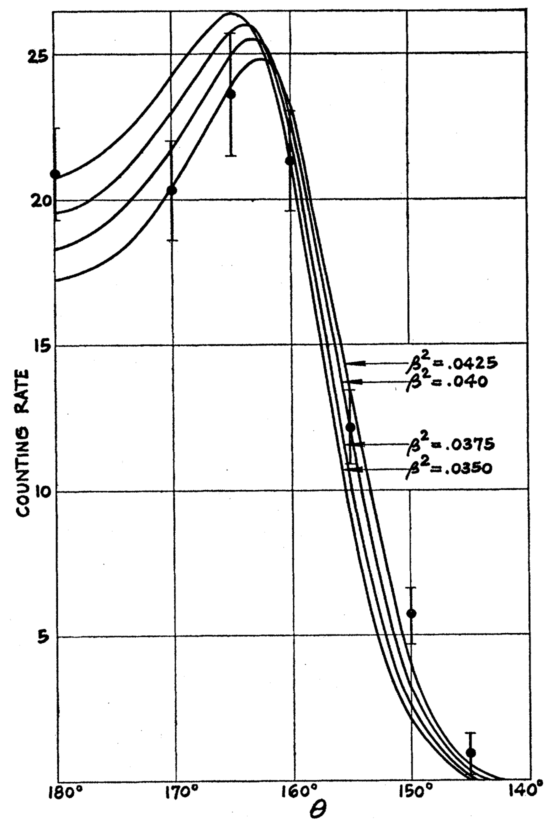

FIG. 6. Calculated and observed coincidence counting rates as a function of θ, the angle between γ-ray detectors. Errors shown are statistical.