# The Stopping Powers of Various Elements for Protons of Energies from 400 to 1050 kev\*

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The proton-stopping powers of the elements nitrogen, neon, argon, krypton, xenon, nickel, and copper are reported in the energy range from 400 kev to 1050 kev. Comparisons are made with results of other experimental work and with theory. The measured stopping powers show a dependence on particle velocity and on the atomic number of the stopping element in general conformity with the theory of N. Bohr. For proton energies below 700 kev, the ratio of atomic stopping power to  $Z^{\frac{1}{2}}$  for nickel and copper is lower than the corresponding ratio for the gases. Energy loss straggling is reported for nickel and copper and is of the same order of magnitude as theory predicts.

## I. INTRODUCTION

HE energy loss of particles as they traverse matter is a topic of great interest in modern physics. Bethe<sup>1</sup> and Bloch<sup>2</sup> have derived expressions for stopping power on the assumption that the particle velocity is much greater than the "velocities" of all electrons in the atoms of the stopping material. In order to extend the range of validity of these expressions to lower particle velocities, correction terms have been computed<sup>3-5</sup> to allow for the deficiency in stopping power of the K electrons. Protons of energies from 400 kev to 1050 kev, the region covered in this report, are so slow that formulas for the stopping power of the heavier elements for these particles require also corrections for deficiencies of electrons of the higher shells. For a few gaseous elements these corrections have been estimated and stopping power values for low-energy protons predicted,<sup>6</sup> but the approximations required to get numerical data make the accuracy of the results somewhat uncertain. Experimental data are thus needed to supplement theoretical calculations.

Bohr<sup>7</sup> has provided a formula applicable to moderately slow protons through heavy media. Although it is not expected to give absolutely accurate predictions, it has been shown to give approximately the correct dependence of stopping power upon proton velocity and atomic number of the medium traversed for protons of 350 to 550 kev passing through metallic foils.<sup>8</sup> The testing of these predictions in stopping power dependence at other energy ranges and through other types of media is desirable.

Another facet of the stopping-power problem which is of interest is the extent of difference to be expected at these low energies between values for media in the condensed state as compared with gaseous media. This aspect of the problem has been widely studied,<sup>9</sup> and is summarized in a review article by Taylor.<sup>10</sup> At high, though still nonrelativistic, particle velocities special stopping effects attributable to the condensed or conducting state of the stopping media are probably small, since the number of outer electrons seriously affected by the state of condensation is a small proportion of the total number of electrons in the atoms of the stopping medium. At low particle velocities the relative importance of the outer electrons to stopping power becomes greater, so that a difference in stopping power between gaseous and nongaseous elements, if it occurs, would be most likely to appear at the lower particle energies.

## **II. OUTLINE OF EXPERIMENTAL TECHNIQUE**

The experiments were carried out by using protons accelerated by a Van de Graaff generator. The basic method was that utilized and described by Madsen and Venkateswarlu.<sup>11</sup> In this technique an energy spectrum for a certain proton reaction with some target material is obtained first without, and then with, a stopping medium inserted in the proton beam. The amount of material inserted is sufficient to slow, but not completely to stop, the protons of the beam. The energy loss suffered by the protons in the stopping material is indicated by the shift in the resonance peaks of the reaction spectrum, whereas the straggling in this energy loss is indicated by the increase in width of the peaks.<sup>12</sup>

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<sup>245 (1937)</sup> 

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<sup>8</sup> S. D. Warshaw, Phys. Rev. 76, 1759 (1949).

<sup>&</sup>lt;sup>9</sup> For example, see Ellis, Rossi, and Failla, Phys. Rev. 86, 562 (1952), and references.

<sup>&</sup>lt;sup>10</sup> A. E. Taylor, Repts. Progr. in Phys. 15, 49 (1952).

<sup>&</sup>lt;sup>11</sup> C. B. Madsen and P. Venkateswarlu, Phys. Rev. 74, 648, 1782 (1948).

<sup>&</sup>lt;sup>12</sup> No resonance peaks, shifted or unshifted, are shown herein since their shapes were quite usual. No significant asymmetry in the energy loss distribution was expected on a theoretical basis, nor was any observed.



FIG. 1. Schematic diagram of the apparatus. Protons from the Van de Graaff generator are deflected to the slit S by the analyz ing magnet M. The value V has a slot covered by a thin foil of either copper or nickel. For calibration runs the value V is lowered so the beam falls directly on the target T. The stopping power of the foil is obtained by raising V into the beam. For determining the stopping powers of gases, the chamber C is filled to a pressure of two to six mm of mercury.

The method as originally described was applied only to foils, but with minor modifications it is readily applicable to gases.

The apparatus utilized is shown schematically in Fig. 1. The proton beam, after passing through the analyzing magnetic field, proceeded through a defining slit into a box which performed the combined functions of gas chamber, target holder, and Faraday cage. This chamber had at the entrance a flap valve covered with a thin foil. This valve could be moved into or out of the beam by a shaft through a gas-tight seal. When the foil was in the beam, the valve was clamped against a gasketed valve seat. When the valve was in this position, the gas chamber would be left evacuated for experiments wherein the energy losses in the foil itself were measured. After the energy losses in the foil were known, appropriate gases could be introduced into the chamber, and energy losses in the gases could be determined. The foils used were either nickel or copper. The gases studied were nitrogen, neon, argon, krypton, and xenon.

The targets used for this work were either of lithium fluoride or of aluminum. The resonance energy levels for the  $(p,\gamma)$  reactions are accurately known for these target elements.<sup>13,14</sup> For study of the foils both types of target were used. Only the lithium fluoride targets were utilized for the study of the stopping powers of gases, because the yield from the shifted aluminum resonances was too low for satisfactory measurements. The targets used were a few kev thick in terms of the energy loss of the entering protons.

The voltage of the cap of the Van de Graaff generator was stabilized in standard manner by detecting any unbalance in proton current intercepted by the defining slits and automatically altering the corona drain to bring the beam back to a balanced position between the slits. The cap potential was determined from the current in the coils of the analyzing magnet, which was calibrated by use of precisely known  $(p,\gamma)$  resonances. The magnet current was provided by a motor-generator set and was regulated electronically. The variation in beam voltage due to magnet current fluctuations was less than 200 volts at one-Mev beam energy.

The proton currents used were about 0.5 to 1.0 microampere. For such currents reaction yields were adequate to give statistically good points on the resonance curves but the currents were not so strong as to cause severe heating of the foils.

# **III. STOPPING MATERIALS**

The copper and nickel foils varied from 0.03 to 0.1 mil in thickness (about 0.67 to 2 mg/cm<sup>2</sup>). They were purchased from the Chromium Corporation of America, Waterbury, Connecticut, which had prepared them by a process of electrolytic deposition. These foils were a little thicker than are commonly used for such lowenergy experiments, but a thicker foil has the advantages of minimizing the effect of errors in thickness estimation and of errors due to minute surface deposits of carbon or oxide. Ease in measurement and handling was also provided. The foils were trimmed to a rectangular shape, weighed in a microanalytical balance, and measured dimensionally under a low-powered microscope.

In order to insure thickness uniformity, a technique was developed which permitted the determination of statistical variations in foil thickness from spot to spot. Figure 2 shows the rather simple apparatus used for this determination. A thin source of polonium alpha particles was screened from a thin-window Geiger counter tube by a metal plate containing a small  $\left(\frac{1}{32}\text{-in}\right)$ . diameter) hole of about the same size as the intense core of the proton beam from the accelerator. A platform containing the source was movable with reference to the Geiger tube. With the foil removed, data were first taken of counting rate as a function of height of the movable platform from the fixed surface. This gave a curve similar to curve (a) at the right side of Fig. 2. The steepness of the curve is determined by the rangestraggling of the alpha particles. The experiment was repeated with a foil interposed over the hole. A curve (b) of similar shape, but displaced by amount A, the air equivalent of the foil for alpha particle stopping, was determined.

The apparatus was then set at height H, in order to



FIG. 2. Apparatus for measuring thickness variations in thin foils and typical curves obtained with the apparatus. Alpha particles from a thin polonium source (Po) pass through a small hole and are counted by the Geiger tube T. Curve (a) at the right is obtained by raising the source toward T. Then the thin foil F is placed over the hole and a curve (b) is obtained. The source is then set at height H and the counting rate determined with many different spots in the foil placed over the hole.

<sup>&</sup>lt;sup>13</sup> F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 24, 321 (1952). <sup>14</sup> Broström, Huus, and Tangen, Phys. Rev. **71**, 661 (1947).

operate on a reasonably linear section of the steep part of the curve, and the counting rate was measured with various portions of the foil positioned over the hole. A change in counting rate  $\Delta c$ , when corrected for the natural statistical variations in source activity, indicated a change in air-equivalent of  $\Delta H$ . The value of  $\Delta H/A$  represented the fractional variation in thickness from the original spot to the new spot studied. The "counting statistics" provided a limit to the accuracy obtainable by this technique. For most of the foils used, a standard deviation of about 1.4 percent was obtained for the thickness variation, though the copper foils seemed on the average a little better than the nickel in this respect. This value was approximately equal to the standard deviation of the radiation counting process and, therefore, to the approximate limit of accuracy.

The chemical purities of the copper and nickel foils were determined by semiquantitative spectrographic analyses. The amounts of impurities were found to be very small, and no correction to the observed stopping powers was indicated. Four nickel foils and two copper foils were used in obtaining stopping power data. Each had a different thickness. Furthermore, each foil was cut into two pieces and in most cases each half was studied. One reason so many foil sections were used is that there was some difficulty with "carbonization" of the foils during bombardment. Foils were checked regularly, both visually and by repeated stopping power measurements, and they were discarded whenever the "carbon" layer became excessive.

The gases nitrogen and argon were purchased from the Linde Air Products Company in ordinary commercial pressurized cylinders with an expected purity of 99.6 percent or better. Because of the availability of these gases in large amounts at above atmospheric pressures, little opportunity for contamination by air leakage into the gas-chamber filling system was expected, and flushing and refilling was simple. Furthermore, the molecular stopping powers of the usual gases of the atmosphere are very close to those of nitrogen and argon, so that a small amount of contaminants would have little effect on stopping power results.

The gases neon, krypton, and xenon were purchased from the Linde Air Products Company and were supplied in glass flasks at approximately atmospheric pressure. After some gas had been used, the pressure was below atmospheric, and there was risk of air contamination during the gas-handling operations. Furthermore, the original purity guaranteed by the company permitted several percent contamination, mostly by other rare gases. For each of these three gases, therefore, a mass-spectrographic analysis was made on the residual gas in the flasks after the stopping power observations had been made. It was found that no appreciable contamination had been suffered by the krypton, enough by the neon to warrant a correction of about 7.5 percent in the final results, and a small

Proton energy (kev)	(a) Nickel Stopping power [kev/ (mg/cm <sup>2</sup> )]	Probable error (%)	Proton energy (kev)	<pre>(c) Nitroger Stopping power [kev/ (mg/cm²)]</pre>	Probable error (%)
527 704 718 739 741 755 757 915 935 941 949 951 971 1000 1007 1046 1047	$\begin{array}{c} (110) (111)$	$\begin{array}{c} 1.5\\ 2.0\\ 3.0\\ 2.0\\ 2.0\\ 1.5\\ 1.5\\ 3.0\\ 2.0\\ 2.0\\ 2.0\\ 2.0\\ 2.0\\ 2.0\\ 2.0\\ 2$	$\begin{array}{c} 408 \\ 410 \\ 454 \\ 502 \\ 503 \\ 542 \\ 648 \\ 648 \\ 648 \\ 716 \\ 717 \\ 750 \\ 914 \\ 942 \\ 973 \\ 974 \\ 1001 \end{array}$	$\begin{array}{c} 400\\ 400\\ 401\\ 371\\ 362\\ 296\\ 278\\ 296\\ 278\\ 275\\ 264\\ 257\\ 238\\ 219\\ 217\\ 207\\ 228\\ 209\\ \end{array}$	$\begin{array}{c} 4.0\\ 4.0\\ 2.5\\ 4.0\\ 4.0\\ 3.0\\ 4.5\\ 4.5\\ 4.5\\ 4.5\\ 4.5\\ 4.5\\ 4.5\\ 4.5$
Proton energy (kev)	(b) Copper Stopping power [kev/ (mg/cm <sup>2</sup> )]	Probable error (%)	Proton energy (kev)	(d) Neon Stopping power [kev/ (mg/cm <sup>2</sup> )]	Probable error (%)
446 532 603 713 755 812 949 996 1006 1050	169 158 150 140 137 132 120 115 112 113	$ \begin{array}{c} 1.5\\ 1.5\\ 1.5\\ 1.5\\ 2.0\\ 1.5\\ 2.0\\ 1.5\\ 2.0\\ 2.0\\ 2.0\\ \end{array} $	 422 424 517 519 668 730 733 929 930 988 992	310 290 280 269 236 217 217 186 191 175 188	$\begin{array}{c} 3.5\\ 3.5\\ 3.5\\ 3.5\\ 4.0\\ 4.0\\ 4.0\\ 4.0\\ 4.0\\ 4.0\\ 4.0\\ 4.0$
Proton energy (kev)	(e) Argon Stopping power [kev/ (mg/cm <sup>2</sup> )]	Probable error (%)	Proton energy (kev)	(g) Xenon Stopping power [kev/ (mg/cm²)]	Probable error (%)
421 476 519 567 662 713 778 929 932 971 989	225 224 215 207 176 187 176 154 155 154 149	$\begin{array}{c} 3.5\\ 2.0\\ 3.5\\ 2.5\\ 4.0\\ 2.5\\ 2.5\\ 4.0\\ 4.0\\ 3.0\\ 4.0\end{array}$	437 441 536 538 679 679 748 750 943 969	121 121 111 115 91 95 87 93 78 77	3.5 3.0 3.5 3.5 3.5 3.5 3.5 3.5 4.0 3.5
Proton energy (kev) 420	(f) Krypton Stopping power [kev/ (mg/cm <sup>2</sup> )] 163	Probable error (%) 3.5			
516	150	3.5			

TABLE I. Summary of data on stopping power for protons.

amount by the xenon requiring correction of about 1.5 percent.

527

665

675

733

743

929 941

989

154

130 129

124

122

107

107

103

4.0

3.5

4.0

and

The scattering of particles in the stopping medium adds to the path length of the particle in the medium and would thus introduce a possible error, particularly in case of the gases, since the amount of material traversed (foil plus gas) was largest in these cases. However, the half-angle subtended by the target (with vertex at the window foil) was small  $(5.6^{\circ})$ . Errors involved in ignoring the scattering factor were determined to be much smaller than the over-all experimental error.

#### **IV. RESULTS**

Table I lists for seven elements the measured values of the stopping power in kev/(mg/cm<sup>2</sup>) for several proton energies. The method by which the proton energy was assigned is described below.

When the average energy loss  $\langle \Delta E \rangle_{Av}$  in the stopping medium is very small compared to the initial proton energy  $E_i$ , it is adequate to assign the observed  $-\langle \Delta E \rangle_{Av} / \Delta x$  as -dE/dx for the arithmetic mean of the initial and exit energies of the beam. However, when the energy loss is appreciable compared to  $E_i$ , the observed rate of energy loss should be assigned to some intermediate energy  $E_p$  other than the average. Warshaw<sup>8</sup> has commented on this matter and has proposed a correction based on the assumption that the stopping power is a linear function of proton energy over a small region.

In view of the great energy losses ( $\sim 100$  to 300 kev) in traversing our unusually large amounts of stopping material, it is important to consider the actual shape of the stopping power curve when determining  $E_{\rm p}$ . In the energy region in question it is approximately true that

$$-dE/dx = kE^{-\gamma},$$
 (1)

where  $\gamma = 0.5$  according to Eq. (3.5.7) of Bohr's<sup>7</sup> paper. The thickness of medium traversed  $\Delta x$  is given by



FIG. 3. Atomic stopping power  $\sigma$  as a function of proton energy for seven elements. Wherever two values from Table I were too close to plot separately, the plotted point represents an average value. The light, dashed lines are the predictions of Hirschfelder and Magee (see reference 6) for Xe and A.

where  $E_i$  and  $E_f$  are the initial and final proton energies, respectively.

If  $E_p$  is the energy of the protons for which  $(E_i - E_f)/\Delta x$  is actually -dE/dx,

$$\left[\frac{dE}{dx}\right]_{E_n} = \frac{E_i - E_f}{\Delta x} \tag{3}$$

$$\Delta x = (E_i - E_f) / k E_p^{-\gamma}. \tag{4}$$

If the expressions for  $\Delta x$  [Eqs. (2) and (4)] are equated and the resultant expression is solved for  $E_p$ , one obtains

$$E_{p} = \left[\frac{1}{\gamma+1} \cdot \frac{E_{i}^{\gamma+1} - E_{f}^{\gamma+1}}{E_{i} - E_{f}}\right]^{1/\gamma}.$$
 (5)

If  $(E_i + E_f)/2 = E_a$  and  $E_i - E_f = \langle \Delta E \rangle_{AV}$ , Eq. (5) may be expanded to give

$$E_{p} = E_{a} \bigg\{ 1 + \frac{(\gamma - 1)}{24} \bigg( \frac{\langle \Delta E \rangle_{A_{V}}}{E_{a}} \bigg)^{2} + \text{other terms} \bigg\}.$$
(6)



FIG. 4. The ratio of atomic stopping power  $\sigma$  to  $Z^{\frac{1}{2}}$  as a function of proton energy for copper and nickel. For comparison, data derived from Kahn (see reference 15) are plotted for several metals to give the dashed curves.

The other terms involve higher powers of  $\langle \Delta E \rangle_{AV}/E_a$  and may be neglected when  $\langle \Delta E \rangle_{AV} \langle E_a$ .

The probable error of each experimental stopping power value has been computed and appears in Table I. For the foils the probable error in  $\langle \Delta E \rangle_{Av}$  was found to be about 2.2 kev and the error in foil thickness 1.1 percent. These values lead to total probable errors in the stopping power results of 1.5 to 3.0 percent. For the gases the error in  $\langle \Delta E \rangle_{Av}$  was taken as 3.0 kev, the total error in equivalent gas thickness, including the effects of impurities, varied from 1.4 to 1.7 percent. The total probable error in the individual values of gas stopping power varied from 2.0 to 4.5 percent. These individual probable errors are only partially independent, since certain measurements were common to calculations of several individual values.

#### **V. DISCUSSION OF RESULTS**

In Fig. 3 the atomic stopping power in  $ev-cm^2$  is plotted as a function of proton energy. Also shown are

the curves for xenon and argon predicted by Hirschfelder and Magee.<sup>6</sup> In both cases the experimental points lie below the theoretical curves by an amount somewhat greater than the computed probable error of the experimental points, the differences being of the order of 5 to 10 percent.

Bohr predicts [reference 7, Eq. (3.5.7)] that stopping power in this energy region is inversely proportional to particle velocity, so that a plot of stopping power versus proton energy should give a straight line on a logarithmic plot, with a slope of -0.5. From Fig. 3 it is clear that our data are in rough, though not precise, agreement with this prediction, as were previous data by Warshaw<sup>8</sup> in a lower energy range.

According to the same equation of Bohr referred to above, the atomic stopping power is proportional to the cube root of the atomic number for the heavier elements. If the values of atomic stopping power divided by  $Z^{\frac{1}{3}}$  are plotted as a function of proton energy for each element, the plots should coincide to the extent that the theoretical prediction is correct. The comparison between the stopping power values for various



FIG. 5. The ratio of atomic stopping power  $\sigma$  to  $Z^{\frac{1}{3}}$  as a function of proton energy for gases.

elements is made easier by such a plot. Figure 4 shows the data for the metals studied. Also included for the sake of comparison are curves for several metals derived from the results of Kahn.<sup>15</sup> It should be noted that our curves for nickel and copper are practically coincident. Kahn's curve for copper lies about 4 percent above ours. Madsen has also done some work on protons through copper,<sup>16</sup> but his curve lies in general about 13 percent below ours. The results on stopping power of metals provided by earlier workers<sup>11,17</sup> on beryllium and gold are not shown here, but are in substantial agreement with the work of Kahn.

In Fig. 5 the atomic stopping power  $\sigma$  divided by the cube root of the atomic number is shown as a function of proton energy for the gases studied. Weyl18 has reported data for protons with energies up to 450 kev



FIG. 6. The ratio of atomic stopping power  $\sigma$  to  $Z^{\frac{1}{2}}$  as a function of atomic number Z at proton energies of 500 and 1000 kev. The open circles represent the results of this paper for gases and the closed circles for metals. The crosses represent the measurements of Kahn (see reference 15) for metals.

passing through argon. His results coincide essentially with ours where the regions of study overlap. Dunbar, Reynolds, Wenzel, and Whaling<sup>19</sup> have studied all the gases reported on herein for proton energies up to 600 kev. Within the region of overlap our data agree well with theirs for nitrogen, neon, and argon. For krypton their results are about 5 percent lower than ours, but for xenon they are about 15 percent higher.<sup>20</sup>

From the curves given in Figs. 4 and 5, data were obtained for Fig. 6 in which stopping power divided by cube root of atomic number is plotted as a function of atomic number. This is done for two values of proton energy: 500 kev and 1000 kev. If  $\sigma/Z^{\frac{1}{3}}$  were constant, as Bohr's theory predicts for heavy elements, the curves would be horizontal lines. The curves fall off for low atomic numbers, but for Z > 25 they show the expected horizontal character.

It is of particular interest to note the discrepancy between the results for the gases and the metals for the proton energy of 500 kev. Figure 3 indicates that this discrepancy begins to be apparent if the proton energy decreases to below about 700 kev. This confirms the conjecture that if differences exist between gaseous and condensed media, they should show up most strongly at low proton velocities, since the relative importance of the outer electrons becomes greater as the proton velocity decreases. Whether the deviation of the metals studied is due to their conduction properties or simply to their condensed state is not known. Future experimental work on nonconducting media in the condensed state would be desirable to answer this question.

<sup>&</sup>lt;sup>15</sup> D. Kahn, Phys. Rev. 90, 503 (1953).

 <sup>&</sup>lt;sup>10</sup> C. B. Madsen (private communication, 1953).
 <sup>17</sup> T. Huus and C. B. Madsen, Phys. Rev. 76, 323 (1949).
 <sup>18</sup> P. K. Weyl, Phys. Rev. 91, 289 (1953).

<sup>&</sup>lt;sup>19</sup> Dunbar, Reynolds, Wenzel, and Whaling, Phys. Rev. **91**, 496 (1953) and **92**, 742 (1953).

<sup>&</sup>lt;sup>20</sup> This discrepancy for xenon was so far outside the probable error limit that a repetition of the measurements of energy loss in xenon and purity of the xenon sample was considered worth-while. The two values of stopping power which were obtained were essentially in agreement with our earlier results.



FIG. 7. Stopping power straggling (energy loss variance per unit thickness) as a function of proton energy. The dotted lines are calculated from theoretical expressions of Livingston and Bethe (see reference 3) and of Bohr (see reference 7).

### VI. STOPPING POWER STRAGGLING

Associated with the idea of stopping power is the concept of "straggling" in energy loss of similar particles in traversing a given amount of stopping material. This effect is due to the statistical nature of the stopping process. This straggling has been treated theoretically for slow particles by Livingston and Bethe<sup>3</sup> and by Bohr.<sup>7</sup> Very little experimental work has been done for low-energy protons. Madsen and Venkateswarlu<sup>11</sup> have measured the straggling in beryllium foils for protons with energies between 400 and 1200 kev.

If  $\Delta E$  is the energy loss of a particular particle in passing through a given layer of material and  $\langle \Delta E \rangle_{Av}$  is the average value of the energy loss for a number of identical particles of the same incident energy, the mean square fluctuation is defined as

$$\Omega^2 = \langle (\Delta E - \langle \Delta E \rangle_{Av})^2 \rangle_{Av}, \tag{7}$$

and the straggling in stopping power is given by  $d(\Omega^2)/dx$ . The straggling can be computed from the measured widths of the resonances in the shifted and unshifted yield curves (see reference 11).

The results on the straggling in energy loss of protons traversing the foils are shown in Table II. It was impossible to compute meaningful values of the straggling in gases from our experimental resonance curves. It is difficult to ascribe probable errors to the values tabulated for the straggling effect. The measurements are themselves of a statistical variable which is hardly greater than the errors themselves in energy losses. Furthermore, the errors are much more likely to be positive than negative, since any effects giving rise to variations in stopping power increase the computed value of straggling. The extent of the errors can be judged best by noting the spread in the individual points in the table.

In Fig. 7 the results are plotted along with calculations based on the theoretical expressions due to Bohr [reference 7, Eq. (3.5.8)] and to Livingston and Bethe [reference 3, Eq. (788)]. (The appropriate electron ionization energies were taken for  $I_n$  and 4/3 was used as the value for each  $\kappa_n$ .) The results are seen to be too erratic to do more than confirm the correctness in order of magnitude of both expressions. The formula of Bohr does seem to predict the general variation of the straggling with proton energy. The variations in foil thickness, even though small, may have a marked effect upon this straggling; such effect is obviously always to add to the observed straggling. This fact, coupled with the results of the thickness variation studies (outlined above) showing that the copper foils seem to be somewhat more uniform in thickness than the nickel foils, would seem to indicate strongly that the correct experimental curve for straggling is somewhat lower than the theoretical curves. The approximations made in developing the formula suggest that the predicted values might be too high for low proton energies.

TABLE II. Summary of data on the stopping power straggling of protons.

(a)	Nickel	(b) Copper		
Proton energy (kev)	Straggling [kev²/(mg/cm²)]	Proton energy (kev)	Straggling [kev²/(mg/cm²)	
527	89	446	34	
704	78	749	63	
739	68	755	57	
741	61	945	65	
757	54	949	72	
941	94	1004	76	
1046	116	1006	96	

### VII. ACKNOWLEDGMENTS

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Note added in proof: The authors have recently received Dr. C. B. Madsen's summary [Kgl. Danske Vid. Sels. Math.-fys. Medd. **27**, 13 (1953)] of his stopping power results for Be, Cu, Ag, and Au. Aside from the previously noted discrepancy for Cu, the results fit well into the  $\sigma/Z^{\frac{1}{3}}$  vs Z curves of Fig. 6. Furthermore, Madsen reports measurements for straggling in Cu which are consistent with those in Table II.