TABLE I. Absorption edges (ry) and oscillator strengths for AgBr.

Band	Ag E (ry)	Ag f	Br E (ry)	Br f
$K(1s)^2$	1880	1.13	993	1.2
$L(2s)^2$	280.9	1.3	130.5	1.4
$(2p)^3$	260.2	1.5	117.8	1.5
$(2p)^3$	247.5	2.5	114.3	2.5
$M(3s)^2$	53.5	1.4	19.48	1.5
$(3p)^3$	45.0	2.4	13.60	2.5
$(3p)^3$	42.7	2.7	13.21	2.8
$(3d)^5$	28.1	9.0	5.27	9.6
$(3d)^5$	27.7		5.13	
$N(4s)^2$	7.67	1.8	1.76	2.3
$(4p)^6$	4.80	6.5	0.42	12.1
$(4d)^{10}$	0.85	14.4		

number of electrons. The Čerenkov radiation may be shown to contribute a very small proportion of the energy loss and to lie entirely in the visible, ultraviolet, and soft x-ray regions. Because of the shift of the oscillator strengths and the breadth of the absorption regions, the total energy loss saturates very rapidly as may be seen from the figure. The relative rise is much greater than that obtained by previous authors[6,7] who calculated the ionization loss by assuming low damping and introducing a mean ionization potential, underestimating the change in the relative magnitude of the contributions of the high-frequency transitions. The numerical value of the plateau loss is only a little over one-half as great as that obtained with the simpler approximation.[7]

The grain density produced by a fast charged particle moving through a photographic emulsion is presumably approximately proportional to the total number of electrons released in the grains. This quantity is, however, difficult to calculate directly. It has therefore been assumed that the grain density is proportional to the total kinetic energy of the primary ejected electrons, given by

$$\Gamma_K(\rho_0)=W_0\int\Sigma_i\,\Phi\frac{\tau_i}{\tau}(E-E_i)dE, \qquad (4)$$

where the E_i are the energies of the absorption edges and the integral is, as before, extended over all transfers less than 5000 ev. It relative value is shown in the figure with the grain density measurements of Daniel *et al.*[8] and of Morrish[9] for comparison. The curve predicts a slightly more rapid rate of rise and a somewhat smaller percentage rise from the minimum to the plateau value than is shown by the measurements. This discrepancy could result from an overestimate of the shift in the oscillator strengths.

A detailed account of this work will be published elsewhere.

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1 E. Fermi, Phys. Rev. **57**, 485 (1940).
2 J. R. Allen, Can. J. Phys. **31**, 218 (1953).
3 A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Co., Inc., New York, 1935), second edition.
4 H. H. Landolt and R. Börnstein, *Zahlenwerte und Funktionen aus Physik, Chemie, Astronomie* (J. Springer, Berlin, 1950), Vol. I.
5 M. Stobbe, Ann. Physik **7**, 661 (1930).
6 O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).
7 M. Huybrechts and M. Schönberg, Nuovo cimento **9**, 764 (1952).
8 Daniel, Davies, Mulvey, and Perkins, Phil. Mag. **43**, 753 (1952).
9 A. H. Morrish, Phys. Rev. **91**, 423 (1953).

μ-Meson Decay, β Radioactivity, and Universal Fermi Interaction

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RECENT experimental data on the μ-meson mass and the μ-meson decay spectrum suggest a reconsideration of the hypothesis of a universal Fermi interaction. The purpose of this letter is to show that the data on μ decay and β radioactivity can be reconciled with a universal Fermi interaction but that the most often proposed interactions are excluded.

The new data to which we refer are: *the μ-meson mass*[1]: $m_\mu=207.0\times0.4m_e$ or $207.1\pm1.1m_e$, for which we adopt:

$$m_\mu=207.0\pm1.0m_e \text{ (standard error)}$$

and *the shape of the energy spectrum of secondary electrons* from decay at rest of μ mesons. If the decay is governed by a Fermi interaction, the shape of the decay spectrum can be characterized by m_μ and a single parameter ρ. If the two neutrinos emitted in μ-meson decay are distinguishable ($\nu\neq$), $0\leq\rho\leq1$. If the two emitted neutrinos are identical ($\nu\equiv$), then $0\leq\rho\leq\frac{3}{4}$.[2] A statistical analysis of the shape of the μ-meson spectrum gives a relation between ρ and m_μ. (The various published values of ρ have then to be adjusted to the new m_μ value.) We adopt here the most recent determination of Vilain and Williams[3]:

$$\rho=0.50\pm0.12 \text{ (standard error).}$$

For the sake of simplicity, we shall apply these new data only to the discussion of the most often proposed linear combinations of the five interactions of β radioactivity, i.e., $\pm S+T\pm P$ or $\pm S+T\mp P$ (the usual notations of β radioactivity are used). These four chosen interactions seem not to be excluded by the β-radioactivity data to date (except that the relative sign of T and P could be determined from the radium E spectrum if the assumptions of Petschek and Marshak[4] are correct).

To compare μ decay and β radioactivity, we have to choose a one-to-one correspondence between the two sets of particles n, p, ϵ, ν, and μ, ϵ, ν, ν. The 4! possible correspondences fall into three essentially different classes of which the following are samples: (a) $pn\epsilon\nu\rightarrow\nu\mu\epsilon\nu$; (b) $pn\epsilon\nu$, $\mu\nu\epsilon\nu$; (c) $pn\epsilon\nu$, $\mu\epsilon\nu\nu$. The examples of (a) and (b) are the correspondences suggested by the triangle of Fermi interactions between the three pairs np, $\epsilon\nu$, $\mu\nu$. Such $\pm S+T\pm P$ or $\pm S+T\mp P$ interactions have been proposed by Caianiello,[5,6] and in previous papers cited in that paper:

	$\pm S+T\pm P$
Pursey[7]:	$-S+T-P$
Peaslee[8]:	$-S+T-P$
Konopinski and Mahmoud[9]:	$\pm S+T+P$
Finkelstein and Kaus[10]:	$-S+T-P$

all with $\nu\equiv$ and correspondence (b). These four chosen β-radioactivity interactions predict values[11] of ρ and τ_μ which have to be compared with experiment. These results are given in Table I

TABLE I. Values of the shape parameter ρ of μ decay and of the ratio λ of the ft values of μ decay and β radioactivity for various universal Fermi interactions.

	$\nu\neq$ (a) ρ	$\nu\neq$ (a) λ	$\nu\neq$ (b) ρ	$\nu\neq$ (b) λ	$\nu\neq$ (c) ρ	$\nu\neq$ (c) λ	$\nu\equiv$ (a) ρ	$\nu\equiv$ (a) λ	$\nu\equiv$ (b) ρ	$\nu\equiv$ (b) λ	$\nu\equiv$ (c) ρ	$\nu\equiv$ (c) λ
$S+T+P$	0	4/3	3/4	4/3	3/4	4/3	0	4/3	0	1/3	0	1/3
$-S+T-P$	3/4	4/3	0	4/3	3/4	4/3	0	1/3	0	4/3	0	1/3
$\pm S+T\mp P$	3/8	4/3	3/8	4/3	3/4	4/3	0.14	0.92	0.14	0.92	0	1/3

but instead of τ_μ, we shall use the parameter λ proposed by Michel,[12] which is essentially the ratio of ft values for μ and β decays.

We adopt here[13] $B=2650\pm10$ percent for the rate constant of β radioactivity. Using the μ-meson lifetime $\tau_\mu=(2.22\pm0.02)\times10^{-6}$ sec,[14] we obtain: $\lambda=1.16\pm0.12$. Table I shows that the closest to a fit is $\rho=3/8$, $\lambda=4/3$. These values are not in very good agreement with experiment. However, they cannot be safely excluded, because systematic errors in the theoretical evaluation of nuclear matrix elements of β radioactivity are possible. Nevertheless, it is clear that *the interactions proposed by the authors listed above are excluded.*

By considering more general interactions, one can obtain a very good fit for ρ and λ. For example, interactions of the type $\pm S+T+pP$ give essentially two possibilities: one with $\nu\neq$, $|p|<\frac{1}{2}$, (a) or (b), e.g., ${}_{b+}{}^{a-}S+T$, $\rho=0.54$, $\lambda=1.17$; the other with $\nu\equiv$, $|p|\sim3.5$, (a) or (b), e.g., ${}_{b-}{}^{a+}S+T\mp3.5P$, $\rho=0.58$, $\lambda=1.10$.

Apart from these possibilities and the above-mentioned borderline case [$\nu\neq$, $\pm S+T\mp pP$ with $p\sim1$, (a) or (b)], no new domains of solutions appear when one considers the general interaction $sS+vV+T+aA+pP$ with $|s|<1.2$, $|v|<0.4$, $|a|<0.2$.

Although the simple connection between Fermi interactions adopted here includes most of the usual choices, it should be emphasized that there exists other possibilities no more and no less arbitrary (see, e.g., Caianiello, Finkelstein and Kaus). A complete discussion is found in the thesis of the first-named author.[5]

[1] Smith, Birnbaum, and Barkas, Phys. Rev. 91, 765 (1953).
[2] L. Michel, Nature 163, 959 (1949).
[3] J. Vilain and R. W. Williams, Phys. Rev. 92, 1586 (1953).
[4] A. Petschek and R. Marshak, Phys. Rev. 85, 698 (1952).
[5] E. Caianiello, Nuovo cimento 10, 43 (1953).
[6] Caianiello's hypotheses are different from ours. In his case the λ values given in Table I must be multiplied by 2, to be compared with experiment.
[7] D. Pursey, Physica 18, 1017 (1952).
[8] D. C. Peaslee, Phys. Rev. 91, 1447 (1953).
[9] E. J. Konopinski and H. M. Mahmoud, Phys. Rev. 92, 1045 (1953). In fact, in their discussion, these authors compare ρ with experiment for an interaction of the type $sS+T+pP$, (a), $\nu\equiv$, with $0.55<|s|<1$.
[10] R. Finkelstein and P. Kaus, Phys. Rev. 92, 1316 (1953).
[11] L. Michel, thesis, Sorbonne, 1953 (unpublished).
[12] L. Michel, *Progress in Cosmic Ray Physics* (North Holland Publ. Company, Amsterdam, 1952), Chapter 3, Eq. (43).
[13] See A. Winther and O. Kofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, No. 14 (1953) for the definition and the determination of B. Their value agrees within the given error with an as yet unpublished value of Feenberg (private communication).
[14] W. E. Bell and E. P. Hincks, Phys. Rev. 84, 1243 (1951).

Coulomb Corrections in Stripping*

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We wish to report some preliminary results from our investigation of the influences of Coulomb forces on the deuteron stripping cross section. It is, in general, still not obvious what new effects should be anticipated, but we can describe one case for which we used numerical methods to carry through a calculation of reliable accuracy. Although some preliminary ideas had suggested that in this case the Coulomb effects might be large, the angular distribution actually is found to agree surprisingly well with the non-Coulomb result.

Our method differs from the now-familiar approach of Butler[1] only in that the various wave functions are replaced by their Coulomb analogs. The Coulomb analog of a plane wave beam of deuterons must be treated approximately, as its exact derivation would require the solution of a three-body dynamical problem. We replace this wave function by $\chi(r)C(\mathbf{K},\mathbf{R})$, where $\chi(r)$ is the undistorted internal deuteron function, and $C(\mathbf{K},\mathbf{R})$ is the Coulomb wave function for a particle of deuteron mass and charge and incident wave vector $\mathbf{K}$. We have also been able to carry through a fairly good calculation of the stripping effects which this approximation omits—those resulting from the distortion of the deuteron by the Coulomb field. For the usual sorts of targets and bombarding energies these "polarization" effects are found small enough to be ignored safely.

Accepting the above approximation for the wave function of the incident deuteron, the only difficult step in the derivation is the computation of the "stripping transform." This is the transform which expresses the incident wave function as expanded into states in which the outgoing particle has definite linear momentum at large distances, and in which the captured particle has definite angular momentum. For example, for (d,p) reactions the stripping transform is

$$\int d\Omega_N d\mathbf{r}_p \chi(|\mathbf{r}_p-\mathbf{r}_N|)C\left(\mathbf{K},\frac{\mathbf{r}_p+\mathbf{r}_N}{2}\right)Y_{L,M}^*(\theta_N,\phi_N)C(-\mathbf{k}_p,\mathbf{r}_p). \quad (1)$$

We have not succeeded in finding any approximations to this integral which are both simple and also sufficiently accurate for the problem. Various approximate evaluations led us to believe that the Coulomb effects might very much change the stripping angular distribution, in contradiction to the familiar good fit between experiment and the non-Coulomb theory. It seemed desirable to check the approximations in one case by making a more accurate calculation, necessarily largely numerical, and quite lengthy.

The method of the more accurate calculation can only be employed for a (d,n) reaction, where $C(-\mathbf{k}_p,\mathbf{r}_p)$ in (1) is replaced by $\exp[-i(\mathbf{k}_N\cdot\mathbf{r}_N)]$. We first consider the integration over $\mathbf{r}_N$:

$$\int d\mathbf{r}_N\chi(r)C(\mathbf{K},\mathbf{R})\exp[-i(\mathbf{k}_N\cdot\mathbf{r}_N)]. \quad (2)$$

In this integral we go over to $\mathbf{r}\equiv\mathbf{r}_p-\mathbf{r}_N$ as the variable of integration, and replace C by the usual form of an exponential times a hypergeometric function. Then the hypergeometric function is written in integral representation. Thus C appears in the form

$$C(\mathbf{K},\mathbf{r}_p-\mathbf{r}/2)=\frac{e^{-\pi n/2}}{\Gamma(-in)}\int_0^1 du u^{-1-in}(1-u)^{in} \times\exp\{i(1-u)[\mathbf{K}\cdot(\mathbf{r}_p-\mathbf{r}/2)]\}\exp(iuK|\mathbf{r}_p-\mathbf{r}/2|). \quad (3)$$

Here $n=2Ze^2M/\hbar^2K$, the usual Coulomb parameter. Now we expand $|\mathbf{r}_p-\mathbf{r}/2|$ in (3), and carry only the first two terms, $r_p-(\mathbf{r}\cdot\mathbf{r}_p)/2r_p$. Having made only this quite reliable approximation it is possible to integrate over $\mathbf{r}$ in (2), leaving to be performed only the one-dimensional integral over u. Both this integral and the spherical harmonic expansion are now done numerically, it being possible to put the analytic expressions into a suitably convenient form for the numerical work.

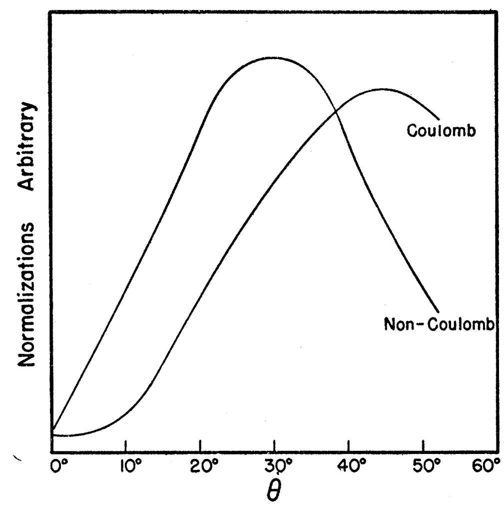

FIG. 1. Angular distributions computed for a (d,n) reaction on a nucleus of $Z\approx15$. Angular momentum transfer, $l=2$. Incident energy, $E_d=8$ Mev. Outgoing energy, $E_n=12$ Mev. Coulomb parameter, $n_d=1.2$.

The particular case that we have worked through involves the capture of $l=2$ protons, an 8-Mev deuteron beam being incident on a nucleus of $Z\approx15$, with the outgoing particles having 12 Mev, approximately corresponding to the $P^{31}(d,p)P^{32}$ reactions of Parkinson *et al.*[2] Figure 1 shows the resulting angular distribution, also including the non-Coulomb curve of the old theory. The peak of the Coulomb curve is somewhat displaced towards larger angles, but otherwise gives an angular distribution which is essentially the same as the non-Coulomb result, and in a region where large Coulomb changes were anticipated ($n=1.2$).

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[1] S. T. Butler, Proc. Roy. Soc. (London) 208, 559 (1951).
[2] Parkinson, Beach, and King, Phys. Rev. 87, 387 (1952); J. S. King and W. C. Parkinson, Phys. Rev. 88, 141 (1952); E. H. Beach and J. S. King, Phys. Rev. 90, 381 (1953).