Cosmic Radiation in the Trapped Orbits of a Solar Magnetic Dipole Field*

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One of the well-known consequences which would follow from the existence of an appreciable solar magnetic dipole field is a diurnal variation in cosmic-ray intensity at intermediate latitudes on the earth. For the purpose of calculating the expected diurnal effect, it is first necessary to determine the extent to which the bounded orbits of the solar field are filled by the mechanism which has been discussed by Kane, Shanley, and Wheeler: namely, the scattering of cosmic-ray particles into bounded orbits as a result of magnetic deflection in the earth's field. A calculation of this effect is carried out here along the lines indicated by Kane, Shanley, and Wheeler but with several modifications. A solar dipole moment of 6.5×10^{33} gauss-cm^3, which is implied by the latitude cutoff at the earth, is adopted for the calculations. The cosmic-ray intensity in the trapped orbits is found to be appreciably smaller than indicated in the earlier calculations. Correspondingly, it is expected that the diurnal effect at the earth will be larger than the currently accepted theoretical values. The apparent experimental absence of the effect, although not conclusive, casts doubt on the existence of a solar dipole field as large as 6.5×10^{33} gauss-cm^3. The present work also provides an estimate of the average time which a cosmic-ray particle would spend in the trapping region. The value 5000 years, previously given, is revised downward by an order of magnitude.

I. INTRODUCTION

DIRECT measurements, based on the observation of Zeeman splitting of solar-spectrum lines, have lead to inconclusive results in the attempt to establish the existence and magnitude of a general solar magnetic field. The early work of Hale and co-workers[1] pointed to a general field of a dipole character with a value of $\sim$50 gauss at the pole. Although some of Hale's observers failed to detect any Zeeman splitting whatever, Cowling[2] and Blackett[3] have argued in favor of Hale's interpretation of his measurements in terms of a dipole field. Thiessen,[4] in 1945, also obtained evidence for a field of $\sim$50 guss at the pole, but in more recent measurements he has failed to find any field as large as 5 gauss.[5] In a set of measurements made between 1940 and 1947, Babcock[6] found fields between 6 and 60 gauss in 18 cases, whereas the remaining 24 cases gave no measureable fields or slightly negative values for the polar field. Measurements carried out in recent years give an upper limit of at most several gauss for the magnetic field.[7]

Further, a theoretical difficulty connected with direct measurements has been pointed out by Alfvén.[8] He argues that the presence of turbulent magnetic fields on the surface of the sun makes it impossible to reach any conclusions about a general solar field on the basis of Zeeman-effect measurements.

An independent approach to the question of a possible solar dipole field consists in studying the effects which such a field would have on the cosmic radiation. The most obvious effect would be to set a lower limit to the magnetic rigidity of particles which can arrive at the earth from infinity. The existence of a cutoff in the primary cosmic-ray spectrum at the earth—as deduced from the occurrence of a "knee" in the latitude effect at high altitudes—was first reported definitely by Cosyns[9] in 1936. Recent measurements have clearly established the location of the "knee" at 58°.[10,11] The corresponding cutoff rigidity in the cosmic-ray spectrum at the earth is 1.5 Bv; and if this is interpreted in terms of a solar dipole field, it leads to a dipole moment of 6.5×10^{33} gauss-cm^3.

No other reasonable interpretation of the cutoff has been put forward. However, the most conclusive test of the solar dipole hypothesis will be given when it becomes possible to determine the cutoff rigidity separately for primary particles of different charge.[10] If it should prove to be that the cutoff rigidity is independent of charge, the case for a solar dipole field would be greatly strengthened.

As a second consequence of a solar dipole field, one expects a diurnal effect in cosmic-ray intensity at the earth. Particles whose magnetic rigidity lies between the solar-imposed cutoff and value $(1+2^{\frac{1}{2}})^2$ times this limit can arrive in the vicinity of the earth only over a limited cone of directions. The further deflection of these particles in the short-scale field of the earth will give rise to a pattern of allowed directions at the earth which, however complicated, has a fixed relation to the earth-sun line. As the earth rotates on its axis, the cosmic-ray intensity will consequently vary at a fixed geographic location. If the intensity within the forbidden cones were indeed zero, the resulting diurnal

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variation on the earth would be very substantial. However, as was first pointed out by Alfvén,[12] particles which approach the earth from allowed directions may be scattered by deflection in the earth's field into the "forbidden" cone. These particles will then find themselves on bounded orbits in the field of the solar dipole and will be removed from the trapping region only by scattering at the earth into unbounded orbits which lead off to infinity or by collisions with bodies in the solar system. The cosmic-ray intensity in the trapping region is therefore determined by the competition between absorption and net scattering.

The details of this process have been very thoroughly discussed by Kane, Shanley, and Wheeler.[13] They found that the intensity in the forbidden directions is appreciable in comparison with that in the allowed directions. The diurnal effect at the earth was therefore expected to be small. It was calculated by Dwight[14] and more recently by Singer[15] and by Dawton and Elliot.[16] In all cases the calculations have been based on the results of KSW.

The calculated magnitude of the diurnal effect ($\sim$2–3 percent at 56° for a solar dipole moment of 6.5×10^{33} gauss-cm^3) was not thought to be inconsistent with the apparent experimental absence of the effect.[16–18]

The purpose of the present work is to consider anew the question of the filling of trapped orbits by the mechanism discussed above. Some of the results obtained by KSW require modification. In the present calculations it is found that the intensities in the trapped orbits are smaller than was indicated by KSW. As a result, the diurnal effect at the earth is expected to be larger than the currently accepted theoretical values, although the discrepancy with experiment is still not entirely conclusive. A second change in the results of KSW has to do with the average time which particles spend in the trapping region. The figure 5000 years, given by KSW as representing a typical value of the lifetime, is revised downward by an order of magnitude in the present calculations. A knowledge of this lifetime is important in connection with certain neglections which are made in the calculations (e.g., neglect of collisions with interplanetary dust) and is of some interest in connection with certain theories on the origin of cosmic rays.[19]

II. DISCUSSION OF THE TRAPPING PROCESS

In accordance with the model which we wish to consider, it is assumed that the dominant magnetic field within the solar system is due to a solar dipole whose moment is taken to be 6.5×10^{33} gauss-cm^3. On a scale of solar-system distances the earth's field is relatively important only over a small region of space. It is treated therefore as an essentially point-like center which scatters particles from one to another of the static trajectories in the field of the solar dipole; in particular, it will scatter particles from unbounded orbits into trapped orbits, and *vice versa*. At the same time, absorption of the trapped particles will occur in collisions with the sun, the earth, and other objects in the solar system.

The possibility that planets other than the earth (e.g., Mars and Venus) have appreciable magnetic moments and thus contribute to the filling of trapped orbits cannot be ruled out. We nevertheless neglect this possibility. We likewise neglect several other effects which might conceivably contribute to the filling of trapped orbits, e.g., direct production of cosmic radiation in the vicinity of the sun, albedo radiation leaving the earth and other bodies in the solar system and going directly into trapped orbits, scattering in the magnetic fields carried by ionized beams of particles from the sun, etc.

For a particle of charge e and momentum p, which moves in the field of the solar dipole M_s, there exists an integral of motion γ (Stoermer's angular momentum parameter) given by

$$-(r/R)\cos\lambda\cos x+(R/r)\cos^2\lambda=-2\gamma, \tag{1}$$

where R is the so-called characteristic radius:

$$R=(eM_s/cp)^{\frac{1}{2}}. \tag{2}$$

Here λ and r are, respectively, the latitude of the particle (relative to the magnetic equator) and its radial distance from the dipole; x is the angle between the velocity vector and the "directrix," a reference vector perpendicular to the meridian plane and pointing east (where we assume that the solar dipole points south). The quantity cp/e is called the magnetic rigidity. It will be assumed that the earth lies in the plane of the sun's magnetic equator so that the coordinates at the earth are $r=r_e$, $\lambda=0°$.

From Eq. (1), one finds that the "allowed" region in the meridian plane ("allowed" regions are characterized by the requirement $|\cos x|\leq 1$) splits up into two parts when $\gamma<-1$, the outer part extending to infinity, the inner part being insulated from infinity by forbidden regions. This leads to the result that particles of characteristic radius R, which come from infinity, can arrive in the vicinity of the earth at angle x only if

$$R/r_e\leq 1+(1+\cos x)^{\frac{1}{2}}. \tag{3}$$

Thus, a complete cutoff is imposed by the solar dipole when $R/r_e>1+2^{\frac{1}{2}}$, whereas all directions of arrival are allowed when $R/r_e<1$. Stated another way, particles of characteristic radius R can arrive from infinity only

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over the cone of angles between $x=0$ and $x=\bar{x}$, where

$$\cos\bar{x} = -2(R/r_e)+(R/r_e)^2. \quad (4)$$

Over this range of directions, particles arrive with full intensity. For $x>\bar{x}$, on the other hand, the solar field imposes a cutoff at the earth, and the intensity would drop to zero except for the fact that trapped orbits (orbits which lie within the inner allowed region) are to some extent filled by scattering at the earth, i.e., a particle approaching the earth along an unbounded orbit may suffer a deflection Δx such that its new value of γ is less than -1. In this case the particle finds itself in a trapped orbit, provided $R/r_e>1$. Conversely, trapped particles approaching the earth may be scattered into unbounded orbits and go off to infinity.

Trapped particles, whose rigidity and angular-momentum parameter lie within specified intervals, will wander about in more or less complicated orbits and in the course of time approach indefinitely close to every point in space consistent with the conservation equation (1). From Liouville's Theorem, it then follows that the intensity is constant throughout this volume of space. In general, the intensity I at any point in the trapping region will depend on R and x but not on azimuth about the directrix.

Consider the group of trapped particles whose characteristic radius and angular momentum parameter lie, respectively, in the specified intervals dR and $d\gamma$. The number of such particles per unit volume of space is given by

$$2\pi(I/v)d(\cos x)dR = 2\pi(I/v)(2R/r\cos\lambda)d\gamma dR, \quad (5)$$

where v is the particle velocity and where we have used the equation

$$d(\cos x) = (2R/r\cos\lambda)d\gamma, \quad (6)$$

which follows from Eq. (1). The total number N of trapped particles of the given class is now obtained by integrating this expression over the total volume of the trapping region. The result is

$$N = 4\pi(I/v)R^3 dR d\gamma(2\pi)g(\gamma), \quad (7)$$

where

$$g(\gamma) = R^{-2}\int_{\text{bounded area}} r dr d\lambda.$$

The integral g represents the area in a meridian plane of the trapping region in units of R^2. One finds for the area the expression

$$g(\gamma) = -2-2\gamma\int_0^{\pi/2}\left[(\gamma^2+\cos^3\lambda)^{\frac{1}{2}} - (\gamma^2-\cos^3\lambda)^{\frac{1}{2}}\right]d\lambda/\cos^2\lambda. \quad (8)$$

This expression differs from the one given by KSW, and the resulting correction in their estimate of the mean lifetime of trapped particles turns out to be quite large. The integral has been evaluated numerically for several values of γ; results are given in Table I.

TABLE I. Values of $g(\gamma)$-area of trapping region in meridian plane, in units of R^2.

$-\gamma$	1.0	1.1	1.2	1.414	$\gg 1$
$g(\gamma)$	0.223	0.106	0.0667	0.0313	$4/35\gamma^4$

III. ABSORPTION CROSS SECTIONS

Absorption of trapped particles, of specified R and γ, will in general occur in collisions with the sun, the earth, and the moon, and also in collisions with Mars and Venus whenever the latter are accessible to particles of the given class. For the bodies which are assumed to have no magnetic field, the absorption cross sections simply correspond to the geometrical cross sections. For example, when Mars is accessible to particles for which R and γ lie in the specified intervals dR and $d\gamma$, losses will occur at the rate

$$(\pi a_M^2)2\pi I d(\cos x)dR = (\pi a_M^2)4\pi I(R/r_M)d\gamma dR, \quad (9)$$

where a_M is the radius of Mars and r_M is the radius of its orbit. Thus Mars, when it can absorb earth-accessible particles, has the same effect as a fictitious planet located in the earth's orbit with a cross section

$$(r_e/r_M)\pi a_M^2 = 2.42\times 10^{17}\ \text{cm}^2. \quad (10)$$

In the same way, we define for Venus the effective cross section

$$(r_e/r_V)\pi a_V^2 = 17.25\times 10^{17}\ \text{cm}^2. \quad (11)$$

The cross section for the moon is simply

$$\pi a^2_{\text{Moon}} = 0.95\times 10^{17}\ \text{cm}^2. \quad (12)$$

The conditions under which Mars and Venus can be reached by earth-accessible particles are discussed by KSW. It turns out that Venus can bring its large cross section to bear only for a small fraction of the particles of interest to us here.

As for the earth, it cannot absorb with its full geometrical cross section particles of rigidity less than 60 Bev. Because of the earth's own dipole field, the allowed cone of arrival at a given latitude on the earth's surface fills a solid angle which depends on the rigidity and the latitude and which is in general less than 2π. The absorption cross section thus depends on the rigidity, and it can be represented by the expression

$$(\pi a_e^2)F, \quad (13)$$

where F is an *accessibility factor* which has been computed by KSW. It rises from the value zero (for zero rigidity) to the value one (for rigidity = 60 Bev).

This brief discussion of the absorption by the planets has been given for the sake of completeness. As it turns out—and this represents an important correction in the results of KSW—the main absorber of trapped particles is the sun. Its effective cross section depends on the magnetic rigidity, but it is always many times larger than the sum of the planetary cross sections. In subsequent calculations, therefore, we neglect the absorption of trapped particles by the planets.

The capture rate at the sun of particles for which γ and R lie within the specified intervals $d\gamma$ and dR is given by

$$\int \text{flux}\cdot\text{direction}\cos\cdot d(\text{surface}) = 2\pi a_s^2 I d\gamma dR \times \int_0^\pi \sin\phi d\phi \int_0^\pi \sin x d(\cos x) d(\sin\lambda)/d\gamma, \quad (14)$$

where a_s is the sun's radius and ϕ is the azimuthal angle about the directrix (direction $\cos = \sin x \sin\phi$). For the case of interest, here, where $a_s \ll R$ and $\gamma \lesssim -1$, one finds, from Eq. (1),

$$d(\sin\lambda)/d\gamma \simeq a_s/R. \quad (15)$$

The capture rate is therefore[19a]

$$4\pi(\pi a_s^2/2)(a_s/R) I d\gamma dR. \quad (16)$$

The equivalent cross section σ_s of a fictitious absorber located on the earth's orbit is obtained by equating the above expression with the expression

$$2\pi\sigma_s I d(\cos x) dR = 2\pi\sigma_s(2R/r_e) I d\gamma dR. \quad (17)$$

This leads to the result

$$\begin{aligned}\sigma_s &= (\pi a_s^2/2)(a_s/r_e)(r_e/R)^2 \\ &= 3.54\times 10^{19}\times (r_e/R)^2 \text{ cm}^2. \quad (18)\end{aligned}$$

IV. TIME OF CIRCULATION OF TRAPPED PARTICLES

Particles are lost from the trapping region both by absorption (mainly at the sun) and by scattering (at the earth) into unbounded orbits. Any discussion of the average time which a particle spends in the trapping region must take into account both of these effects. Indeed, for any single case the lifetime inside the trapping region can only be found by a detailed computation of the individual orbit. A simpler approach, which leads to an estimate of the average behavior for particles of a given class (specified R and γ), consists in the following.

Suppose that the trapped orbits are filled to their equilibrium intensity and that the scattering into the trapping region suddenly ceases (i.e., assume that the intensity of particles approaching the earth from infinity suddenly vanishes). A measure of the mean lifetime T is then given by the expression

$$T = -N/(dN/dt), \quad (19)$$

where N is the number of particles of the given class in the trapping region [Eq. (7)] and $-dN/dt$ is the rate of loss of these particles due to absorption and outscattering.

Assume for the moment that scattering at the earth is negligible, so that losses occur only by absorption. The rate of loss is given by

$$-dN/dt = 2\pi\sigma(2R/r_e) I d\gamma dR, \quad (20)$$

where σ is the sum of the absorption cross sections. From Eqs. (7), (19), and (20) we then find for the lifetime due to absorption only

$$T_{\text{abs}} = 2\pi(r_e/c)R^2 g(\gamma)/\sigma. \quad (21)$$

The absorption lifetime depends on the parameters R and γ; i.e., it depends on the magnetic rigidity and the angle x which the particles make with respect to the directrix at the earth [x is uniquely specified by R and γ accordinging to Eq. (1)]. Values of the absorption lifetime are given in Table II for several choices of x and R.

TABLE II. Absorption mean lifetime, in units of 100 years, for various values of the characteristic radius R and angle of inclination x at the earth's orbit.

$\cos x$ \ R/r_e	2.414	2.000	1.000
1.0	47.2	...	...
0.0	13.6	22.2	...
−1.0	6.62	5.24	1.39

When the effect of scattering is taken into account, the definition of lifetime according to Eq. (19) becomes ambiguous. In each passage near the earth, a given particle may be scattered into an unbounded orbit, or it may be scattered back into the trapping region. The out-scattering cross section depends on the parameters R and γ. If a particle is scattered back into the trapping region, its value of γ will in general be changed, so that in each successive passage near the earth the probability of outscattering is changed. Any attempt to follow the course of these successive scatterings would be equivalent to the prohibitive task of computing individual orbits. An additional difficulty is that the scattering cross section diverges for small angles of scatter. Thus, the out-scattering cross section diverges for those particles which require only a small deflection to go into unbounded orbits (these are the particles for which $\gamma \approx -1$). For such particles, the mean lifetime, as defined by Eq. (19), becomes very small; but the quasi-ergodic hypothesis, and the treatment of the earth's field as a point scattering center, are approximations which are then no longer valid.

Despite these difficulties, it is relatively easy to set a lower limit to the out-scattering cross section for par-

[19a] An extra factor of one-half has been inserted into expression (16) in order to take into account, in an approximate manner, the effect of the sun's shadow in reducing the opening of the allowed cone at the sun. This estimate is based on curves given in the following references: T. H. Johnson, Revs. Modern Phys. **10**, 193 (1938); R. A. Alpher, J. Geophys. Research **55**, 437 (1950). See also Fig. 5 of reference 13.

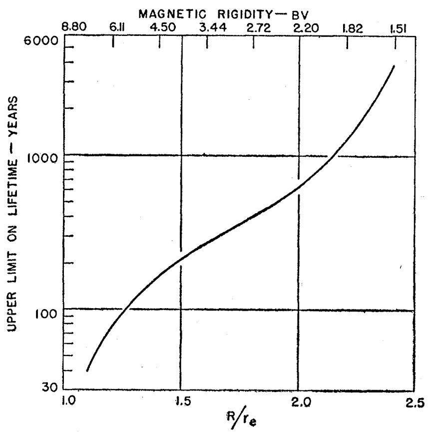

FIG. 1. Upper limit on lifetime in trapped orbits as a function of magnetic rigidity.

ticles of given magnetic rigidity and thereby obtain an *upper* limit on the lifetime as defined by Eq. (19). According to KSW, the differential cross section for scattering through an angle θ can be approximated by the expression

$$d\sigma_{sc}/d\Omega=(\pi/32)(eM_e/cp)[(\sin\tfrac{1}{2}\theta)^{-3}+2]-(1/4\pi)\pi a_e^2F, \quad (22)$$

where M_e is the earth's dipole moment and F is the accessibility factor defined in connection with Eq. (13). For the cases of interest to us, the second term is small compared to the first, and it will be neglected in subsequent calculations. Suppose now that a trapped particle approaches the earth at an angle x_0 with respect to the directrix and suppose that its direction of motion after the scattering is specified by the angles x and ϕ, where ϕ is the azimuthal angle about the directrix, relative to the azimuth before scattering. The angle of scattering θ can be expressed in terms of x_0, x, and ϕ. The total out-scattering cross section is then given by

$$\sigma_{sc}(x_0,R)=\int(d\sigma_{sc}/d\Omega)d\phi\,\sin x dx, \quad (23)$$

where the integration is carried out over all values of ϕ and over values of x between $x=0$ and $x=\bar{x}(R)$ [see Eq. (4)]. This is the range of x for which particles of characteristic radius R are unbounded.

Because of the term $(\sin\frac{1}{2}\theta)^{-3}$ which appears in Eq. (22), it is clear that σ_{sc} takes on its minimum value when $x_0=\pi$. Our procedure for setting an upper limit on the lifetime consists then in the following. Equation (21) is modified by including σ_{sc} in addition to σ (absorption) in the denominator. The scattering cross section depends on x_0 and R, or, alternatively, on γ and R. Since in fact γ varies in successive approaches to the earth, we obtain an upper limit on the lifetime, for any choice of R, by choosing that value of γ which maximizes T. The term $g(\gamma)$ in Eq. (21) favors small values of $|\gamma|$, whereas σ_{sc} favors large values ($|\gamma|$ has its largest value when $x_0=\pi$, which, as we have seen, minimizes σ_{sc}). Since the evlauation of σ_{sc} is simple only in the special case $x_0=\pi$, we obtain an upper limit on T, which is larger than could be obtained by the maximization procedure, by simply assigning to g its maximum possible value 0.223 and to σ_{sc} its minimum value (which corresponds to $x_0=\pi$).

For the case $x_0=\pi$ we have

$$\sin\tfrac{1}{2}\theta=[(1+\cos x)/2]^{\frac{1}{2}}. \quad (24)$$

From Eqs. (22) and (23)—neglecting the second term on the right-hand side of Eq. (22)—we then obtain the result

$$\sigma_{sc}(x_0=\pi;R)=4\pi(\pi/32)(eM_e/cp)\times[2\sqrt{2}(1+\cos\bar{x})^{-\frac{1}{2}}-(1+\cos\bar{x})], \quad (25)$$

where $\cos\bar{x}$ is related to R by Eq. (4).

The upper limit on the lifetime is plotted as a function of R/r_e in Fig. 1. It is seen to depend fairly strongly on the magnetic rigidity, but in all cases the upper limit on T is smaller than the figure 5000 years given by KSW as representing the order of magnitude of the lifetime for a typical case. Indeed, taking into account the fact that the present calculations provide only an upper limit, it seems more reasonable to take the lifetime for a typical case to be of the order of a few hundred years. Our neglect of losses due to collisions with interplanetary dust thus becomes even more easily justifiable than it was in the calculations of KSW.

V. EQUILIBRIUM INTENSITIES OF TRAPPED PARTICLES

Under equilibrium conditions, the intensities of trapped particles are determined by the competition between absorption and scattering. The appropriate mathematical technique for handling this problem has been given by KSW. In general, the intensity I depends on the magnetic rigidity and the angle of inclination x at the earth's orbit, but it is independent of the azimuthal angle about the earth's directrix. When $x<\bar{x}$ [see Eq. (4)], the intensity has the standard value I_0 associated with particles of the given rigidity at great distances from the solar system. For inclinations between $\bar{x}$ and π, on the other hand, the intensity has a lower value, which it is now our purpose to determine.

Consider the particles of a given magnetic rigidity. When the losses from a fixed solid angle element $d\Omega_1$, at $x=x_1$, balance the gains from all other solid angle elements $d\Omega_2$, then the intensity $I(x)$ at the earth's orbit satisfies the integral equation

$$\int_2 d\Omega_2 I(x_2)(d\sigma_{sc}/d\Omega)_{21}=I(x_1)\left[\sigma+\int_2 d\Omega_2(d\sigma_{sc}/d\Omega)_{12}\right], \quad (26)$$

for x_1 between $\bar{x}$ and π. Insofar as we shall consider absorption only by the sun, $\sigma=\sigma_s$ is independent of x, as can be seen from Eq. (18). The cross section for scattering from direction 2 to direction 1, $(d\sigma_{sc}/d\Omega)_{21}$, is of course equal to the cross section for scattering in the reverse direction.

The integral equation is transformed to a more convenient form by expanding I into a series of Legendre polynomials:

$$I(x)=\sum C_L P_L(\cos x). \tag{27}$$

From the law of addition of associated Legendre functions, KSW shows that the integral equation now reduces to the equation

$$\sum S_L C_L P_L(\cos x)+\sigma \sum C_L P_L(\cos x)=0 \tag{28}$$

for $x>\bar{x}$. The scattering coefficient S_L is defined by

$$S_L=2\pi\int_0^\pi [1-P_L(\cos\theta)](d\sigma_{sc}/d\Omega)\sin\theta d\theta. \tag{29}$$

Neglecting the second term on the right-hand side of Eq. (22), we find for the scattering coefficients

$$S_L=\begin{cases}0, & L=0\\ (2L+1)(\pi^2/4)(eM_e/cp), & L>0.\end{cases} \tag{29'}$$

Equation (28) is of course supplemented by the equation

$$\sum C_L P_L(\cos x)=I_0 \tag{30}$$

for $x<\bar{x}$.

Consider now the quantity

$$J=2\sum S_L C_L{}^2/(2L+1)+\sigma\int_{\bar{x}}^\pi I^2(x)\sin x dx. \tag{31}$$

Suppose that I is given the variation $\delta I(x)$ for $x>\bar{x}$, whereas $\delta I(x)=0$ for $x<\bar{x}$. This will produce the variation

$$\delta J=2\int_{\bar{x}}^\pi [\sum S_L C_L P_L(\cos x)+\sigma I(x)]\delta I(x)\sin x dx. \tag{32}$$

We have made use here of the equation

$$\delta C_L=\tfrac{1}{2}(2L+1)\int_{\bar{x}}^\pi P_L(\cos x)\delta I(x)\sin x dx. \tag{33}$$

By virtue of Eq. (28), the expression in brackets in Eq. (32) vanishes for the true solution $I(x)$, i.e., J takes on a stationary value for the true solution. Thus, by representing $I(x)$ as an empirical function with a certain number of adjustable parameters and adjusting the parameters to give J a stationary value, we obtain the best approximation to the exact solution which is attainable with a function of the given form.

An additional result which proves to be useful is the following. Multiplying through by $I(x)$ in Eq. (28), integrating over x from $\bar{x}$ to π, and making use of the equation

$$2C_L/(2L+1)=\int_{\bar{x}}^\pi IP_L\sin x dx+I_0\int_0^{\bar{x}} P_L\sin x dx, \tag{34}$$

we find that

$$\sigma I_0\int_x^\pi I\sin x dx =2\sum S_L C_L{}^2/(2L+1)+\sigma\int_{\bar{x}}^\pi I^2\sin x dx. \tag{35}$$

The right-hand side is just the quantity J, which takes on a stationary value.

For the trial function we adopt the expression, used by KSW,

$$I/I_0=\begin{cases}1-K_2(y-\bar{y})^2-K_3(y-\bar{y})^3, & y<\bar{y}\\ 1, & y>\bar{y},\end{cases} \tag{36}$$

where $\bar{y}=\cos\bar{x}$, $y=\cos x$. The magnetic rigidity determines the value of $\bar{y}$, in accordance with Eq. (4). Solutions were obtained for the four cases: $\bar{y}=0.9$, 0.5, 0.0, -0.5. The requirement that J be stationary with respect to variations in K_2 and K_3 leads to the best choice of these parameters for each case. For $\bar{y}=0.9$, the Legendre coefficients C_L higher than the third were negligible. With decreasing $\bar{y}$, however, it is necessary to include additional coefficients in evaluating J. Thus, for $\bar{y}=-0.5$ it was necessary to evaluate twelve terms to obtain sufficiently accurate results. The adequacy of the solutions was tested by means of Eq. (35), which holds only for the exact solution. In every case the two sides of Eq. (35) agreed to within three significant figures for the adjusted trial solution.

The calculated intensity given by Eq. (36) reaches a minimum value for some negative value of $\cos x$ and then rises again as $\cos x$ further decreases to -1. This latter behavior is anomalous, since the intensity is expected to decrease continuously as $\cos x$ decreases. The situation would presumably be improved by using more adjustable parameters in the trial solution. In any case, we arbitrarily cut off the rise in $I(x)$ beyond the minimum (dotted lines in Fig. 2). The results of the computations are shown in Fig. 2, where the intensity in the trapped orbits is plotted against $\cos x$ for four different magnetic rigidities. The value 6.5×10^{33} gauss-cm^3 was taken for the solar dipole moment. The best choices of the parameters K_2 and K_3 are given in Table III.

One additional point requires mentioning. It is obvious on physical grounds that, as $\cos\bar{x}$ approaches very

TABLE III. Values of the parameters K_2 and K_3.

$\cos\bar{x}$	K_2	K_3
0.9	0.239	0.109
0.5	0.328	0.186
0.0	0.712	0.601
-0.5	3.20	5.40

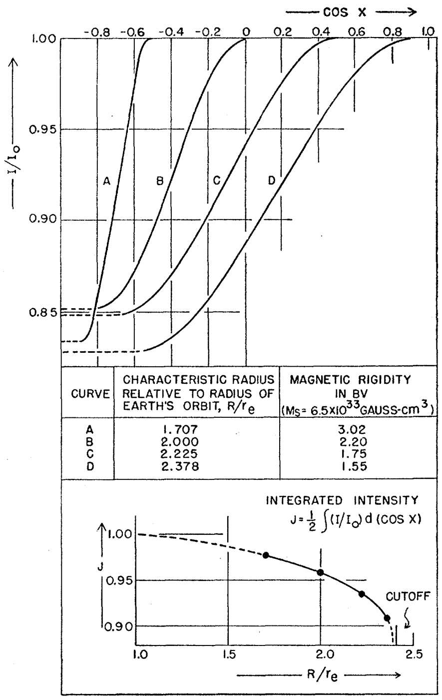

FIG. 2. Intensity of trapped particles at the earth's orbit, relative to the standard intensity at infinity, as a function of the angle of inclination with respect to the earth's directrix. The lower diagram gives the omnidirectional flux at the earth's orbit, relative to the standard value at infinity, as a function of magnetic rigidity.

close to unity (i.e., as the magnetic rigidity approaches the absolute cut-off value imposed by the solar dipole), the intensity in the trapped orbits must approach zero. This sharp drop in intensity has not yet occurred, however, for $\cos\bar{x}=0.9$.

VI. DISCUSSION

Kane, Shanley, and Wheeler have computed the intensities in trapped orbits for two choices of the solar dipole moment: $M_s=1.0\times10^{34}$ and 4.2×10^{33} gauss-cm³. One can interpolate from their curves to find the results appropriate to a dipole moment 6.5×10^{33} gauss-cm³. One finds that the intensities reported here are appreciably smaller. To this extent, calculations of the diurnal effect based on the results of KSW require modification. Dwight's work[14] was based on a cosmic-ray differential spectrum of the form $I_0\approx(cp/e)^{-2.75}$. The exponent in this spectrum, over the latitude-sensitive range of magnetic rigidities, is now believed to be closer to -2.1,[20,21] so that, as pointed out by Singer,[15] Dwight's results lead to an over-estimate of the diurnal effect. According to the calculations of Singer,[15] which were based on the curves of KSW, the maximum variation in intensity over a diurnal cycle is $\sim$2.5 percent for $\lambda=56°$ and $M_s=6.5\times10^{33}$ gauss-cm³. A similar estimate is given by Dawton and Elliot.[16] In the light of the present results, this figure must be revised upwards to perhaps 7 or 8 percent.

The recent measurements of Dawton and Elliot[16] (18 day-night balloon flights at $\lambda=57°$) have failed to confirm the existence of a regular diurnal effect larger than 1.4 percent, although irregular variations of the order of 6 percent are frequently found. Other observers have likewise failed to detect a diurnal effect of the kind we are discussing here.[17,18] However, because of the large, irregular intensity variations in the low-energy part of the primary spectrum,[11,16,22] one cannot conclude with complete certainty that the experimental results are inconsistent with the 7 or 8 percent diurnal variation which would be expected on the basis of a solar dipole moment of 6.5×10^{33} gauss-cm³.

Nevertheless, it should be noted that for this value of the dipole moment a larger diurnal effect is predicted for latitudes somewhat above 56–57°, where the detailed experiments have to date not been carried out. The calculations indicate that at $\lambda=60°$ the effect should be about 12 percent.[23] It should be possible to reach a definite experimental decision about an effect of this size.

In addition to the experimental difficulties connected with the large, irregular intensity variations, one might attempt to explain away the discrepancies between experiment and theory by supposing that the trapped orbits are filled to an even larger extent than reported here, by additional mechanisms which have not been considered in the present work. Some examples of possible such mechanisms have been mentioned briefly at the beginning of Sec. II.

To take one example, large increases in cosmic-ray intensity at the earth are occasionally observed at times of large solar flares.[24] The increases are most pronounced at the low-energy end of the cosmic-ray spectrum. It is clear from the magnitude of the effect and the close correlation with solar disturbances that cosmic radiation is being produced on or near the sun at these times. Many of the particles which are involved in this effect would certainly go into trapped orbits if the solar moment were as large as 6.5×10^{33} gauss-cm³.

These large increases in cosmic-ray intensity are of course infrequent (only four such events have been reported over a period of many years). However, it has recently been found that the smaller but more common solar flares are often accompanied by (small) cosmic-ray intensity increases.[25] It does not appear unreasonable then to suppose that production of low-energy cosmic radiation on or near the sun is sufficiently significant so that it would contribute appreciably to the filling of trapped orbits in a solar dipole field. This suggestion, however, meets with two difficulties. (1) It

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would be expected that particles with rigidities below the solar-imposed cutoff would be involved. There is no obvious reason why such particles would not be able to reach the earth as easily as particles of slightly higher rigidity. If sufficiently abundant, they would destroy the latitude cutoff at the earth. The other mechanisms mentioned at the beginning of Sec. II also meet with this difficulty. (2) Certain characteristics of the cosmic-ray effects associated with solar flares point to the fact that the new particles approach the earth preferentially along the earth-sun line.[25] On the basis of this observation, an argument has been made that the solar dipole moment cannot be larger than about 5×10^{32} gauss-cm^3.[26]

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VII. SUMMARY AND ACKNOWLEDGMENTS

The cosmic-ray evidence on the question of a possible solar dipole field can be briefly summarized as follows. The latitude cutoff at the earth, if interpreted in terms of a solar magnetic dipole, implies a dipole moment of 6.5×10^{33} gauss-cm^3; the apparent absence of a diurnal effect, although not yet conclusive, at least suggests an upper limit on the dipole moment which is somewhat smaller than the above value; the characteristics of the cosmic-ray intensity increases associated with solar flares imply a much smaller upper limit on the dipole moment.

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Solar Magnetic Moment and Diurnal Variation in Cosmic-Ray Intensity*

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The expected diurnal variation in cosmic-ray intensity at geomagnetic latitude 60° has been calculated assuming a solar magnetic dipole moment of 6.5×10^{33} gauss-cm^3. The calculation is based on new estimates of the intensity of cosmic radiation in the trapped orbits of the solar dipole field. The method of Dwight is followed, but with an important modification. The magnitude of the expected diurnal variation turns out to be about 12 percent.

I. INTRODUCTION

The existence of a "knee" in the cosmic-ray latitude effect at high altitudes implies, as is well known, a cutoff in the primary spectrum of cosmic radiation incident on the earth. Recent measurements have clearly established the location of the knee at $\lambda=58°$.[1,2] In terms of magnetic rigidity, the corresponding cutoff is 1.5 Bv. As first pointed out by Janossy, this cutoff can most easily be understood if one assumes the existence of a solar magnetic dipole, which would deflect away from the earth particles of rigidity below the cutoff. If the knee at 58° is explained in this way, the solar dipole moment must have the value 6.5×10^{33} gauss-cm^3.

Although no other detailed explanation of the cutoff has been put forward, doubt has been cast on the existence of a solar moment of this magnitude. Direct measutements, based on the observation of Zeeman-splitting of solar spectrum lines, have in recent years set an upper limit on the dipole moment which is one order of magnitude smaller than the above value;[3] and the diurnal effect in cosmic-ray intensity at intermediate latitudes on the earth, which would be expected on the basis of a solar dipole moment of the above magnitude, has not been found experimentally.[4–6]

The apparent experimental absence of the diurnal effect, however, has not generally been considered as conclusive evidence against a solar dipole moment of 6.5×10^{33} gauss-cm^3. The theoretically expected effect for this dipole moment, according to the calculations of Singer[7] and of Dawton and Elliot,[6] is 2–3 percent at $\lambda=56°$, whereas the extensive experimental measurements of Dawton and Elliot set an upper limit of 1.4 percent for the diurnal variation. Because of the frequent occurrence of large (5–10 percent), irregular intensity variations,[2,6,8] which might well mask a

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