domains of solutions appear when one considers the general interaction $sS+vV+T+aA+pP$ with $|s|<1.2$, $|v|<0.4$, $|a|<0.2$.

Although the simple connection between Fermi interactions adopted here includes most of the usual choices, it should be emphasized that there exists other possibilities no more and no less arbitrary (see, e.g., Caianiello, Finkelstein and Kaus). A complete discussion is found in the thesis of the first-named author.[5]

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Coulomb Corrections in Stripping*

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(Received November 20, 1953)

We wish to report some preliminary results from our investigation of the influences of Coulomb forces on the deuteron stripping cross section. It is, in general, still not obvious what new effects should be anticipated, but we can describe one case for which we used numerical methods to carry through a calculation of reliable accuracy. Although some preliminary ideas had suggested that in this case the Coulomb effects might be large, the angular distribution actually is found to agree surprisingly well with the non-Coulomb result.

Our method differs from the now-familiar approach of Butler[1] only in that the various wave functions are replaced by their Coulomb analogs. The Coulomb analog of a plane wave beam of deuterons must be treated approximately, as its exact derivation would require the solution of a three-body dynamical problem. We replace this wave function by $\chi(r)C(\mathbf{K},\mathbf{R})$, where $\chi(r)$ is the undistorted internal deuteron function, and $C(\mathbf{K},\mathbf{R})$ is the Coulomb wave function for a particle of deuteron mass and charge and incident wave vector $\mathbf{K}$. We have also been able to carry through a fairly good calculation of the stripping effects which this approximation omits—those resulting from the distortion of the deuteron by the Coulomb field. For the usual sorts of targets and bombarding energies these "polarization" effects are found small enough to be ignored safely.

Accepting the above approximation for the wave function of the incident deuteron, the only difficult step in the derivation is the computation of the "stripping transform." This is the transform which expresses the incident wave function as expanded into states in which the outgoing particle has definite linear momentum at large distances, and in which the captured particle has definite angular momentum. For example, for (d,p) reactions the stripping transform is

$$\int d\Omega_N d\mathbf{r}_p \chi(|\mathbf{r}_p-\mathbf{r}_N|)C\left(\mathbf{K},\frac{\mathbf{r}_p+\mathbf{r}_N}{2}\right)Y_{L,M}^*(\theta_N,\phi_N)C(-\mathbf{k}_p,\mathbf{r}_p). \quad (1)$$

We have not succeeded in finding any approximations to this integral which are both simple and also sufficiently accurate for the problem. Various approximate evaluations led us to believe that the Coulomb effects might very much change the stripping angular distribution, in contradiction to the familiar good fit between experiment and the non-Coulomb theory. It seemed desirable to check the approximations in one case by making a more accurate calculation, necessarily largely numerical, and quite lengthy.

The method of the more accurate calculation can only be employed for a (d,n) reaction, where $C(-\mathbf{k}_p,\mathbf{r}_p)$ in (1) is replaced by $\exp[-i(\mathbf{k}_N\cdot\mathbf{r}_N)]$. We first consider the integration over $\mathbf{r}_N$:

$$\int d\mathbf{r}_N\chi(r)C(\mathbf{K},\mathbf{R})\exp[-i(\mathbf{k}_N\cdot\mathbf{r}_N)]. \quad (2)$$

In this integral we go over to $\mathbf{r}\equiv\mathbf{r}_p-\mathbf{r}_N$ as the variable of integration, and replace C by the usual form of an exponential times a hypergeometric function. Then the hypergeometric function is written in integral representation. Thus C appears in the form

$$C(\mathbf{K},\mathbf{r}_p-\mathbf{r}/2)=\frac{e^{-\pi n/2}}{\Gamma(-in)}\int_0^1 du u^{-1-in}(1-u)^{in} \times\exp\{i(1-u)[\mathbf{K}\cdot(\mathbf{r}_p-\mathbf{r}/2)]\}\exp(iuK|\mathbf{r}_p-\mathbf{r}/2|). \quad (3)$$

Here $n=2Ze^2M/\hbar^2K$, the usual Coulomb parameter. Now we expand $|\mathbf{r}_p-\mathbf{r}/2|$ in (3), and carry only the first two terms, $r_p-(\mathbf{r}\cdot\mathbf{r}_p)/2r_p$. Having made only this quite reliable approximation it is possible to integrate over $\mathbf{r}$ in (2), leaving to be performed only the one-dimensional integral over u. Both this integral and the spherical harmonic expansion are now done numerically, it being possible to put the analytic expressions into a suitably convenient form for the numerical work.

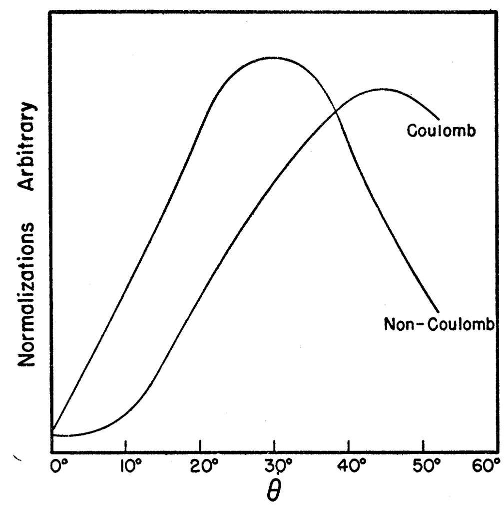

FIG. 1. Angular distributions computed for a (d,n) reaction on a nucleus of $Z\approx 15$. Angular momentum transfer, $l=2$. Incident energy, $E_d=8$ Mev. Outgoing energy, $E_n=12$ Mev. Coulomb parameter, $n_d=1.2$.

The particular case that we have worked through involves the capture of $l=2$ protons, an 8-Mev deuteron beam being incident on a nucleus of $Z\approx 15$, with the outgoing particles having 12 Mev, approximately corresponding to the $P^{31}(d,p)P^{32}$ reactions of Parkinson *et al.*[2] Figure 1 shows the resulting angular distribution, also including the non-Coulomb curve of the old theory. The peak of the Coulomb curve is somewhat displaced towards larger angles, but otherwise gives an angular distribution which is essentially the same as the non-Coulomb result, and in a region where large Coulomb changes were anticipated ($n=1.2$).

We are very grateful to Professor H. A. Bethe for his generous help and encouragement, and to Max Goldstein of the Los Alamos staff for performing the numerical calculations.

* Supported by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
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