The Scattering of 45-Mev Positive Pions by Hydrogen*

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Standard Ilford G.5 plates were exposed in an improvised 46-Mev external pion beam of the Chicago cyclotron. Scatterings of the pions by the hydrogen in the emulsion were observed by scanning for recoil protons and examining the proton beginnings for an incoming and scattered pion. Energy-momentum conservation permits separation of similar looking nonhydrogen events from the pion-proton scatterings. The average pion energy in the plates was determined to be 45±2 Mev from 15 events where the recoil proton stops in the emulsion. A total cross section of (12±3) millibarns has been obtained from 37 pion-proton scatterings. No scatterings were observed less than 79°. Only four are less than 90°. A phase-shift analysis of the data gives $\alpha_3=-(5.7\pm1.2)°$, $\alpha_{33}=(4.4\pm1.1)°$, and $\alpha_{31}=(2.4\pm1.8)°$. Since these particular phase shifts happen to give very little Coulomb interference, these data do not determine the absolute signs of the phase shifts. Upon comparison of these results with scattering data at other energies, α_3 appears to vary as the first power of the pion momentum and α_{33} with the third power at pion energies less than 80 Mev.

I. INTRODUCTION

FOR pion energies less than 80 Mev, $\lambda\!\!\!^{-}$ is greater than the range of nuclear forces ($r_0=1.4\times10^{-13}$ cm). Pion-proton scatterings in the low-energy region should be simpler to interpret than the higher energy scattering results. For example, in the energy region where $\lambda\!\!\!^{-}\gg r_0$, quantum mechanics gives for the phase shift of l-wave scattering:

$$\delta_l\propto\eta^{2l+1}, \tag{1}$$

where η is the pion momentum in units of $M_\pi c$. So far (π^+-p) angular distributions below 80 Mev have been obtained at 53,[1] 78,[2] 58,[3] and 40 Mev.[4]

The earlier scattering results[1,5] indicated that the s-wave phase shift might not behave according to Eq. (1) in the 50-Mev region. The early results of Anderson *et al.*[5] at 110 and 135 Mev gave an increase in α_3 (the s-wave phase shift) much more rapid than the first power of η, and the 53-Mev results of Fowler *et al.*[1] gave a symmetric angular distribution indicating zero s-phase shift at this energy. An explanation for such an anomalous s-wave behavior was given by Marshak[6] by postulating an attractive s-wave interaction with a strong, short-range repulsive core. This would force α_3 to change sign at some intermediate energy. However, the present experiment along with the more recent 40-Mev[4] and 58-Mev[3] results gives strong evidence that α_3 is proportional to the first power of the pion momentum at these energies. If α_3 had changed sign, there would be predominant scattering in the forward direction. In this experiment 33 scatterings were observed greater than 90° and 4 less than 90°. This is an even stronger backward scattering than observed at the higher energies.

These 37 events were fitted by an extended maximum likelihood method described in Sec. IV to give $\alpha_3=-(0.100\pm0.023)$, $\alpha_{33}=(0.077\pm0.020)$, and $\alpha_{31}=(0.042\pm0.035)$. α_{33} is the phase shift for scattering into the $P_{\frac{3}{2}}$ state and α_{31} for scattering into the $P_{\frac{1}{2}}$ state. Because $\eta<1$, states of higher angular momentum are neglected.

The resulting phase shifts happen to give a small nuclear scattering in the region 0° to 60° and thus a Coulomb interference perhaps even smaller than the pure Rutherford scattering. This makes determination of the signs of the phase shifts unfeasible for this energy region.

Details of exposure and beam analysis are given in Sec. II. As described in Sec. III the plates were area scanned under 830× magnification for single proton beginnings. Then each proton beginning was examined for an incoming and scattered pion. Three-pronged events of this type, but not due to hydrogen, were rejected on the basis of two tests: coplanarity of the three prongs, and angular correlation of the two scattering angles to satisfy energy-momentum conservation. The resulting total cross section of 12±3 mb and the corresponding phase shifts are calculated in Sec. IV. The angular distribution is shown in Fig. 2.

II. EXPOSURE

The lowest-energy pions able to escape the field of the Chicago cyclotron are 29 Mev. The channel in the 12-foot iron shield closest to this escape orbit is the channel normally used for external scattered protons. This channel gives pions of 46 Mev according to an absorption curve obtained by using two scintillation counters preceding the absorber and two others following in quadruple and double coincidence. This technique of beam analysis is described in reference 2.

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[1] Fowler, Fowler, Shutt, Thorndyke, and Whittemore, Phys. Rev. **86**, 1053 (1952).
[2] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).
[3] Bodansky, Sachs, and Steinberger, Phys. Rev. **90**, 996 (1953).
[4] J. P. Perry and C. E. Angell, Phys. Rev. **91**, 1289 (1953).
[5] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952).
[6] R. Marshak, Phys. Rev. **88**, 1208 (1952).

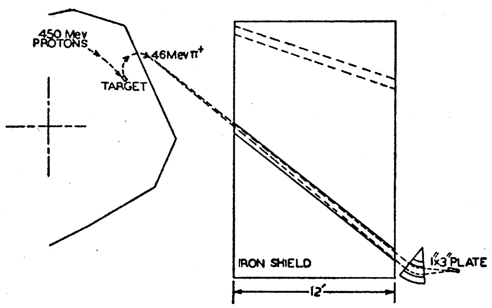

FIG. 1. Experimental setup for obtaining 46-Mev external pi-plus beam using proton channel of the Chicago cyclotron.

Of course a somewhat lower-energy pion beam of less path length could be obtained if a channel were cut as indicated by the dotted lines in Fig. 1. Analysis of the absorption curve gives a value of 81 ± 3 percent for the pion content of the beam used. This value was independently checked by counting grain density of 73 successive tracks over a length of 600μ. The tracks cluster into two groups of grain densities: 13 tracks about an average of 250 grains and 60 tracks at 330 grains. Muon decay electrons gave a value of 177 ± 8 grains. If the lighter tracks are assumed to be muons, this gives a value of 82 ± 5 percent for the pion content.

The beam was carefully collimated to reduce high-energy background protons. The low-energy protons of the same momentum as the pions were filtered out by the Bakelite wall of the plate holder. In an effort to achieve parallel pion tracks the plates were exposed in the vertical plane. In the plates used, 50 percent of the tracks were parallel to within $\pm2°$. The flux of tracks was 4.1×10^5 mesons/cm^2.

III. SCANNING

The scanning technique is exactly the same as that described by one of the authors.[7] At this energy discovery of events is easier than at 120 Mev since all recoil protons are quite heavy and the 45-Mev pions are about twice minimum ionization. Scanning efficiency is estimated to be greater than 90 percent on the basis of a double scanning. The first scanning was accomplished by scanner X whose efficiency was about 80 percent based on 30 of these (π^+-p) collisions. Scanner Z then completely rescanned this area. In no case did scanner Z fail to find an event which was found previously by scanner X. In addition certain test portions of the plate (about 20 percent of the scanned area) were scanned a third time by scanner Y and again all events were seen.

Events which occur near the surfaces of the emulsion, or are close to being straightforward or backward scatterings are somewhat more difficult to see, and are thus excluded from the cross-section calculations, with the appropriate geometrical corrections being made. The regions excluded are those within 15μ of the surfaces of the emulsion, and events where the scattering angle is less than 30° or greater than 160°.

The probability that a nonhydrogen event be counted as a pion-proton collision is very small in this case.[7]

IV. RESULTS

If L_π is the total pion path scanned, n_H the density of hydrogen in the plates, and $dN/d\omega$ the number of events per unit solid angle, then

$$\frac{d\sigma}{d\omega}=\frac{1}{L_\pi n_H}\frac{dN}{d\omega}.$$

The value $n_H=3.34\pm0.1\times10^{22}$ protons/cm^3 is specified by Ilford, where the uncertainty is that due to moisture content of the emulsion. The value $L_\pi=(1.045\pm0.040)\times10^5$ cm is based on flux determination using 1357 tracks. In Table I the region $30°<\chi<160°$ is divided into four regions of equal solid angle, $\Delta\omega=2.84$ steradians. χ is the scattering angle in the center-of-mass system. These four points of the angular distribution are shown in Fig. 2. The corresponding total cross section is 12 ± 3 mb. The error of ±3 mb is obtained by combining the statistical error of 36 events with the uncertainty in total pion track scanned and the uncertainty of scanning efficiency (assumed to be 0.95 ±0.05).

TABLE I. Number of events in four equal units of solid angle.

Limits of χ	30° to 66°	66° to 92°	92° to 119°	119° to 160°
No. of events	0	5	10	21
$(d\sigma/d\omega)\times10^{27}$	<0.13	0.48 ± 0.21	0.96 ± 0.30	2.00 ± 0.55

The following is a list of values of χ for all 37 events observed. Parentheses containing the energy (in Mev) of the incoming pion follow those events in which the recoil proton stops in the emulsion: 79.5° (47.0 Mev), 84, 84.5 (47.2), 87, 92, 97.5 (43.8), 102.5, 111, 113.5, 115(47.2), 115, 116 (49.0), 116, 117.5, 117.5, 121 (47.7), 124(48.6), 126.5, 128.5(43.3), 132, 133.5, 137, 140.5-(45.3), 142, 142.5(48.2), 143(47.8), 145, 145, 146, 148(46.5), 148, 150, 152(46.5), 158(48.1), 158, 160-(38.6), and 162.

A theoretical curve is fitted to these 36 data using a statistically efficient method which makes equal use of each scattering angle. The theoretical form for the differential cross section is given in Eq. (2) which is a close approximation including Coulomb interference when the phase shifts are small.[8]

$$f(\chi)=\lambdabar^2\left[\alpha_3+(2\alpha_{33}+\alpha_{31})\cos\chi-\frac{e^2}{2\hbar c\beta}\frac{1}{\sin^2\frac{1}{2}\chi}\right]^2+\lambdabar^2(\alpha_{33}-\alpha_{31})^2\sin^2\chi. \quad (2)$$

[7] Jay Orear, Phys. Rev. 92, 156 (1953).

[8] J. Ashkin and L. Smith, "Coulomb Interference Effects in the Scattering of Mesons by Protons," Technical Report No. 1, Carnegie Institute of Technology, Feb. 2, 1953 (unpublished).

The maximum-likelihood method was used to obtain the phase shifts and their errors. According to statistics, no other method of analysis can give the phase shifts with greater accuracy in the limit of large N.[9]

The relative probability that N events be found at N scattering angles χ_i is

$$P=\exp\left\{-n_{\mathrm{H}}L_\pi\int_{30^\circ}^{160^\circ} f(\chi)d\omega\right\}\prod_{i=1}^{N} f(\chi_i). \qquad (3)$$

The maximum-likelihood theorem states that the best values of the phase shifts are those which maximize P. The theorem also states that the standard deviation in the value of α so obtained approaches the minimum possible error:

$$[\langle(\Delta\alpha)^2\rangle_{\mathrm{Av}}]^{\frac{1}{2}}=\left[\int fd\omega \Big/ N\int f^{-1}(\partial f/\partial\alpha)^2 d\omega\right]^{\frac{1}{2}}. \qquad (4)$$

The error ellipsoid in the three-dimensional space of α_3,

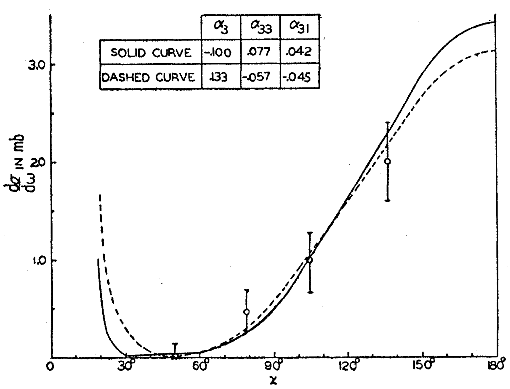

FIG. 2. $d\sigma/d\omega$ in mb/sterad corresponding to dividing the data into four equal units of solid angle. The solid curve is the best-fit theoretical form assuming repulsive s wave. The dashed curve is the best fit assuming attractive s wave.

α_{33}, and α_{31} was calculated using expressions of the form of Eq. (4). The extreme points of this ellipsoid in the α directions were taken as the errors in the α's. These calculations were made assuming both sets of signs for the phase shifts. For repulsive s-wave the calculations gave $\alpha_3=-0.100\pm0.023$, $\alpha_{33}=0.077\pm0.020$, and $\alpha_{31}=0.042\pm0.035$. The attractive s-wave calculations gave

[9] We are indebted to Mr. George Backus at the University of Chicago for proving that the maximum-likelihood theorem may be extended to a probability function of the form in Eq. (3).

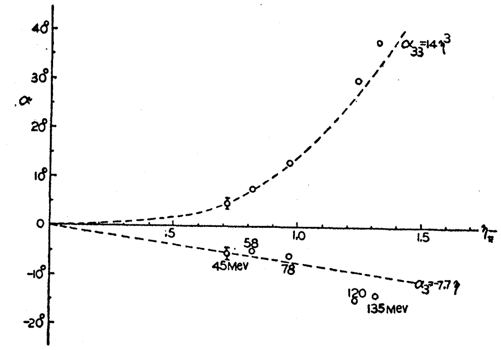

FIG. 3. α_3 and α_{33} *vs* η of the pion in the center-of-mass system. Present 45-Mev results are plotted along with those of 58 Mev (see reference 3), 78 Mev (see reference 2), 120 Mev (see reference 2), and 135 Mev (see reference 2).

$\alpha_3=0.133\pm0.021$, $\alpha_{33}=-0.057\pm0.020$, and $\alpha_{31}=-0.045\pm0.035$. The two curves are plotted in Fig. 2.

V. CONCLUSIONS

The quantity P in Eq. (3) is proportional to the probability that the experiment turn out as it did given $f(\chi)$. In this experiment the ratio of the two P values corresponding to the two sets of phase shifts is 1.4 in favor of the repulsive s-wave. Thus whether one assumes attractive or repulsive s-wave, the probability of the experiment turning out the way it did is about the same. This is because α_{33} is close to $-1/2\alpha_3$ and according to Eq. (2) there should be almost no interference term. For this reason it is extremely difficult to determine the signs of the phase shifts from Coulomb interference in this energy region. Determination of the signs of α should be possible using this plate technique at higher energies. Preliminary results on 120-Mev π^+ tend to favor repulsive s-wave.[10]

The values of α_3 and α_{33} are plotted *vs* η in Fig. 3 along with the values obtained by others. It appears that for data below 80 Mev α_3 can be fitted quite well by $\alpha_3=-7.7\eta$ and α_{33} by $\alpha_{33}=14\eta^3$ in degrees. This good fit helps verify both Eq. (1) and the results of others.[2–4]

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[10] Jay Orear, Bull. Am. Phys. Soc. **28**, No. 6, 14 (1953).