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Isotopic Spin and Odd-Odd $N=Z$ Nuclei*

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The lowest-lying states of isotopic spin $T=0$ and $T=1$ are surveyed for odd-odd nuclei with $N=Z$ by considering β-decay schemes and Coulomb energy differences. For $A\geq 26$ the lowest $T=0$ and $T=1$ states appear to be very close together. The analysis permits certain predictions regarding the decay schemes of of these nuclei. Further experimental information is needed on the higher members of this series, from Sc^{42} to Cu^{58}. Predicted β-decay schemes for some corresponding $Z=N+2$ nuclei are also summarized.

I. INTRODUCTION

INTEREST in isotopic spin[1] as a relatively valid quantum number, at least for light nuclei, has been stimulated recently by the experimental evidence for associated selection rules in nuclear reactions.[2] The odd-odd $N=Z$ nuclei are especially favorable subjects for the study of isotopic spin, since they alone have more than one T value among their low-lying states. A striking affirmation of this fact has been the recent experimental discovery[3] of a 1.4-second ground state in Cl^{34}, which must be interpreted[4] as indicating that this nucleus has $T=1$ in its ground state. This is contrary to the usual expectation that the ground state has the minimum T value possible, in this case $T=0$.

It thus becomes an intriguing question to determine the relation between the lowest $T=0$ and $T=1$ states for all light odd-odd $N=Z$ nuclei and look for a systematic behavior. The present note attempts to survey the possibilities in this regard. The stable nuclei of this category—Li^6, B^{10}, N^{14},—have been well explored.[5]

We therefore consider mainly the unstable examples: F^{18}, Na^{22}, Al^{26}, P^{30}, Cl^{34}, and K^{38}. The uncertain cases V^{46}, Mn^{50}, Co^{54}, and Cu^{58} are also mentioned.

In Sec. II, likely assignments of spin and isotopic spin for odd-odd ground states are made on the basis of β-decay schemes and ft values; in Sec. III we study the energies of the lowest $T=0$ and $T=1$ states in odd-odd nuclei as function of mass number. In Sec. IV the previous considerations are employed to make tentative predictions regarding energies and decay schemes of some odd-odd excited states; these are extended in Sec. V to the β decay of the analogous even-even nuclei with $Z=N+2$.

II. BETA-DECAY SCHEMES AND ft VALUES

The odd-odd nuclei above N^{14} are all unstable, going by positron decay to an even-even nucleus with $T=1$, $J=0$ in its ground state and $T=1$, $J=2$ (generally) in its first excited state. If allowed beta-decay occurs to either of these states, we can make use of the selection rules involved to assign quantum numbers to the odd-odd ground state. There is no parity change in allowed transitions, and in addition the Fermi matrix element has selection rules $\Delta J=\Delta T=0$, the Gamow-Teller matrix element has selection rules $\Delta J=\Delta T=0, \pm 1$ (no $J=0\rightarrow 0$ and of course beta-decay is incompatible with $T=0\rightarrow 0$ in any case).

The introduction of one more rule about nuclear levels allows the assignment of unique T, J values to the odd-odd ground states. This rule is that for odd-odd $N=Z$ nuclei with neutrons and protons in equivalent orbits, the lowest-lying $T=0$ levels have odd J, while $T=1$ levels have even J. This rule is strictly true for j-j coupling states of lowest seniority; and even in L-S coupling there are many more low-lying states in accord with the rule than in violation of it. Some empirical evidence in support of a general principle of this sort follows from the $J=0-2-4$ and $0-2-2$ level sequences in many even-even nuclei with $T=0$.

The above rules may be summarized in the statement that

$$|\Delta J|=|\Delta T|=0 \text{ or } 1 \tag{1}$$

for allowed beta-transitions between odd-odd and even-even nuclei where neutrons and protons occupy equivalent orbits. The half-lives are given by[6]

$$\log ft=3.7-\log|M|^2, \tag{2}$$

where $|M|^2$ is a nuclear matrix element which includes statistical factors. We may argue that $|M|^2$ will generally be a maximum for $\Delta J=\Delta T=0$ transitions, because the two states will be exactly similar and have perfectly overlapping wave functions. For the other case, $|\Delta J|=|\Delta T|=1$ the overlap of nuclear wave

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[1] E. P. Wigner, Phys. Rev. **51**, 106 (1937); R. K. Adair, Phys. Rev. **87** 1041 (1952).

[2] We do not discuss the less restrictive condition of isotopic parity or charge symmetry of nuclear forces [N. M. Kroll and L. L. Foldy, Phys. Rev. **88**, 1177 (1952)], because charge independence or isotopic spin conservation seems to be at least approximately valid from the equivalence of n-p and p-p scattering at low energies and the equivalent energy levels in the $T=1$ triads at $A=6$, 10 and 14.

[3] W. Arber and P. Stähelin, Helv. Phys. Acta **26**, 433 (1953).

[4] D. C. Peaslee, *Nuovo cimento* **10**, 1349 (1953); O. Kofoed-Hansen and A. Winther (unpublished). The latter authors also suggest a reduction in the $T=1$ and $T=0$ spacing of odd-odd $N=Z$ nuclei as A increases. See also, O. Kofoed-Hansen, Phys. Rev. **92**, 1075 (1953) and P. Stähelin, Phys. Rev. **92**, 1076 (1953).

[5] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

[6] A. Winther, Physica **18**, 1079 (1952). In accordance with the evidence that the Fermi and Gamow-Teller coupling constants are equal, we write $|M|^2=|\int 1|^2+|\int \boldsymbol{\sigma}|^2$.

TABLE I. Ground states of odd-odd $N=Z$ nuclei.

Parent nucleus	Beta-decay final state	log ft	$\lvert M\rvert^2$	T	J, parity
F^{18}	0+	3.6	1.3	0	1+
Na^{22}	2+	7.4	$\ll 1$	0	3+
Al^{26}	0+	3.5[a]	1.5	1 (or 0)	0+ (or 1+)
P^{30}	0+	4.9	$\ll 1$	0	1+
Cl^{34}	0+	3.4	2	1	0+
K^{38}	2+	5.0	$\ll 1$	0	3+
Sc^{42}	...	...	...	...	...
V^{46}	(0+)	≥ 3.4	≤ 2	...	...
Mn^{50}	(0+)	≥ 3.4	≤ 2	...	...
Co^{54}	(0+)	≥ 3.5	≤ 1.6	...	...
Cu^{58}	(0+)	≥ 4.7	$\ll 1$	(0)	(1+)

[a] Using β^+ maximum energy of 3.3 Mev [L. Katz (private communication)].

functions may be poor and $|M|^2$ may be considerably reduced. Using these principles, we obtain Table I.

For all the nuclei in Table I a $\Delta T=0$ transition with perfect overlap has $|M|^2=|\int 1|^2=2(|\int \boldsymbol{\sigma}|^2=0)$, regardless of whether there is L-S or j-j coupling. This expectation is borne out by the measurement on Cl^{34} and the known $\Delta T=0$ transitions in C^{10} and O^{14}. It is the basis for the probable assignment of Al^{26}. For $\Delta T=1$ transitions, $|M|^2=|\int \boldsymbol{\sigma}|^2=2(|\int 1|^2=0)$ in these same nuclei only for perfect L-S coupling; the value of $|M|^2=1.3$ for F^{18} suggests that L-S coupling is predominant but not without some j-j admixture. We therefore take $|M|^2<2$ to mean $|\Delta J|=|\Delta T|=1$ for this transition; this same assignment is made to all cases with $|M|^2 \ll 1$. An unequivocal value of J comes from considering decay schemes.[7]

The case of Al^{26} is of considerable interest in that it probably resembles Cl^{34} in having a $T=1$ ground state; this "ground state" may be isomeric (see Sec. IV).

III. ENERGIES OF LOWEST $T=0$ AND $T=1$ STATES

It is also of interest to study the energies of the lowest $T=0$ and lowest $T=1$ states in odd-odd $N=Z$ nuclei as function of mass number. All energies are taken for neutral atoms relative to the ground state of the neighboring even-even $N=Z-2$ isobar. Energies of odd-odd ground states (and the lowest $T=0$ state in Cl^{34}) are taken from decay schemes.[7] Energies of lowest $T=1$ states in the lightest nuclei are obtained from a survey of the excited states.[8] The results of this investigation are shown in Fig. 1. The energy $E_{T=1}$ of the lowest $T=1$, $J=0$ state in an odd-odd nucleus, relative to the ground state of the neighboring $N=Z+2$ isobar, can also be estimated from the formula for neutral atoms

$$E_{T=1}=0.60(A-2)A^{-\frac{1}{3}}-0.78 \text{ Mev}. \tag{3}$$

Here the nucleus is assumed to be a uniformly charged sphere of radius $R=1.4\times10^{-13}A^{\frac{1}{3}}$ cm. Figure 1 shows agreement to within a few tenths of an Mev between empirical results and Eq. (3). The experimental values of $E_{T=1}$ seem to increase with A slightly faster than formula (3) indicates; this is probably due to the neglect of exchange terms[9] in (3).

For $A\leq 22$, the odd-odd ground state lies far below the calculated energy for the $T=1$ state. The resulting assignment $T=0$ for ground state of these nuclei is, of course, in agreement with considerations from decay schemes (Sec. II). The ground states of Al^{26}, P^{30}, Cl^{34}, and K^{38} are close to the calculated [Eq. (3)] energies for $T=1$ states. Definite assignments of T for these cases have, however, been made by a study of decay schemes in Sec. II. The situation for the heavier nuclei is still uncertain experimentally.

IV. SOME PREDICTED EXCITED STATES OF ODD-ODD $N=Z$ NUCLEI

On the basis of the above consideration, it is possible to make some tentative predictions regarding level schemes and energies of some odd-odd $N=Z$ nuclei. We consider here only the fairly well established cases F^{18}, Na^{22}, Al^{26}, P^{30}, and K^{38}. In all of these cases except Al^{26}, the ground state appears to have $T=0$, as was pointed out above. The position of the lowest $T=1$ state, at excitation energy $E_{T=1}-E_{T=0}$ can be estimated crudely on the basis of the theoretical formula [Eq. (3)]. Perhaps slightly better estimates of $E_{T=1}$ can be made for these nuclei by adjusting the theoretical curve slightly to conform with experimentally known values of this energy for Li^6, B^{10}, N^{14}, Al^{26}, and Cl^{34}. This gives excitation energies 1.1, 0.8, 0.5, and 0.1 Mev (accurate to, say, 0.2 Mev), for the lowest $T=1$ state of F^{18}, Na^{22}, P^{30}, and K^{38}, respectively.

On the basis of these estimates, the following tentative predictions[10] regarding decay schemes can be made:

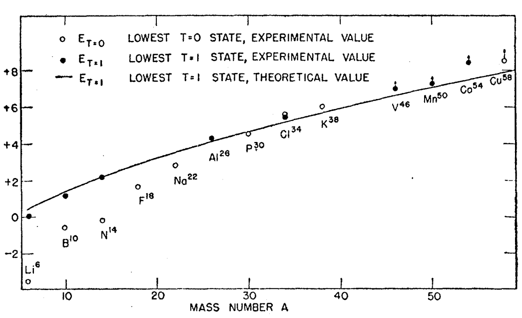

FIG. 1. Energies (neutral atom) of lowest $T=0$ and $T=1$ states in odd-odd $N=Z$ nuclei, relative to ground state of neighboring even-even $N=Z+2$ isobar.

[7] Decay schemes except Cl^{34} (references 3, 4) taken from R. W. King, Ph.D. Dissertation, Washington University, St. Louis, Missouri, 1952 (unpublished), and Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

[8] Li^6, B^{10}, and N^{14} from F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. *24*, 321 (1952).

[9] See L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 801 (1953).

[10] *Note added in proof.*—Experimental investigations of some of these cases have already been made [P. Stähelin, Helv. Phys. Acta **26**, 691 (1953)]. The results are in general agreement with the remarks of this section.

F^{18}

The lowest $T=1$, $J=0+$ state probably occurs at about 1.1 Mev above the $T=0$, $J=1+$ ground state and is expected to decay to this ground state by $M1$ radiation of very short half-life. The known excited state at 1.050 Mev is produced by a $Ne^{20}(d,\alpha)$ reaction[9] and accordingly must have $T=0$ (See Adair, reference 1). Thus, the 1.050-Mev state and the lowest $T=1$ state in F^{18} appear to be distinct but close together.

Na^{22}

The $T=1$, $J=0$ state probably occurs at about 0.8 Mev above the $T=0$, $J=3$ ground state. The decay of the $T=1$ state depends on whether there is a $T=0$, $J=1$ state below it. If there is, the decay will go to the Na^{22} ground state by an $M1-E2$ cascade of very short half-life. If there is not, the $T=1$ state will decay principally to the Na^{22} ground state by $M3$ radiation with partial half-life of 10^{-2} to 1 second; in addition, it should some of the time decay instead to the Ne^{22} ground state by positron emission of $E_{max}=2.5$ Mev, $\log ft=3.4$, partial half-life$=10$ sec.

P^{30}

The $T=1$ state probably occurs at about 0.5 Mev above the $T=0$, $J=1$ ground state and is expected to decay to the ground state by $M1$ radiation of very short half-life.

K^{38}

In this nucleus the $T=0$ and $T=1$ states are very close together, probably within 0.2 Mev of each other. Although the $T=0$, $J=3^+$ level seems likely to be the lower one, this is not entirely certain. In any case, if there is no intervening $J=1^+$ level between the 0^+ and 3^+ levels, there will be only a relatively small probability of a connecting $M3$ transition between them. For the most part these two states will undergo independent β decay, regardless of their exact order. The $T=1$ state will decay directly to the A^{38} ground state with a maximum energy of about 5 Mev and a half-life of about 0.5 sec.

Al^{26}

It is tempting to speculate at this point that Al^{26} may resemble K^{38} in having two lowest levels that undergo independent β decay. At present there is no known exception to the rule[11] that the J of an odd-odd $N=Z$ nucleus obeys j-j coupling near the close of a subshell. Thus the best estimate for the lowest $T=0$ state in Al^{26} appears to be $J=5^+$. Regardless of their exact order, two such lowest states would undergo independent beta decay. The Al^{26} "ground" state (probably $T=1$, $J=0^+$) decays to the ground state of Mg^{26} in about 6 sec. A $T=0$, $J=5^+$ state would decay to the first excited state of $Mg^{26}(J=2^+,\ E_{ex}=1.87$ Mev) with a half-life on the order of 10^4-10^6 years. If the second excited state of $Mg^{26}(E_{ex}=2.97$ Mev) has $J=4^+$, an allowed decay can occur with a half-life on the order of a day to a month. This decay would be about half K capture and half positron emission.

[11] R. W. King and D. C. Peaslee, Phys. Rev. **90**, 1001 (1953).

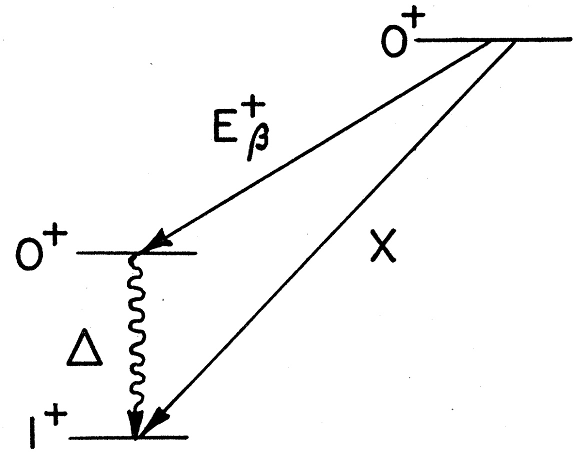

FIG. 2. General decay scheme for $Z=N+2$, $A=4n+2$ nuclei.

The above discussion suggests that it is possible to make predictions regarding the quantum numbers and decay schemes of the lowest lying states in odd-odd $N=Z$ nuclei. Naturally it would be of great interest to obtain more detailed experimental information on the decay schemes of other odd-odd $N=Z$ nuclei such as Sc^{42}, V^{46}, Mn^{50}, Co^{54} and Cu^{58}. These nuclei can all be produced by (p,n) reactions on stable targets.

V. DECAY SCHEMES OF SOME $Z=N+2$ NUCLEI

The odd-odd $N=Z$ nuclei discussed above appear as daughter products in the positron decay of nuclei with $A=4n+2$, $Z=N+2$. Experimental production of this series appears possible for $A\leq 42$. All members of the series should be particle stable by at least 5 Mev, including Ne^{18} and Ti^{42}. They can be formed by (He^3,n) on abundant target nuclei of the α-particle type (with the exception of Ca^{38}) in reactions with Q values close to zero. They are also formed (except for Ti^{42}) from the same abundant α-particle nuclei by $(\gamma,2n)$ reactions with thresholds on the order of 30 Mev. It may therefore be of interest to extend the predictions of the previous section to include the β decay schemes of these nuclei.

The $Z=N+2$ nuclei of this series are even-even with ground states $T=1$, $J=0$, from which they make superallowed transitions with $\log ft\approx 3.4$ to the corresponding states in the $N=Z$ nuclei. There will be in addition a branch of fractional intensity x (Fig. 2) to the $T=0$, $J=1$ state of the daughter nucleus. This transition has an ft value different from that for the $0-0$ transition by a factor ranging from about 2 to $\frac{1}{6}$, to judge by corresponding decays from F^{18} and P^{30} (corrected by a factor $3=2J_f+1$ for the $0\rightarrow 1$ transition). For three cases (Ne^{18}, S^{30}, Ti^{42}) the situation is already

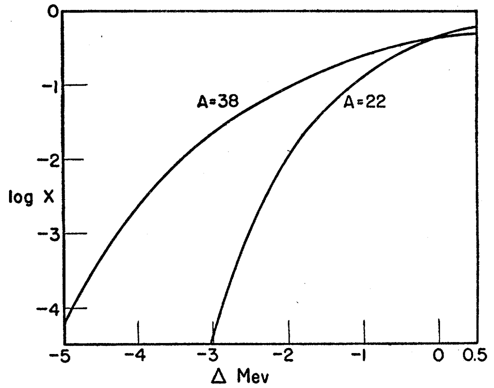

FIG. 3. Fractional intensity x of $0-1$ transition in β decay of $Z=N+2$, $A=4n+2$ nuclei.

defined by present information. The values of Δ in Fig. 2 are 1.1, 0.3 and ~ 0 Mev, respectively, where the last is an extrapolation from all known $A=4n+2$ nuclei. Using for the Coulomb energies formula (3) adjusted to fit the experimental points of Fig. 1, we obtain the results in Table II. Here E_{β^+} is the estimated energy of the $0-0$ transition, $E_\gamma=|\Delta|$ the energy of the connecting γ ray between 0^+ and 1^+ states of the final nucleus. The total half-life t and fractional intensity $x(0\leq x\leq 1)$ of the $0-1$ branch were obtained from the values of the f function for β^+ decay.[12]

For the four remaining cases we do not know the separation Δ of the 1^+ and 0^+ states in the daughter nucleus. Δ is certainly negative for $A=26$, 34 and very likely negative for $A=38$; for $A=22$ it is less certain but still probably negative. Although the negative Δ militates against the $0-1$ transition, measurement of the associated γ energy could provide valuable information about the 1^+-3^+ spacing of these particularly simple levels. The half-lives are the same order of magnitude as indicated in Table II. The fractional intensity in the $0-1\,\beta$ transition is shown in Fig. 3 as a function of Δ for $A=22$ and $A=38$. These curves are uncertain by a factor of order 2 because of uncertainty in the nuclear matrix elements.

TABLE II. Decay-scheme parameters.

Parent nucleus	E_{β^+} (Mev)	t (sec)	E_γ (Mev)	x
Ne^{18}	2.2	2	1.1	0.9
S^{30}	4.0	1.5	0.3	0.2
Ti^{42}	5.5	0.2	~ 0	~ 0.6

Comparison with the spacings in these same nuclei of the $J=0$, $J=2$ levels known from β decay suggests that $|\Delta|\lesssim 2$ Mev in this region. Observation of the weak $0-1\,\beta$ transition with the following γ ray does not then appear to be beyond the range of experimental feasibility.

In cases where the two lowest $N=Z$ states are $J=0^+$ and 3^+ ($A=38$, 34, probably 22) the γ decay from the 1^+ level will comprise an interesting competition between 1^+-0^+ (pure $M1$) and 1^+-3^+ (pure $E2$).

[12] E. Feenberg and G. Trigg, Revs. Modern Phys. 22, 399 (1950).