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Energy Loss and Straggling of Electrons*

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The energy-straggling distribution and the most probable energy loss for 15.7-Mev electrons in Cu is calculated. Corrections for the resonance effect, bremsstrahlung, the polarization effect, and multiple scattering are included. The results are in very satisfactory agreement with recent experiments. The discrepancy in the most probable energy loss in Au is also discussed.

RECENTLY very accurate experiments on the energy loss and straggling of 15.7-Mev electrons were carried out.[1] They show good agreement with theory for Al and lighter elements, but they disagree with theoretical expectations for Au.[2] No comparison of the Cu data with theory could be made since calculations for this element were not available. In this note we want to report on such calculations.

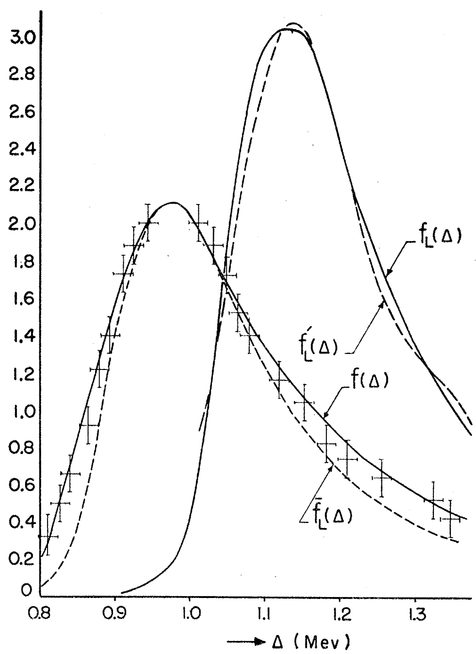

FIG. 1. Energy straggling distributions of 15.7-Mev electrons in Cu as a function of the energy loss Δ. Solid curves: Landau distribution $f_L(\Delta)$ and final theoretical distribution $f(\Delta)$, both normalized to unity. Dashed curves: Landau distribution $f_L(\Delta)$ with maximum coinciding with $f(\Delta)$ for the purpose of shape comparison and Blunck-Leisegang approximation $f_L'(\Delta)$ normalized to unity. (See reference 4.) The experimental results are from references 1 and 8.

The basic Landau distribution for energy straggling[3] was corrected for the following effects in this order:

(a) The resonance effect. The Williams-Landau theory does not take into account correctly those large but infrequent losses which result from ionization energies in the vicinity of S_0, the separation energy between soft and hard collisions.

(b) Bremsstrahlung. The Landau curve involves collision loss only.

(c) The polarization effect (density effect).

(d) Multiple scattering.

The first two effects will mainly broaden the distribution but will shift its maximum only slightly. The last two effects will only shift the whole distribution but not affect its shape.

For Cu the K-electron ionization energy $I_K < S_0 = 0.0589$ Mev. The Landau curve can be corrected, therefore, for the resonance effect by folding it into a gaussian distribution whose half-width is a certain average over the ionization energies of the atom.[4] The bremsstrahlung correction is then applied by folding the resultant distribution into the radiation loss distribution. The latter can easily be found to be of the form

$$f_B(\Delta)d\Delta = B\exp(-B\Delta)(B\Delta)^{xA-1}d\Delta/\Gamma(xA),$$

where Δ is the energy loss, x is the target thickness, and Γ is the gamma function. The constants A and B are found by approximating the cross section for bremsstrahlung $\sigma(\Delta)$ for small Δ,

$$\sigma(\Delta)d\Delta = A\exp(-B\Delta)d\Delta/\Delta.$$

This expression is a very good approximation in the Δ region of interest.[5]

The corrections (a) and (b) shift the maximum of the distribution from 1.130 Mev to 1.155 Mev. The polarization effect has recently been calculated very

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[1] Goldwasser, Mills, and Hanson, Phys. Rev. **88**, 1137 (1952).

[2] C. N. Yang and J. M. Kennedy (unpublished numerical work for Au).

[3] E. J. Williams, Proc. Roy. Soc. (London) **A125**, 445 (1929); L. Landau, J. Phys. (U.S.S.R.) **8**, 201 (1944).

[4] O. Blunck and S. Leisegang, Z. Physik **128**, 500 (1950). It should be pointed out here that the approximation to the Landau curve in terms of four gaussians is not at all as accurate (less than 2 percent) as claimed by these authors. The approximate curve, $f_L'(\Delta)$ is shown for comparison in Fig. 1.

[5] This approximation was first applied to electron straggling by W. Schultz, Z. Physik **129**, 530 (1951). For further details of this and the other corrections and for numerical values the reader is referred to the B.A. thesis of one of us (C.W.) (unpublished).

accurately by Sternheimer.[6] From his paper one finds for our case -0.187 Mev for this correction, yielding 0.968 Mev for the most probable energy loss, Δ_p. Finally, a small correction for multiple scattering should be applied. This can be done by replacing the target thickness by an effective thickness[7] which in this case increases Δ_p by 0.9 percent or 0.009 Mev. Thus, the final theoretical value in Cu is

$$\Delta_p = 0.977 \text{ Mev}.$$

The theoretical straggling distribution, $f(\Delta)$, is shown in Fig. 1 together with the uncorrected Landau curve $f_L(\Delta)$ (both normalized to unity). For the purpose of comparing the shapes of these two distributions, $f_L(\Delta)$ is also plotted in a reduced vertical scale and a shifted energy scale so that its maximum coincides with the maximum of $f(\Delta)$. The experimental points with errors of ± 0.015 Mev in energy and ± 5 percent in counting rate are also shown.[8] Both the shape and the most probable energy loss are seen to be in very satisfactory agreement.

If we use for Au the polarization correction of Sternheimer[6] (-0.134 Mev) instead of the approximate Fermi value (-0.055 Mev), we find $\Delta_p = 0.966$ Mev compared with an experimental value of 0.902 Mev. The reduction of the discrepancy from 0.143 Mev to 0.064 Mev is appreciable but not sufficient. However, the good agreement for all the other elements including Cu suggests that further experiments and a careful check of the calculations will yield agreement also for Au.

We are grateful to Dr. Yang for letting us see the unpublished calculations on Au by him and Dr. Kennedy and for two profitable discussions.

[6] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952), and **91**, 256 (1953).

[7] C. N. Yang, Phys. Rev. **84**, 599 (1951).

[8] We are grateful to Dr. Hanson for communicating these results to us.

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One-to-Two Millimeter Wave Spectroscopy. IV. Experimental Methods and Results for OCS, CH_3F, and H_2O†

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Design details are given of the harmonic generator and detector which made possible precision spectroscopy in the one-to-two mm wave region. The signal-to-noise ratio obtained on OCS rotational lines is 7 to 1 at 1.0-mm wavelength; 30 to 1 at 1.1 mm; and better than 100 to 1 at 1.4 mm and above. A useful tuning and measuring technique is made possible by the ability of the new system to detect several klystron harmonics at once and, hence, spectral lines in several different regions at the same time. The applications of the methods in the measurement of centrifugal distortion of molecules is illustrated with OCS, for which $D_J = 1.310 \pm 0.010$ kc/sec is obtained, and with CH_3F, for which $D_J = 57.8 \pm 1.0$ kc/sec and $D_{JK} = 445 \pm 4$ kc/sec are obtained. A new water-vapor line, the $2_{2,0} \rightarrow 3_{1,3}$ rotational line, has been measured at 183 311.30$\pm$0.30 Mc/sec.

INTRODUCTION

THE present paper is the fourth in a series reporting measurements of spectral lines in the one-to-two mm wave region. The experimental methods which made possible high-resolution spectroscopy in this essentially uncharted region of the electromagnetic spectrum will be described here in some detail. These methods were described earlier by one of us (W.G.) at the North Carolina Meeting of the American Physical Society (March, 1953) and briefly in the first paper of this series.[1]

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[1] W. C. King and W. Gordy, Phys. Rev. **90**, 319 (1953).

EXPERIMENTAL METHODS

Harmonic Generator and Detector

The instrument and techniques developed earlier in this laboratory[2] extended high-resolution spectroscopy down to approximately 2 mm. Like the earlier methods, the present ones employ crystal harmonic generators driven by klystrons as the source of energy and crystal rectifiers for detection. The improvement in performance results principally from the changes in details of design of these components. The limit of applicability of the earlier harmonic generators and detectors which employed commercial, coaxially mounted crystals was found to be near 2-mm wavelength. Some improvement in the multiplier design

[2] Smith, Gordy, Simmons, and Smith, Phys. Rev. **75**, 260 (1949); Gilliam, Johnson, and Gordy, Phys. Rev. **78**, 140 (1950).