If this is done, one arrives at a problem that was solved ten years ago by Wannier,[3] and the resulting theory of susceptibility is essentially that already given by Dingle.[4]

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The Density Effect for the Ionization Loss at Low Energies*

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THE density effect for the ionization loss of charged particles has been evaluated recently for a number of substances.[1] At low energies, the density effect is given by[1]

$$\Delta\frac{dE}{dx}=\frac{2\pi n_0e^4}{mv^2}\sum_j f_j\ln\left(\frac{l_j^2}{\nu_j^2}\right), \tag{1}$$

where n_0 is the electronic density, f_j and ν_j are the oscillator strength and the atomic frequency [in units $\nu_p\equiv(n_0e^2/\pi m)^{\frac{1}{2}}$] for the jth transition; l_j is given by

$$l_j=(\nu_j^2+f_j)^{\frac{1}{2}}. \tag{2}$$

In the experiment of Bakker and Segrè[2] on the stopping power for 340-Mev protons, this density effect was included, so that this experiment measures the ionization potential,[3]

$$I_{BS}=h\nu_p\prod_j l_j^{f_j}, \tag{3}$$

rather than the ionization potential for the isolated atom, $I=h\nu_p\Pi_j\,\nu_j^{f_j}$. When the values of I_{BS} are used to calculate the ionization loss, the density effect correction is given by

$$\delta=\sum_j f_j\ln\left(\frac{l_j^2+l^2}{\nu_j^2}\right)-l^2(1-\beta^2)-\sum_j f_j\ln\frac{l_j^2}{\nu_j^2}, \tag{4}$$

where l is determined by the equation:

$$\beta^{-2}-1=\sum_j f_j/(\nu_j^2+l^2). \tag{5}$$

The first two terms of (4) give the correction which would have to be applied if the atomic ionization potential were used [see Eq. (46) of A]. The last term is due to the density effect already included in I_{BS}. Equation (4) can be written

$$\delta=\sum_j f_j\ln\left(\frac{l_j^2+l^2}{l_j^2}\right)-l^2(1-\beta^2), \tag{6}$$

where the l_j are such that Eq. (3) is satisfied. This procedure was used in A to calculate δ and gives exact results for the case of solids. However, for gases the density effect at low energies is negligibly small so that the atomic ionization potential I should be used rather than I_{BS}. In A the values of the ionization potential for gases were obtained by interpolation of I_{BS} for neighboring substances in the periodic table. The correction $I_{BS}-I$ is very small. In view of (1) and (2), I/I_{BS} is given by $\exp(-D/2)$, where

$$D=\sum_j f_j\ln\left(1+\frac{f_j}{\nu_j^2}\right), \tag{7}$$

and $(2\pi n_0e^4/mv^2)D$ is the amount by which dE/dx for gases exceeds the value calculated using I_{BS}. D was calculated for some of the substances listed in Table I of A, using the ionization potentials and the f_j which are given in this table. The results are: $D(\text{Li})=0.34$, $D(\text{C})=0.22$, $D(\text{Al})=0.056$, $D(\text{Fe})=0.14$, $D(\text{Cu})=0.13$, $D(\text{Ag})=0.09$, $D(\text{Sn})=0.05$, $D(\text{W})=0.07$. By interpolation one finds: $D(\text{N}_2)=0.20$, $D(\text{O}_2)=0.17$, $D(\text{Ne})=0.13$, $D(\text{A})=0.09$, $D(\text{Kr})=0.11$, $D(\text{Xe})=0.05$.

It should be emphasized that these values of D are considerably uncertain because of the sensitivity of D to the distribution of the low frequencies ν_j which correspond to excitation of the outer electron shells. An alternative method of obtaining D is to deduce the effective ionization potential I_j of the outermost electron shell for the gas from the observed index of refraction n in the optical region,[4] which is given by:

$$n=1+\frac{f_j}{2[I_j/(h\nu_p)_{\text{gas}}]^2}, \tag{8}$$

where $f_j=N_j/Z$ and N_j is the number of valence electrons which was taken as the number of electrons with the highest principal quantum number. Thus, for[4] Kr, $n=1.00043$, $f_j=8/36$, and $h\nu_p=0.085$ ry lead to $I_j=1.37$ ry. The density effect which would be measured for this dispersion oscillator in a solid is given by:

$$D=f_j\ln\left\{1+\frac{f_j}{[I_j/(h\nu_p)_{\text{solid}}]^2}\right\}, \tag{9}$$

where $(h\nu_p)_{\text{solid}}$ is the average of $h\nu_p$ for the neighboring solids measured by Bakker and Segrè.[2] Equation (9) gives: $D(\text{N}_2)=0.53$, $D(\text{O}_2)=0.48$, $D(\text{Ne})=0.24$, $D(\text{Kr})=0.26$, $D(\text{Xe})=0.17$. A comparison of these values with those obtained above indicates the uncertainty in D. However, it should be noted that even with the larger values obtained from the index of refraction the correction is quite small. D may be compared with the square bracket of Eq. (11) of A for dE/dx which is $\sim$20. Thus, $D=0.5$ corresponds to $a\sim 2.5$ percent increase of dE/dx. This correction is hardly outside the limits of error owing to the uncertainty of the experimental values[2,5] of I.

In view of the smallness of D and the uncertainty about its value, it seems questionable whether this correction should be applied at present to the ionization loss for gases.[6] For high energies ($p/\mu c\gtrsim 100$) when the density effect for the gas is important, D is smaller than Eq. (7) and becomes zero when the ionization loss has attained saturation (dE/dx independent of I).

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Coulomb Excitation of Heavy and Medium Heavy Nuclei by Alpha Particles*

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WE wish to report some preliminary results concerning the Coulomb excitation of some 35 nuclei between $Z=20$ and $Z=90$ by both alpha particles and protons with energies up to 3.8 Mev. Recent work on this process induced by protons in tantalum, tungsten, and a few other heavy elements[1,2] has pointed up the great usefulness of this method in studying transition probabilities between low-lying nuclear energy levels. It seemed desirable to extend the scope of these investigations, especially since such very definite predictions are made concerning the properties of many of these excited states by the collective model of the nucleus.[3,4]

Because of their higher charge and lower velocity for a given energy, alpha particles are eminently suited for the electric excitation of nuclei of lower atomic number since the condition for the simplified classical treatment of the process,[5,6] $2Z_1Z_2e^2/\hbar v$