would be expected that particles with rigidities below the solar-imposed cutoff would be involved. There is no obvious reason why such particles would not be able to reach the earth as easily as particles of slightly higher rigidity. If sufficiently abundant, they would destroy the latitude cutoff at the earth. The other mechanisms mentioned at the beginning of Sec. II also meet with this difficulty. (2) Certain characteristics of the cosmic-ray effects associated with solar flares point to the fact that the new particles approach the earth preferentially along the earth-sun line.[25] On the basis of this observation, an argument has been made that the solar dipole moment cannot be larger than about 5×10^{32} gauss-cm^3.[26]

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VII. SUMMARY AND ACKNOWLEDGMENTS

The cosmic-ray evidence on the question of a possible solar dipole field can be briefly summarized as follows. The latitude cutoff at the earth, if interpreted in terms of a solar magnetic dipole, implies a dipole moment of 6.5×10^{33} gauss-cm^3; the apparent absence of a diurnal effect, although not yet conclusive, at least suggests an upper limit on the dipole moment which is somewhat smaller than the above value; the characteristics of the cosmic-ray intensity increases associated with solar flares imply a much smaller upper limit on the dipole moment.

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Solar Magnetic Moment and Diurnal Variation in Cosmic-Ray Intensity*

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The expected diurnal variation in cosmic-ray intensity at geomagnetic latitude 60° has been calculated assuming a solar magnetic dipole moment of 6.5×10^{33} gauss-cm^3. The calculation is based on new estimates of the intensity of cosmic radiation in the trapped orbits of the solar dipole field. The method of Dwight is followed, but with an important modification. The magnitude of the expected diurnal variation turns out to be about 12 percent.

I. INTRODUCTION

The existence of a "knee" in the cosmic-ray latitude effect at high altitudes implies, as is well known, a cutoff in the primary spectrum of cosmic radiation incident on the earth. Recent measurements have clearly established the location of the knee at $\lambda=58°$.[1,2] In terms of magnetic rigidity, the corresponding cutoff is 1.5 Bv. As first pointed out by Janossy, this cutoff can most easily be understood if one assumes the existence of a solar magnetic dipole, which would deflect away from the earth particles of rigidity below the cutoff. If the knee at 58° is explained in this way, the solar dipole moment must have the value 6.5×10^{33} gauss-cm^3.

Although no other detailed explanation of the cutoff has been put forward, doubt has been cast on the existence of a solar moment of this magnitude. Direct measutements, based on the observation of Zeeman-splitting of solar spectrum lines, have in recent years set an upper limit on the dipole moment which is one order of magnitude smaller than the above value;[3] and the diurnal effect in cosmic-ray intensity at intermediate latitudes on the earth, which would be expected on the basis of a solar dipole moment of the above magnitude, has not been found experimentally.[4–6]

The apparent experimental absence of the diurnal effect, however, has not generally been considered as conclusive evidence against a solar dipole moment of 6.5×10^{33} gauss-cm^3. The theoretically expected effect for this dipole moment, according to the calculations of Singer[7] and of Dawton and Elliot,[6] is 2–3 percent at $\lambda=56°$, whereas the extensive experimental measurements of Dawton and Elliot set an upper limit of 1.4 percent for the diurnal variation. Because of the frequent occurrence of large (5–10 percent), irregular intensity variations,[2,6,8] which might well mask a

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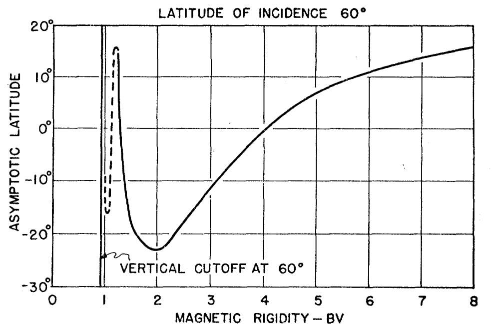

FIG. 1. Latitude of asymptotic velocity vector as a function of magnetic rigidity for particles which arrive vertically at the earth at magnetic latitude 60°.

small diurnal effect, the discrepancy between theory and experiment has not hitherto appeared to be serious. More recently, however, several important modifications have been introduced into the theory of the diurnal effect,[9] and it has been indicated that the magnitude of the effect at 56° should be more nearly 7–8 percent for the case of a solar dipole moment of 6.5×10^{33} gauss-cm^3. It seems unlikely that an effect of this size could have escaped detection.

In the present paper we present detailed results on the predicted diurnal effect at $\lambda=60°$ (see Fig. 3). At this latitude the effect is even larger than at 56°. The maximum intensity variation over a diurnal cycle is expected to be about 12 percent. A decision as to whether or not an effect of this magnitude exists can surely be made experimentally.

II. OUTLINE OF THE THEORY

The calculation of the diurnal effect due to a solar dipole field comes in two parts.

(1) The effect of the solar field is to establish a cone of directions ("allowed" cone) in which particles of given magnetic rigidity can arrive in the vicinity of the earth from infinity. The cosmic-ray intensity in the "forbidden" directions would be zero except for the fact that particles which approach the earth from allowed directions may be scattered by the earth's field into the forbidden directions. The intensity in the forbidden cone is thus determined by the competition between this scattering effect and the effect of absorption of the "trapped" particles in collisions with the sun, the earth, and other objects in the solar system. In calculations dealing with this process, the earth's field is treated as a point-like center which scatters particles from one to another of the static trajectories in the field of the solar dipole. The calculations lead to an estimate of the intensity in the forbidden directions, expressed as a function of magnetic rigidity and angle of inclination with respect to the earth's "directrix," a reference vector tangent to the earth's orbit.

(2) In the second part of the theory of the diurnal effect, one takes into account the details of the short-scale deflection in the field of the earth's dipole. The resulting pattern of forbidden cones at the earth has a fixed relation to the earth-sun line. The reduced intensity in the forbidden directions (relative to the full intensity in the allowed directions) gives rise to a diurnal effect at a fixed geographic location on the earth as the earth rotates on its axis.

A thorough discussion of part (1) of the theory was given by Kane, Shanley, and Wheeler.[10] They found that the intensity in the forbidden directions is appreciable in comparison with that in the allowed directions. The diurnal effect at the earth was therefore expected to be small. Recently, however, several corrections have been made in this part of the theory, and it has been found that the intensity in the forbidden directions is considerably smaller than indicated in the earlier results.[9]

An excellent discussion of part (2) of the theory is given by Dwight.[11] In his numerical work he proceeds from the results of Kane, Shanley, and Wheeler and assumes a cosmic-ray differential spectrum of the form $(cp/e)^{-2.75}$, where (cp/e) is the magnetic rigidity. The more recent calculations of Singer[7] and of Dawton and Elliot[6] are likewise based on the results of Kane, Shanley, and Wheeler; but a more realistic primary spectrum has been adopted and several improvements made in the approximations which are involved in the calculations.

In the present work we assume a primary spectrum of the form $(cp/e)^{-2.0}$, which is consistent with the results of a number of recent investigations,[12] and we proceed from the new results on the trapped-orbit

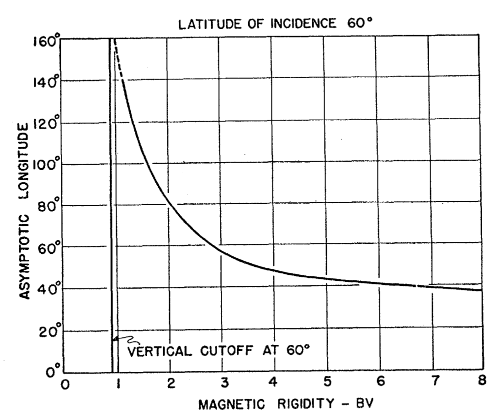

FIG. 2. Longitude of asymptotic velocity vector as a function of magnetic rigidity for particles which arrive vertically at the earth at magnetic latitude 60°.

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intensities.[9] We specifically assume a solar dipole moment of 6.5×10^{33} gauss-cm^3 and carry out the calculations for the diurnal effect of vertically-incident cosmic radiation at magnetic latitude 60°. It is assumed that the earth's dipole is perpendicular to the plane of the solar magnetic equator. The latitude 60° is believed to be near the optimum location for the diurnal effect because of a solar dipole moment of the above magnitude.

The procedure which is followed in the present work is similar to that which has been followed in previous calculations (in particular, see reference 11). We therefore omit all discussion of the details, except for several points wherein we have introduced modifications in the procedure.

The basic curves of the present calculations are shown in Figs. 1 and 2, which give respectively, the latitude and longitude of the "asymptotic velocity vector" as a function of magnetic rigidity for particles which arrive vertically at the earth at latitude 60°. (The definition of asymptotic velocity vector is given in reference 11.) These curves were constructed from data provided in the following sources: calculated trajectories given by Stoermer,[13] interpolation from the curves of Dwight,[11] and orbits obtained in the terrella experiments of Malmfors.[14]

The present calculations differ from earlier work in the following respects.

(1) As a matter of convenience, we imagine that the intensity in the allowed directions is zero and that the intensity in the forbidden directions is equal to the difference between the full intensity and actual intensity in these directions. In this way we calculate directly the *defect* in cosmic-ray intensity as a function of local time. This procedure somewhat simplifies the computations.

(2) In previous work, the dependence of trapped-particle intensity on the angle of inclination with respect to the earth's directrix has not been taken into account in detail. Instead, it has been the practice to assume that the trapped orbits are uniformly filled to an average intensity (averaged over the angle of inclination). We instead take into account this dependence on the angle of inclination as well as the dependence on magnetic rigidity (see Fig. 2 in reference 9). As it turns out, this refinement has a large effect on the final diurnal curve.

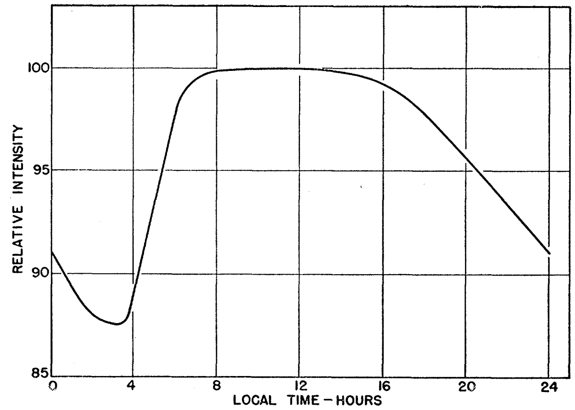

FIG. 3. Diurnal variation of vertically-incident cosmic-ray intensity at magnetic latitude 60°, a solar dipole moment of 6.5×10^{33} gauss-cm^3 being assumed.

III. SUMMARY AND ACKNOWLEDGMENTS

The final result of the calculations is shown in Fig. 3, where we plot the vertical cosmic-ray intensity at latitude 60° as a function of local solar time, on the assumption of a solar dipole moment of 6.5×10^{33} gauss-cm^3 and a primary differential spectrum of the form $(cp/e)^{-2.0}$. The magnitude of the variation in intensity over a diurnal cycle is about 12 percent, an effect which is considerably larger than previous calculations have indicated. This difference results in part from the recent improvements which have been made in the estimate of trapped-orbit intensities[9] and in part from the refinement which has been introduced in the present work and which has been discussed in the preceding paragraph.

We wish to express our thanks to Professor J. A. Simpson for his kind assistance and encouragement. One of us (SBT) wishes especially to thank Professor Simpson for the many courtesies extended to him during the course of an extended visit in the summer of 1953 at the Institute for Nuclear Studies, University of Chicago.

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