Level Densities of Nuclei from the Inelastic Scattering of 18-Mev Protons*

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The spectra of protons inelastically scattered from Al, Fe, Ni, Cu, Ag, Sn, Pt, and Au have been measured. The energy distribution of the scattered protons is represented only approximately by a Maxwellian distribution. The relative level densities of the target nuclei have been calculated from the proton spectra. The scattering cross sections are compared with the predictions of the statistical model. The results indicate that the observed energy variation of the level densities may not depend on the excitation energy of the residual nucleus primarily, but, instead, there is some evidence that it is a function of the energy of the emitted particle.

I. INTRODUCTION

SINCE Weisskopf and Ewing[1] proposed the statistical model for nuclear reactions, several experiments have been performed to test this theory. Many of the more recent measurements support its predictions.[2–6] Paul and Clarke[7] carried out an extensive survey of (n,p), (n,α), and $(n,2n)$ reactions. They were able to show that the $(n,2n)$ reaction cross sections are approximately in agreement with theory. However, the (n,p) and (n,α) reaction cross sections for the heavy elements are orders of magnitude larger than expected. Similar observations were made in the study of (γ,p) reactions.[8] Most of these experiments consisted of the measurement of reaction cross sections which are not sensitive to the assumptions incorporated into the statistical model. Usually, it is not possible to determine which of these assumptions is responsible for any disagreement between experimental and theoretical results.

The basic assumption underlying the theory is the Bohr assumption, which states that in a nuclear reaction a compound nucleus is formed which decays in all energetically possible modes independent of its formation. This hypothesis makes it possible to separate the formation of the compound system and its decay. The decay is assumed to be a free competition between all the different possible processes of de-excitation. The reaction cross section σ_{ab} for a reaction $A(a,b)B$ can then be written

$$\sigma_{ab}=\sigma_a\Gamma_b/\textstyle\sum_i\Gamma_i. \tag{1}$$

Here σ_a is the cross section for the formation of a compound nucleus C by the bombardment of A with a; Γ_i is the width for the emission of a particle i. The sum is extended over all possible particles which may energetically be emitted. Weisskopf calculated the probability per unit time $\eta=\Gamma/\hbar$ for the emission of a particle by using the principle of microscopic reversibility:

$$\eta_b=\frac{M}{\pi^2\hbar^3\omega_C(E_0)}\int_{\epsilon=0}^{\epsilon_{\max}}S_b\omega_B(E)\epsilon d\epsilon. \tag{2}$$

M is the mass of particle b; ω_C is the level density of compound nucleus C at an excitation energy E_0; ϵ is the energy of particle b; S_b is the cross section for the formation of nucleus C with excitation energy E_0 by bombarding B, which is in a state with excitation energy E, by particle b, which has kinetic energy ϵ; E is the excitation energy of B after b has been emitted; $E=E_0-Q_b-\epsilon$, where Q_b is the binding energy of b; $\epsilon_{\max}=E_0-Q_b$ is the maximum kinetic energy b can obtain. The integrand

$$nd\epsilon=\text{const}S_b\omega_B(E)\epsilon d\epsilon \tag{3}$$

represents the energy distribution of emitted particles.

The principle of detailed balancing relates the cross section S_b with the density of states ω_B. Generally, the cross section for the formation of a particular nuclear state depends on the properties of that state. In this theory S_b is assumed to be a monotonic function of the kinetic energy of particle b

$$S_b=\sum_{l=0}^{\infty}(2l+1)\pi\lambda^2T_l(\epsilon),$$

in which T_l is a transmission coefficient. This applies only if the nuclear states are densely packed.Furthermore, this assumption does not take account of the width of a level. Hence, the transition to each final state becomes equally probable, e.g., an n-fold degenerate level will be counted n times. The density of states ω_B and the actually observed level density ρ_B should then be related by $\omega_B=f\rho_B$, where f has the meaning of an oscillator strength and contains the square of the matrix element for the particular transition.

The density of states at a residual excitation energy E of the nucleus B is assumed to be only a function of the excitation energy of a particular residual nucleus. The possibility that ω_B depends also on the kinetic energy of the bombarding particle should not be ex-

* This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
[1] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).
[2] B. C. Diven and G. M. Almy, Phys. Rev. **80**, 407 (1950).
[3] P. R. Byerly and W. E. Stephens, Phys. Rev. **81**, 473 (1951).
[4] Nabholz, Stoll, and Wäffler, Phys. Rev. **86**, 1043 (1952).
[5] Brolley, Fowler, and Schlacks, Phys. Rev. **88**, 618 (1952).
[6] Bleuler, Stebbins, and Tendam, Phys. Rev. **90**, 460 (1953).
[7] E. B. Paul and R. L. Clarke, Can. J. Phys. **31**, 267 (1953).
[8] O. Hirzel and H. Wäffler, Helv. Phys. Acta **20**, 373 (1947).

cluded, since the maximum angular momentum of C depends on the energy of the bombarding particle. Therefore, higher angular momenta states in B may be excited with increasing E_0. ω is generally called "level density," although it may not represent the density of actual levels.

Little is known about ω. Weisskopf assumes $\omega = b\exp[(aE)^{\frac{1}{2}}]$ and calculates a and b from the known levels at low excitation energy and at the neutron dissociation energy. Formula 3 shows that it is possible to obtain the level density function experimentally from the energy distribution of the emitted particles.

The observation of the energy spectrum of emitted particles gives a direct measurement of the differential cross section $\partial^2\sigma/\partial\epsilon\partial\Omega$. Hauser and Feshbach[9] have shown that the cross section should not depend on the angle of observation in the energy region where the statistical theory applies. However, angular anisotropy for emitted particles has already been observed in γ-induced reactions.[3,10–12] Courant[13] predicted an anisotropic angular distribution on the basis of a direct dipole interaction between incident γ ray and the subsequently emitted nucleon.

The spectra of the products of heavy-particle-induced reactions in the intermediate energy range have been investigated for inelastic scattering of 14-Mev neutrons by Stelson and Goodman,[14] Whitmore and Dennis,[15] and Graves and Rosen.[16] (p,n) reactions were investigated by Gugelot.[17] Deuteron-induced reactions are not considered here, since the stripping process, which does not involve a compound nucleus, complicates the discussion. From the observed spectra, the energy variation of ω_B can be calculated. All the results obtained so far show that the spectra are approximately of the form $nd\epsilon = \text{const}e^{-\epsilon/T}\epsilon d\epsilon$. The parameter T varies little over the whole periodic system and is of the order of 1 Mev. As a result of the experimental difficulties in the detection of neutrons, the uncertainties of these results are large, and the investigated energy range is small. For this reason, an attempt has been made here to measure the spectra of inelastically scattered protons for which these experimental difficulties do not exist. The calculation of the relative level density from proton measurements requires only a knowledge of the barrier penetrability S_p in Eq. (3). For a nuclear radius of $1.5\times10^{-13}A^{\frac{1}{3}}$ cm, S_p is given by Blatt and Weisskopf[18] and more recently by Shapiro.[19]

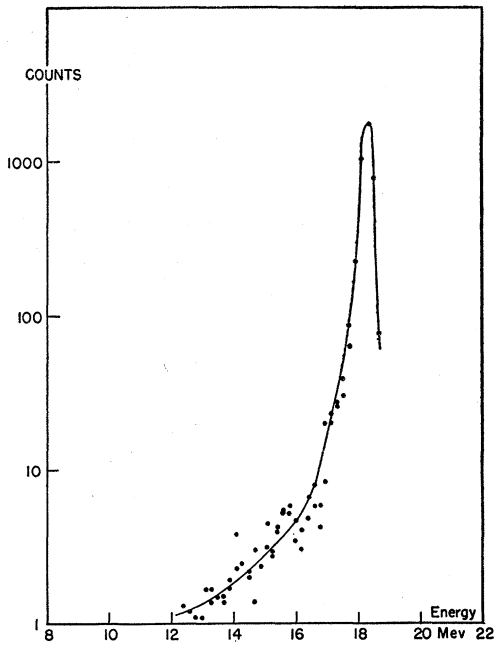

FIG. 1. The energy distribution of the protons in the incident beam.

II. EXPERIMENTAL SETUP

The focused and collimated 18.3-Mev proton beam of the cyclotron entered a 12-inch diameter scattering chamber with Al windows at 30, 60, 90, 120, and 150 degrees to the incident beam. The scattering chamber and NaI (Tl) scintillation detector are described by Likely and Franzen.[20] The beam was collected in a 5-inch long graphite Faraday cup. A 1-μf polystyrene precision condensor was connected between the cup and ground. The potential of the condensor could be measured by means of a quadrant electrometer, whose maximum deflection corresponded to about 1 volt. Calibration of the electrometer was carried out with a Rubicon potentiometer. The performance of the Faraday cup was checked by counting the number of scattered protons per unit charge collected as a function of the bias voltage on the cup. The counting rate stayed constant within 1 percent for a variation of the bias voltage between +6 and −6 volt.

The scattering foils used were 0.1- to 0.15-mil thick foils of Al, Fe, Ni, Cu, Ag, Sn, Pt, and Au. No correction for multiple scattering is necessary for this foil thickness. The foils were always mounted in such a way that their normal bisected the angle smaller than 90° between beam direction and detector.

The pulses from the RCA 5819 multiplier (in later experiments a Dumont K1186 was used) were amplified by a Los Alamos type 501 amplifier and displayed on

[9] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[10] Toms, Carroll, Rosenblum, and Stephens (to be published); M. E. Toms and W. E. Stephens, Phys. Rev. **88**, 160 (1952); Stephens, Toms, Carroll, and G. K. Rosenblum, Phys. Rev. **89**, 893 (1953).
[11] W. A. Butler and G. M. Almy, Phys. Rev. **89**, 893 (1953).
[12] Mann, Halpern, and Rothman, Phys. Rev. **87**, 146 (1952).
[13] E. D. Courant, Phys. Rev. **82**, 703 (1952).
[14] P. H. Stelson and C. Goodman, Phys. Rev. **82**, 69 (1951).
[15] B. G. Whitmore and G. E. Dennis, Phys. Rev. **84**, 296 (1951).
[16] E. R. Graves and L. Rosen, Phys. Rev. **89**, 343 (1953).
[17] P. C. Gugelot, Phys. Rev. **81**, 51 (1951).
[18] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (J. Wiley and Sons, Inc., New York, 1952), p. 352.
[19] M. M. Shapiro, Phys. Rev. **90**, 171 (1953).
[20] J. G. Likely and W. Franzen, Phys. Rev. **87**, 666 (1952).

a Tectronix 513D oscillograph which was fed through a line voltage regulator. The pulses were photographed on 35-mm film moving with a speed up to 5 cm/sec. The film was projected on a ground glass screen in front of which 65 regularly spaced microswitches were placed, each connected to a register.[21] The whole row of switches could be moved to follow the base line of the oscilloscope trace. The linearity of the complete system was checked by means of calibrated test pulses. It was not possible to obtain an energy resolution better than 3 percent (width at half-maximum of the elastic peak), whereas a multichannel discriminator showed that the actual resolution was 2 percent.

The pulse-height distributions were corrected for background, for the energy loss of the scattered protons in the Al windows, scattering foil and air path (total absorption about 25 mg/cm^2), and center-of-mass motion for the target elements up to Cu. Background runs were taken after each measurement. In general, three background runs were made: (1) with the scattering foil in place, but with an aluminum absorber of sufficient thickness to stop all protons between chamber window and detector, to detect the total γ-ray background; (2) with the same arrangement but without the scattering foil to detect the γ-ray background which originated in the collimators and collector cup; and (3) without scattering foil and without Al absorber to detect slit-scattered protons. There was no slit scattering observable in any run. At a pulse height corresponding to about 5 Mev, the γ-ray background increased very rapidly with decreasing pulse height, so that the proton spectra could not be taken below this energy.

To ascertain that no degraded protons reached the target, the energy distribution of the protons in the incident beam was measured by having the detector in place of the collector cup and operating the cyclotron at very low intensity. The results of this measurement are shown in Fig. 1. At 13 Mev the intensity is less than 0.1 percent of the main beam. The low-energy tail is probably produced by slit scattering. A gold collimator was used to define the beam. The actual measurements have been repeated at least twice for each element with slightly different collimator arrangements.

III. RESULTS

The results of the scattering experiments are presented as relative level-density functions. From Eq. (3) it is found that the quantity $n/\epsilon S_p$ is proportional to the level density of the residual nucleus. n is the experimentally observed number of protons per unit energy interval having kinetic energy ϵ; S_p is the cross section for the formation of a compound nucleus by protons as given by Weisskopf and Blatt.[18] The nuclear radius was assumed to be $1.5\times10^{-13}A^{\frac{1}{3}}$ cm.

[21] R. Britten, Phys. Rev. 88, 283 (1952).

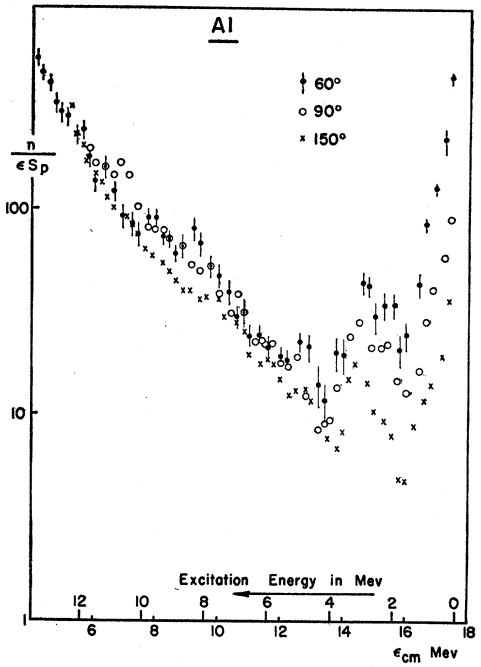

Fig. 2. The relative level density $\omega=n/\epsilon S_p$ of Al, obtained from the scattered proton energy distribution at 60°, 90°, and 150° to the incident beam.

A. Aluminum

Spectra were taken at 60°, 90°, and 150° to the incident beam. The results are shown in Fig. 2. The abscissa gives the energy in the center-of-mass system and the excitation energy of the residual nucleus. The ordinate is proportional to the level density in the residual nucleus for a region in which the compound nucleus theory holds. At zero excitation energy the elastic scattering is apparent. Up to 4-Mev excitation energy, the intensity fluctuates due to unresolved levels. Above 4 Mev the level structure is so dense that the spectrum becomes smooth, so that from here on the statistical model is applicable. The angular distribution of the scattered protons is seen to be isotropic, in agreement with the calculations by Hauser and Feshbach.[9] The spectrum is not purely Maxwellian. Defining nuclear temperature as $1/T=d\ln\omega/dE$, we find that T varies between 1.3 Mev and 2.7 Mev. Table I compares the values for T obtained from different experiments.

B. Iron, Nickel, and Copper

The results for these medium heavy elements are given in Figs. 3–5. The relative values for the level density are similar for the three elements considered. This is to be expected on the basis of the statistical model. However, in disagreement with this model is an anisotropic angular distribution of the emitted protons. More fast protons are emitted at 60° than at 150° to the incident beam, showing that a fraction of the re-

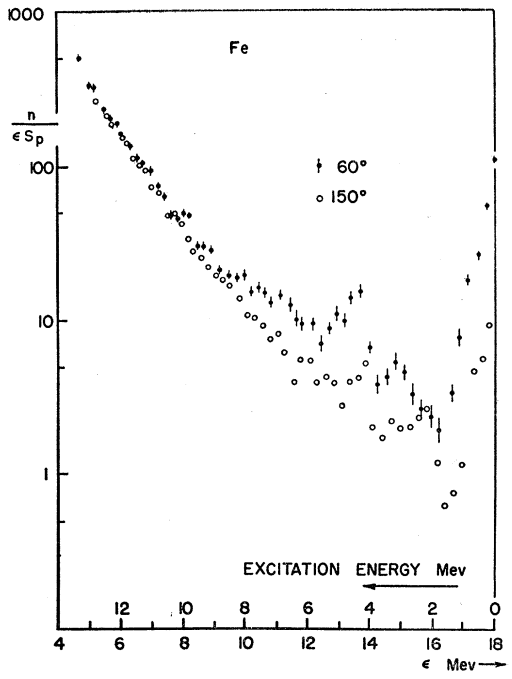

FIG. 3. The relative level density of Fe.

actions are due to interactions which cannot be described by the compound-nucleus theory.

Assuming that the backwards-scattered protons conform to the statistical description, we will confine the discussion in particular to the 150° data. This spectrum deviates again from a Maxwellian distribution: T increases with decreasing excitation energy. At $E=12$ Mev we find $T=1.3$ Mev. T increases to about 3 Mev at low excitation energies.

Table II compares data from different reactions.

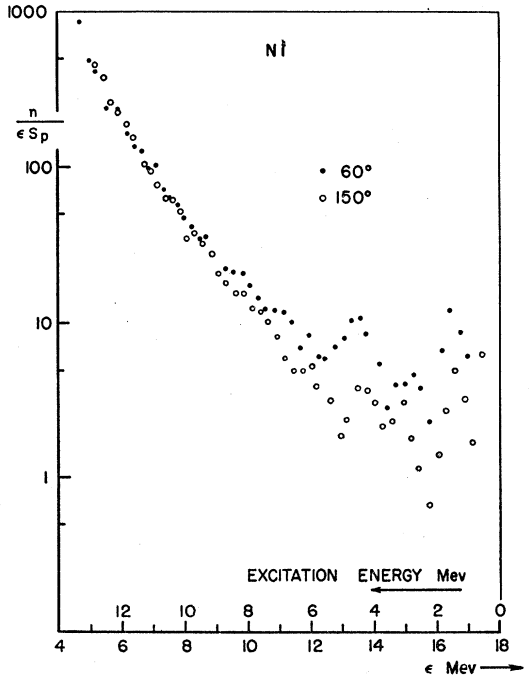

FIG. 4. The relative level density of Ni.

C. Silver and Tin

The relative level densities for silver and for tin are shown in Figs. 6 and 7. The protons having an energy larger than 8 Mev are emitted anisotropically. For these elements this effect is even more pronounced than for the medium heavy ones. Little difference is apparent between the results for Ag and those for Sn. The magic character of the proton number of Sn does not seem to have influence on the spectrum.

From the 150° data for Ag one finds that T changes from 0.8 Mev to about 3.5 Mev. The T values for Sn are approximately the same.

The measurement for Ag at 150° has been repeated for the normal incident proton energy of 18.3 Mev.

TABLE I. Values for $T=1/(d\log\omega/dE)$. ϵ_{max} is the maximum energy with which a particle can be emitted. $\Delta\epsilon$ is the energy interval of the emitted particle over which T is computed. ΔE is the interval of the residual excitation energy after emission of a particle with kinetic energy in the interval $\Delta\epsilon$.

Reaction	ϵ_{max}	$\Delta\epsilon$	ΔE	T	Ref.
Al(n,n)	13	1–2	12–9	1.0	a
Al(n,n)	14	1–4	13–10	1.1	b
Al(p,p)	28	10–25	18–3	3–6.5	c
Al(p,p)	17	5–7 8–12	12–10 9–5	1.3 2.7	d
Al(p,n)	11	2–5	9–6	1.3	e

[a] See reference 16.
[b] See reference 15.
[c] See reference 21.
[d] This paper.
[e] See reference 17.

Immediately afterwards the proton energy was reduced to 16.2 Mev and a run was made under the same conditions. These results are presented in Fig. 8. The ordinate of this plot is on an absolute scale, $\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega$ being the cross section for inelastic scattering per Mev per steradian, ϵ the scattered proton energy, and S_p the formation cross section as defined by Blatt and Weisskopf. The slopes at equal excitation energy for both measurements do not agree: for $\epsilon_{max}=16$ Mev and $10<E<6$ Mev one finds $T=1.3$ Mev, whereas for $\epsilon_{max}=18$ Mev in the same excitation energy interval $T=1.8$ Mev. For the higher incident energy, $T=1.3$ Mev is obtained for $12<E<8$ Mev. Table III summarizes all available data.

D. Platinum and Gold

Figure 9 shows the results obtained from the inelastic scattering of protons from Pt. The relative level density of Au is very similar to that of Pt; it is therefore not reproduced. Several Pt runs have been carried out

with different beam and detector collimators. All results agreed with one another. The intensity of the scattered beam measured at 60° is almost uniformly three times as great as at 150°. A small admixture to the incident beam of a few percent of protons of degraded energy could have produced an apparent increase of the forward scattering, since the 60° elastic-scattering cross section is very much larger than that for 150° scattering. At 18 Mev measured values for the elastic-scattering cross section at 60° and 150° are 170 mb/sterad and 4 mb/sterad, respectively.[22] However, the energy distribution of the incident beam (Fig. 1) accounts for only 8 percent of the 60° scattered intensity at a proton energy of 12 Mev. Therefore, this amount of background cannot appreciably reduce the 60° intensity relative to the 150° intensity of scattered protons.

T varies between 0.7 Mev and about 2.8 Mev for the 150° data. The value for T at large excitation energy is just above the neutron binding energy. Table IV compares then the ratio of the level densities at the neutron binding energy and at 1-Mev excitation. In column 2 resolved levels are used, in column 3 the results of this experiment are used. For Al the agreement between the two ratios is fair, while for the other elements, the discrepancy amounts to at least a factor 100.

TABLE II. Values for T for Fe, Ni, and Cu. The symbols and references are the same as in Table I.

Reaction	ϵ_{max}	$\Delta\epsilon$	ΔE	T	Ref.
Fe(n,n)	14	1–4	13–10	0.76	a
Fe(n,n)	15	1–3	14–12	0.6	b
Fe(p,p)	18	5–8 10–16	13–10 8–2	1.36 2.6	d
Fe(p,n)	10	2–6	8–4	0.95	e
Ni(p,p)	18	5–8 10–16	13–10 8–2	1.2 2.6	d
Cu(n,n)	14	1–4	13–10	0.77	a
Cu(p,p)	18	5–8 10–16	13–10 8–2	1.2 2.6	d

in the residual nucleus cannot be very reliable, because the correction for the barrier penetration is large.

From the scattering of 14 Mev neutrons from Au, Graves and Rosen find $T=0.7$ Mev. From the Au(p,n)Hg reaction $T=0.8$ Mev was deduced.

IV. DISCUSSION

Several features of these results have to be discussed. The first and most general property of the deduced relative level density is that the density increases exponentially with excitation energy. Its variation with energy at high excitations is even more rapid than thermodynamical models predict. Secondly, the level densities of neighboring elements are very similar.

Since it is possible to obtain relative level densities only with these experiments, a normalization is necessary. A suitable excitation energy to fix the absolute value of ω is at about 1 Mev. At that excitation one knows that the level spacing is between 0.5 and 0.1 Mev. Another point where the level density is known

[22] P. C. Gugelot, Phys. Rev. **87**, 525 (1952).

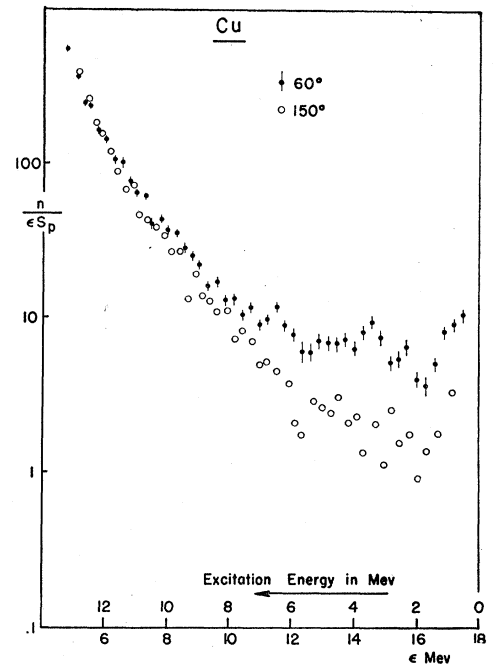

FIG. 5. The relative level density of Cu.

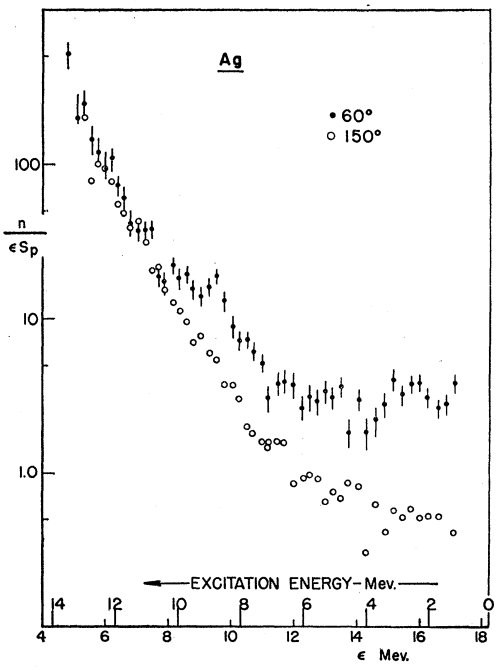

FIG. 6. The relative level density of Ag.

An explanation for this may be connected with the explanation for the observed anisotropic angular distribution of the emitted protons. The anisotropic emission cannot be brought into agreement with the common picture of the compound nucleus. It is therefore necessary that direct interactions take place—possibly, interactions between the incoming particle and the nucleons forming the surface of the target nucleus.[23] Transparency[24,25] of the whole nucleus is probably ruled out, since no angular anisotropy is found for the light elements for which the transparency would be larger than for the heavy ones. It may well be that even for backward scattering the direct interactions with the surface give a noticeable contribution. The measured spectra may therefore be composed of two partial spectra, the first arising from surface effects, the second from the decay of the compound nucleus. Experimental data on (p,α) or (n,α) reactions may give more information on the origin of the angular distribution, since it seems improbable that α particles will be emitted by direct interactions.

TABLE III. Values for T for Ag and Sn. The symbols and references are the same as in Table I.

Reaction	ϵ_{max}	$\Delta\epsilon$	ΔE	T	Ref.
Ag(n,n)	14	1–4	13–10	0.76	a
Ag(p,p)	18	5–8 11–15	13–10 7–3	1.0 3.5	d
Ag(p,p)	16	6–10	10–6	1.3	d
Rh(p,n)	14	2–6	12–8	0.85	e
Sn(n,n)	14	1–4	13–10	0.56	a
Sn(p,p)	18	5.5–8 11–14	12.5–10 7–4	0.9 2.8	d

Another possible argument to explain the small ratio for the level densities at the neutron dissociation energy and near the ground state is the following: the transition probabilities from the compound state to low-lying levels in the residual nucleus could be very large compared to the transition probabilities to highly excited states. This argument cannot be ruled out on the basis of available evidence, but the size of the variation and the fact that no such variation is required to explain our results for aluminum throws some doubt on this argument.

Another explanation for the small variation of ω between 0 and 8 Mev is given by Hurwitz and Bethe[26] and is discussed by Weinberg and Blatt.[27] They assume that the excitation energy of a particular nucleus should not be measured from the ground state but from a state which is above the ground state and varies smoothly from element to element. The ground state

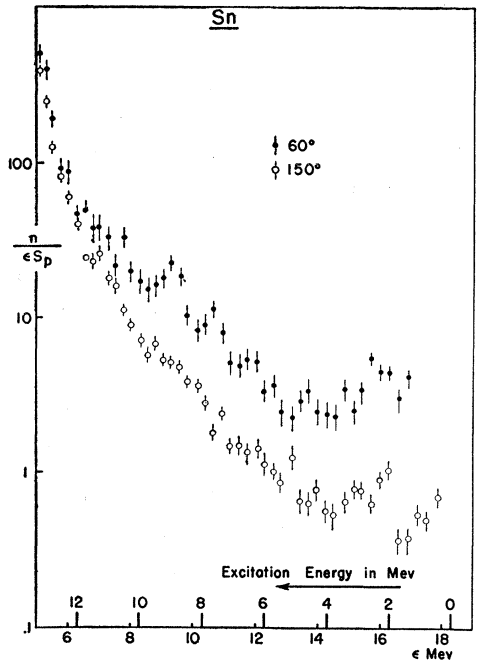

FIG. 7. The relative level density of Sn.

may be very much lower than this level for particularly stable nuclei. However, the similarity of the Ag and magic Sn spectra and of the Fe, Cu, and magic Ni spectra seems to refute this hypothesis.

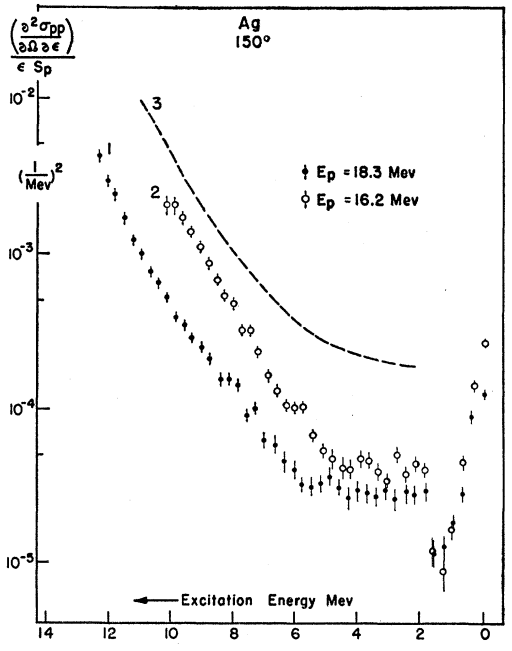

FIG. 8. The relative level density of Ag obtained from the scattering of 18.3-Mev and of 16.2-Mev protons. The values for $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ are absolute. Curve 3 presents the calculated value for $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ for 16.2-Mev incident protons. The level density used in the calculation is that obtained from the 18-Mev experiment.

[23] H. McManus and W. T. Sharp, Phys. Rev. **87**, 188 (1952).
[24] R. Serber, Phys. Rev. **72**, 1114 (1947).
[25] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[26] H. Hurwitz and H. A. Bethe, Phys. Rev. **81**, 898 (1951).
[27] I. G. Weinberg and J. M. Blatt, Am. J. Phys. **21**, 124 (1953).

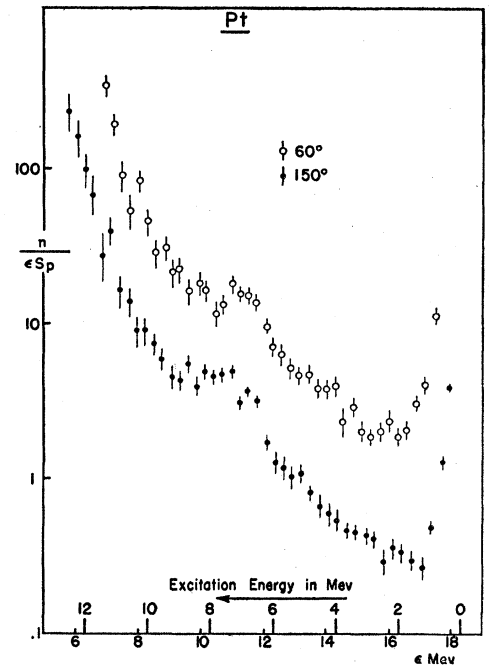

FIG. 9. The relative level density of Pt.

Another effect which may have to be considered to explain the observed angular anisotropy for the scattered protons is the electric excitation of nuclei by protons. This effect has been discussed by Weisskopf,[28] Mullin and Guth,[29] and Huby and Newns.[30] Total cross sections for this interaction can be as large as 10^{-28} cm^2. However, the angular distribution calculations are not very reliable. It is therefore not possible to conclude whether or not the electric interaction has to be taken into account. More information on the high-energy part of the spectrum of inelastically scattered neutrons may lead to a clarification of this point.

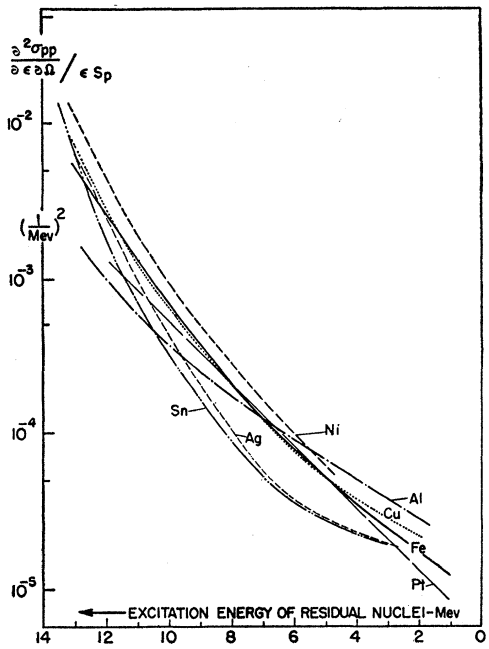

FIG. 10. A summary of the 150° experimental data.

Adhering to the cross-section calculations based on the statistical model but inserting for the level density the functions found in this paper from the 150° scattering, one may recalculate the theoretical cross sections for (n,p) and $(n,2n)$ reactions. These values for σ_{theor} may be compared with the experimental results from Paul and Clark.[7] This has been done for Rh (using the level density from Ag) and Pt. For Rh the following results are obtained:

$\sigma_{\text{theor}}(n,p)=10^{-26}$ cm^2, observed $\sigma_{\text{exp}}(n,p)=13\times10^{-26}$ cm^2. Using Weisskopf's level density: $\sigma_{\text{theor}}(n,p)=0.1\times10^{-26}$ cm^2. For Pt: $\sigma_{\text{theor}}(n,p)=3.7\times10^{-27}$ cm^2, which compares with $\sigma_{\text{exp}}(n,p)=3.10^{-27}$ cm^2, whereas the theoretical level density formula gave $\sigma_{\text{theor}}(n,p)=0.02$ 10^{-27} cm^2. The cross sections for $(n,2n)$ reactions are practically independent of the shape of the level density function.

TABLE IV. Ratio of level densities at the neutron binding energy and at 1-Mev excitation energy. Column 2, from neutron resonance experiments. Column 3, from the relative level densities in this paper.

Element	$\omega(Q_n)/\omega(1)$	
Al	10	7
Fe	—	13
Ni	5.10^2	9
Cu	2.10^2	7
Ag	2.10^3	8
Sn	—	9
Pt	10^3	11

The energy dependence of the level density functions has to be considered next. Excluding that part of the level density function at high energy of excitation, it was found that ω increased more rapidly with energy than $\exp[(aE)^{\frac{1}{2}}]$, as postulated by Weisskopf. In addition, the variation of $T=(d\log\omega/dE)^{-1}$ over the periodic system seems to be small, smaller than the variation of T with E for a given element. A comparison of all the data collected in these experiments is shown in Fig. 10. The ordinate is on an absolute scale. It is remarkable that the value of $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ does not vary appreciably from one element to another.

According to formulas 1 and 2, the plotted quantity can be written as

$$\frac{\partial^2\sigma_{pp}}{\partial\epsilon\partial\Omega}\Big/\epsilon S_p=\frac{\sigma_p}{4\pi}\omega(E)\Big/\sum_i\int_0^{\epsilon_{\max^i}}S_i\omega(\epsilon_{\max^i}-\epsilon)\epsilon d\epsilon.$$

An isotropic angular distribution for the emitted protons is assumed. The sum has to be carried out over all

[28] V. F. Weisskopf, Phys. Rev. **53**, 1018 (1938).
[29] C. J. Mullin and E. Guth, Phys. Rev. **82**, 141 (1951).
[30] R. Huby and H. C. Newns, Proc. Phys. Soc. (London) **A64**, 619 (1951).

energetically possible reactions. By far the most probable reaction is, however, the (p,n) reaction; however, for Al the inelastic scattering of protons should also be considered. Setting $\omega(0)=1$ one may calculate

$$\frac{\sigma_p}{4\pi}\Big/\sum_i\int_0^{\epsilon_{\max}{}^i} S_i\omega(\epsilon_{\max}{}^i-\epsilon)\epsilon d\epsilon,$$

and this expression should then be equal to the ordinate of the measured curves at $E=0$. This has been carried out for those elements for which the threshold of the (p,n) reaction is known. The following results were obtained:

Al^{27}, 4.1×10^{-5} (Mev)$^{-2}$ calculated, and 1.6×10^{-5} (Mev)$^{-2}$ experimental;

Fe^{56}, 1.2×10^{-5} (Mev)$^{-2}$ calculated, and 10^{-5} (Mev)$^{-2}$ experimental;

Ag^{107}, 0.8×10^{-5} (Mev)$^{-2}$ calculated, and 1.5×10^{-5} (Mev)$^{-2}$ experimental.

The agreement is good, taking into account the rough assumptions. These results may show that cross section calculations on the basis of the statistical model give reliable results, provided the right level density functions are used. The results are very sensitive to $\epsilon_{\max}$; a variation of $\epsilon_{\max}$ for Ag by 0.2 Mev gives a 26 percent variation in the computed values. However, the meaning of these estimates is rather doubtful. To be specific, it does not seem possible to use the same level density function for different excitation energies. Curves 1 and 2 of Fig. 8 show $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ observed from the bombardment of Ag by 18.3-Mev and 16.2-Mev protons. The calculated value for $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ for 16.2-Mev incident protons using the level density function ω obtained from the 18.3-Mev data is shown in Fig. 8, curve 3. This curve has to be compared with the experimental curve 2 of Fig. 8. The calculated value for $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ for 16.2-Mev incident protons agrees again with the experimental curve if one computes its value by using ω from the 16.2-Mev experiment. This may indicate that ω is not only a function of the excitation energy of a particular residual nucleus, but also a function of the excitation energy of the compound nucleus. With this evidence it is necessary to consider the results of all available experiments compiled in Tables I to III. Although the values for T are not very precise, it is still possible to conclude that the slopes of the level density function obtained from the various observations on a particular nucleus at a given excitation energy do not agree with one another. Neither can one find a definite trend of $T(E)$ with the excitation energy of the compound nucleus. The different values of T at high excitation energy obtained from (n,n) or from (p,p) scattering could be produced by a wrong Coulomb barrier penetration correction, due to either a depression of the barrier caused by surface vibration of an excited nucleus[31] or an incorrect estimate of the nuclear radius. However, the disagreements are already obvious for Al, for which the penetration correction is small.

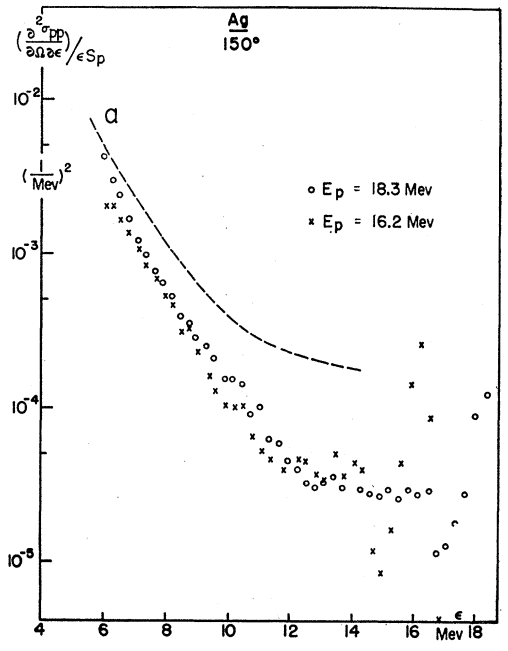

FIG. 11. $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ *vs* energy of the emitted protons obtained from the scattering of 18.3-Mev and 16.2-Mev protons by Ag. Curve *a* presents the calculated value of $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ for 16.2-Mev incident protons. The level density used in the calculation is that obtained from the 18-Mev experiment. Curve *a* is equivalent with curve 3 of Fig. 8.

There seems to be some evidence from columns 3 and 5 of Tables I–III that T is a function of the energy of the emitted particle only. This conclusion is also borne out by the Ag experiments, shown in Fig. 8. Here, the two curves overlap if they are plotted as a function of ϵ, as shown in Fig. 11. From the theory of nuclear reactions one would expect that the quantity $(1/\epsilon S_p)\times(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ obtained for 18-Mev incident protons is 1/9 of the same quantity obtained from the 16-Mev measurements at equal excitation energy E. The proportionality factor is independent of E, since the derivative of the level density with excitation energy E should only be a function of the excitation energy. Figure 8 shows that this is not the case, but that there are too many fast protons in the spectrum obtained with 18-Mev incident protons as compared to the 16-Mev spectrum. Curve *a* of Fig. 11 shows $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$, which is calculated for 16.2-Mev incident protons using ω obtained from the 18.3-Mev experiment. This curve corresponds to curve 3 of Fig. 8. At $\epsilon=5$ Mev the calculated curve *a* and the experimental curves approach each other, as one expects on the basis of the theory for

[31] E. Bagge, Ann. Phys. **33**, 389 (1938).

a Maxwellian distribution of emitted particles.[32] It is possible to make noncompound nuclear effects responsible for the emitted high-energy protons. These direct interactions may be a function of the energy of the incoming particle, so that the observed level density has only a meaning at high excitation energy of the residual nucleus. However, the slopes of the level density obtained by Britten[21] by the inelastic scattering of 32-Mev protons by Al are in good agreement with our slopes for Al, if the "level density" is assumed to be a function of ϵ and not of E. In addition, $(1/\epsilon S_p) \times (\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ from the 32-Mev data is 7×10^{-5} Mev^{-2}, and that quantity from the 18-Mev data is 8.5×10^{-5} Mev^{-2} at $\epsilon=11.5$ Mev, in fair agreement. The error stays approximately constant over the range of ϵ for which the two measurements overlap. No correlation between the slopes of the two experimental curves is obtained, if ω is considered to be a function of the excitation energy E, as shown in Table I.

Figure 12 shows $T=-[d\log\omega(\epsilon)/d\epsilon]^{-1}$ taken from all available results on Al and Ag as a function of the particle energy. This plot shows better agreement between the different experiments than the values given for $T(E)$ in the tables. It should be kept in mind that the estimation of slopes from these spectra cannot be very precise.

It is now possible to understand the small variation of $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ for different elements as shown in Fig. 10. It is necessary to calculate

$$\omega(\epsilon)\Big/4\pi\int_0^{\epsilon_{\max}}\omega(\epsilon)\epsilon d\epsilon,$$

assuming that only neutron emission is probable; $\epsilon_{\max}$ represents the maximum energy with which a neutron can be emitted. To evaluate the integral the spectral distribution may be approximated by a Maxwellian distribution $\omega=e^{-\epsilon/T}$, where T is taken at small values of ϵ from which one obtains the main contribution to the integral. This yields for $(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)/\epsilon S_p\sim\omega(\epsilon)/4\pi T^2$; T does not vary much for different elements and is about 0.8 Mev. Thus $\omega(0)/4\pi T^2$ is of order 0.1 Mev^{-2}, in agreement with a reasonable extrapolation to $\epsilon=0$

[32] The fact that $(1/\epsilon S_p)(\partial^2\sigma_{pp}/\partial\epsilon\partial\Omega)$ depends only on ϵ if the level density is $\omega=a\exp(E/T)$, in which T is constant, is also shown by K. J. Le Couteur (private communication).

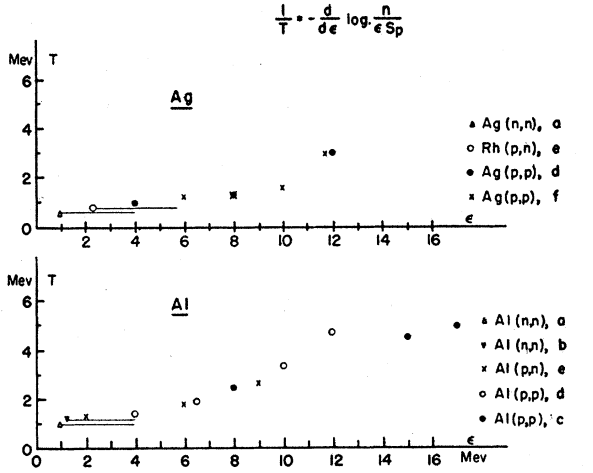

FIG. 12. $T=-[d\log\omega(\epsilon)/d\epsilon]^{-1}$ for Ag and Al *vs* energy of emitted protons. a: Graves and Rosen (see reference 16); b: Stelson and Goodman (see reference 15); c: Britten (see reference 21); d: this paper; 18.3-Mev incident protons; e: Gugelot (see reference 17); f: this paper; 16.2-Mev incident protons.

of the experimental plots. The ordinate is no longer a sensitive function of the threshold of the (p,n) reaction.

Using the hypothesis presented, one is able to recalculate the theoretical cross sections for (n,p) reactions which were measured by Paul and Clarke.[7] The cross section for $Ag(p,n)$ comes out to be about 10 mb with an uncertainty of a factor 2 resulting from the necessary extrapolation of the neutron spectrum to $\epsilon=0$. The measured cross section is 130 mb.

The evidence that ω is only a function of ϵ may have been caused by a misrepresentation of the form of the spectrum as given by Eq. (3), or by the fact that direct interactions are much more likely than had been expected. It cannot be concluded from the small amount of data available whether ω is a function of ϵ alone or a function of ϵ and the energy of the incoming particle. It will be necessary to obtain more experimental information from scattering and reaction experiments carried out in different energy ranges in order to check the evidence presented here and to be able to construct a model for the reaction mechanism.

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