If this is done, one arrives at a problem that was solved ten years ago by Wannier,[3] and the resulting theory of susceptibility is essentially that already given by Dingle.[4]

[1] W. Band, Phys. Rev. **91**, 249 (1953).
[2] R. Courant and D. Hilbert, *Methoden der Mathematischen Physik* (J. Springer, Berlin, 1931), second edition, Vol. I, Chap. 6, Sec. 4.
[3] G. H. Wannier, Phys. Rev. **64**, 358 (1943).
[4] R. B. Dingle, Proc. Roy. Soc. (London) **A212**, 47 (1952).

The Density Effect for the Ionization Loss at Low Energies*

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THE density effect for the ionization loss of charged particles has been evaluated recently for a number of substances.[1] At low energies, the density effect is given by[1]

$$\Delta\frac{dE}{dx}=\frac{2\pi n_0e^4}{mv^2}\sum_j f_j\ln\left(\frac{l_j^2}{\nu_j^2}\right), \tag{1}$$

where n_0 is the electronic density, f_j and ν_j are the oscillator strength and the atomic frequency [in units $\nu_p\equiv(n_0e^2/\pi m)^{\frac{1}{2}}$] for the jth transition; l_j is given by

$$l_j=(\nu_j^2+f_j)^{\frac{1}{2}}. \tag{2}$$

In the experiment of Bakker and Segrè[2] on the stopping power for 340-Mev protons, this density effect was included, so that this experiment measures the ionization potential,[3]

$$I_{BS}=h\nu_p\prod_j l_j^{f_j}, \tag{3}$$

rather than the ionization potential for the isolated atom, $I=h\nu_p\prod_j\nu_j^{f_j}$. When the values of I_{BS} are used to calculate the ionization loss, the density effect correction is given by

$$\delta=\sum_j f_j\ln\left(\frac{l_j^2+l^2}{\nu_j^2}\right)-l^2(1-\beta^2)-\sum_j f_j\ln\frac{l_j^2}{\nu_j^2}, \tag{4}$$

where l is determined by the equation:

$$\beta^{-2}-1=\sum_j f_j/(\nu_j^2+l^2). \tag{5}$$

The first two terms of (4) give the correction which would have to be applied if the atomic ionization potential were used [see Eq. (46) of A]. The last term is due to the density effect already included in I_{BS}. Equation (4) can be written

$$\delta=\sum_j f_j\ln\left(\frac{l_j^2+l^2}{l_j^2}\right)-l^2(1-\beta^2), \tag{6}$$

where the l_j are such that Eq. (3) is satisfied. This procedure was used in A to calculate δ and gives exact results for the case of solids. However, for gases the density effect at low energies is negligibly small so that the atomic ionization potential I should be used rather than I_{BS}. In A the values of the ionization potential for gases were obtained by interpolation of I_{BS} for neighboring substances in the periodic table. The correction $I_{BS}-I$ is very small. In view of (1) and (2), I/I_{BS} is given by $\exp(-D/2)$, where

$$D=\sum_j f_j\ln\left(1+\frac{f_j}{\nu_j^2}\right), \tag{7}$$

and $(2\pi n_0e^4/mv^2)D$ is the amount by which dE/dx for gases exceeds the value calculated using I_{BS}. D was calculated for some of the substances listed in Table I of A, using the ionization potentials and the f_j which are given in this table. The results are: $D(\mathrm{Li})=0.34$, $D(\mathrm{C})=0.22$, $D(\mathrm{Al})=0.056$, $D(\mathrm{Fe})=0.14$, $D(\mathrm{Cu})=0.13$, $D(\mathrm{Ag})=0.09$, $D(\mathrm{Sn})=0.05$, $D(\mathrm{W})=0.07$. By interpolation one finds: $D(\mathrm{N_2})=0.20$, $D(\mathrm{O_2})=0.17$, $D(\mathrm{Ne})=0.13$, $D(\mathrm{A})=0.09$, $D(\mathrm{Kr})=0.11$, $D(\mathrm{Xe})=0.05$.

It should be emphasized that these values of D are considerably uncertain because of the sensitivity of D to the distribution of the low frequencies ν_j which correspond to excitation of the outer electron shells. An alternative method of obtaining D is to deduce the effective ionization potential I_j of the outermost electron shell for the gas from the observed index of refraction n in the optical region,[4] which is given by:

$$n=1+\frac{f_j}{2[I_j/(h\nu_p)_{\mathrm{gas}}]^2}, \tag{8}$$

where $f_j=N_j/Z$ and N_j is the number of valence electrons which was taken as the number of electrons with the highest principal quantum number. Thus, for[4] Kr, $n=1.00043$, $f_j=8/36$, and $h\nu_p=0.085$ ry lead to $I_j=1.37$ ry. The density effect which would be measured for this dispersion oscillator in a solid is given by:

$$D=f_j\ln\left\{1+\frac{f_j}{[I_j/(h\nu_p)_{\mathrm{solid}}]^2}\right\}, \tag{9}$$

where $(h\nu_p)_{\mathrm{solid}}$ is the average of $h\nu_p$ for the neighboring solids measured by Bakker and Segrè.[2] Equation (9) gives: $D(\mathrm{N_2})=0.53$, $D(\mathrm{O_2})=0.48$, $D(\mathrm{Ne})=0.24$, $D(\mathrm{Kr})=0.26$, $D(\mathrm{Xe})=0.17$. A comparison of these values with those obtained above indicates the uncertainty in D. However, it should be noted that even with the larger values obtained from the index of refraction the correction is quite small. D may be compared with the square bracket of Eq. (11) of A for dE/dx which is $\sim$20. Thus, $D=0.5$ corresponds to $a\sim2.5$ percent increase of dE/dx. This correction is hardly outside the limits of error owing to the uncertainty of the experimental values[2,5] of I.

In view of the smallness of D and the uncertainty about its value, it seems questionable whether this correction should be applied at present to the ionization loss for gases.[6] For high energies ($p/\mu c\gtrsim100$) when the density effect for the gas is important, D is smaller than Eq. (7) and becomes zero when the ionization loss has attained saturation (dE/dx independent of I).

I would like to thank Dr. A. O. Hanson and Dr. G. N. Whyte for pointing out the existence of the correction for the ionization potential of gases.

* Work done under the auspices of the U. S. Atomic Energy Commission.
[1] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952). Unless otherwise indicated, we use here the same notation as in that paper, which will be referred to as A.
[2] C. J. Bakker and E. Segrè, Phys. Rev. **81**, 489 (1951).
[3] Goldwasser, Mills, and Hanson, Phys. Rev. **88**, 1137 (1952).
[4] H. H. Landolt and R. Bornstein, *Physikalisch-Chemische Tabellen* (Julius Springer, Berlin, 1923), fifth edition, Vol. 2, p. 961.
[5] R. Mather and E. Segrè, Phys. Rev. **84**, 191 (1951); D. C. Sachs and J. R. Richardson, Phys. Rev. **89**, 1163 (1953).
[6] We note that the Lorentz term and the damping effect (see reference 1) introduce additional corrections which may be of the same order as D.

Coulomb Excitation of Heavy and Medium Heavy Nuclei by Alpha Particles*

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WE wish to report some preliminary results concerning the Coulomb excitation of some 35 nuclei between $Z=20$ and $Z=90$ by both alpha particles and protons with energies up to 3.8 Mev. Recent work on this process induced by protons in tantalum, tungsten, and a few other heavy elements[1,2] has pointed up the great usefulness of this method in studying transition probabilities between low-lying nuclear energy levels. It seemed desirable to extend the scope of these investigations, especially since such very definite predictions are made concerning the properties of many of these excited states by the collective model of the nucleus.[3,4]

Because of their higher charge and lower velocity for a given energy, alpha particles are eminently suited for the electric excitation of nuclei of lower atomic number since the condition for the simplified classical treatment of the process,[5,6] $2Z_1Z_2e^2/\hbar v$

$\gg 1$ (Z_1 and Z_2 are the charges of projectile and target, v is their relative velocity), is well satisfied down to the lower end of the periodic table. A further advantage in the use of helium ions lies in the fact that they turn out to be relatively much less effective in exciting the troublesome characteristic K x-radiation as compared with the nuclear gamma radiation in the targets; for instance, at 3 Mev the relative yields of K x-rays and 137-kev gamma rays from tantalum are 2.27 for alpha particles and 14.8 for protons. Background problems are reduced by several orders of magnitude compared with proton excitation,[7] permitting us to operate with solid angles approaching 2π.

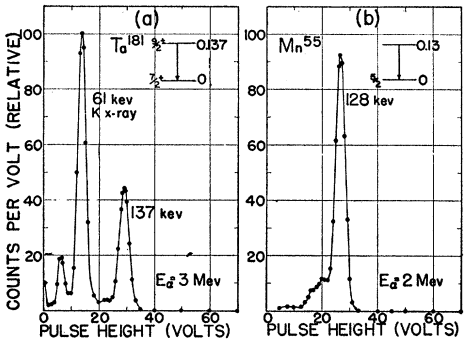

FIG. 1. (a) Pulse-height distribution of 137-kev gamma radiation and 61-kev K x-radiation from Ta^{181} bombarded by 3-Mev alphas. No absorber. (b) Pulse-height distribution of 128-kev gamma radiation from Mn^{55} bombarded by 2-Mev alphas. For previous level information, see reference 8.

Our experimental set-up is very simple. The beam from our electrostatic generator strikes the target which is thick to the incident particles but thin to the emerging radiation. Either a 1-in. or 2-in. thick NaI(Tl) crystal (depending on the energy of the gamma radiation under study), separated from the target by about 0.040 in. aluminum and mounted on a Dumont 6292 photomultiplier tube, serves as the gamma-ray detector. This entire assembly is surrounded by a 1-in. layer of lead. The output is fed through a conventional linear amplifier to a single-channel pulse-height analyzer. A one-volt channel width is used throughout. The known gamma-ray lines of ionium (Th^{230}) at 68 kev and 142 kev, In^{114} at 190 kev, Cd^{111} at 170 kev and 247 kev, Na^{22} at 0.511 Mev (annihilation) and 1.28 Mev, and Cs^{137} at 662 kev are used for energy calibration of the system. We are able satisfactorily to detect radiation down to about 10 kev. Figure 1(a) shows the pulse-height distribution we obtain with a 5-mil tantalum target bombarded with 3-Mev alphas, exhibiting the K x-ray line at about 61 kev and the 137-kev gamma ray. Figure 1(b) gives a similar plot for a metallic 35-mil manganese target showing a line at 128 kev.[8] Space does not permit our including the spectra for all the targets we have examined; we shall do so in our more complete publication.

TABLE I. Survey of levels below 500 kev observed by Coulomb excitation with 3-Mev alphas. Approximate intensity relative to Ta^{181} (*uncorrected* for relative abundance and internal conversion).

Element[a]	Abundance %	E_γ (observed) kev	Approx. int.	E_γ (known)[b] kev
$_3Li^{7}$,[c]	92.5	478	···	478
$_9F^{19}$,[c]	100	108, 196	···	110, 190
		1280	···	1280
$_{11}Na^{23}$,[c]	100	430	···	440
$_{21}Sc^{45}$	100	388	0.1	450
$_{22}Ti^{47}$	7.8	155, 433	0.5, 0.08	185
$_{23}V^{51}$	99.8	320	0.24	320
$_{25}Mn^{55}$	100	128	10	130
$_{26}Fe^{57}$	2.2	122	0.14	14, 117, 131
$_{30}Zn^{67}$	4.1	93, 182	0.02, 0.06	92, 182
$_{32}Ge^{73}$	7.7	68	0.23	54
$_{33}As^{75}$	100	68, 199, 283	0.02, 0.4, 0.3	67, 202, 281
$_{34}Se^{77}$	7.6	(155), 237, 440	0.15, 0.2	160, 237
$_{37}Rb^{85,87}$	72, 28	150	0.05	150, 410
$_{38}Sr$?	···	362	0.07	?
$_{40}Zr$[d] (+$_{72}Hf^{176,180}$)	1.5[d]	90[d]	0.1[d]	89, 93[d]
$_{42}Mo^{95}$	15.7	198	0.05	200
$_{45}Rh^{103}$	100	305, 370	0.1, 0.04	40
$_{46}Pd^{105}$	22.6	68?	0.02	63
$_{48}Cd^{111}$	12.8	300?	0.01	247, 340
$_{49}In^{115}$,[e]	95.8	500	···[e]	500[e]
$_{51}Sb^{123}$	42.8	~160	0.02	153
$_{55}Cs^{133}$	100	85	0.6	81
$_{59}Pr^{141}$	100	~150	0.01	145
$_{62}Sm^{152}$	26.6	119	2.5	122
$_{70}Yb^{170}$	3.0	82	0.6	84
$_{73}Ta^{181}$	100	137	*1.00*	137
$_{74}W^{182,184,186}$	85.4	~120	1.3	102, 113, 124[f]
$_{81}Tl^{203}$	29.5	220?	0.003	279
$_{90}Th^{232}$	100	50	0.20	50

[a] The following elements did not show lines below 500 kev with alphas: Mg, Al, Si, P, S, Cl, K, Ca, Cr, Co, Ni, Cu, Ga, Nb, Ag, Te, Pt, Au, Pb, Bi.
[b] See reference 9.
[c] Via compound nucleus. For $Z<20$ see text.
[d] All entries refer to Hf impurity (3%).
[e] With protons only.
[f] T. Huus and J. H. Bjerregaard, Phys. Rev. **92**, 1579 (1953).

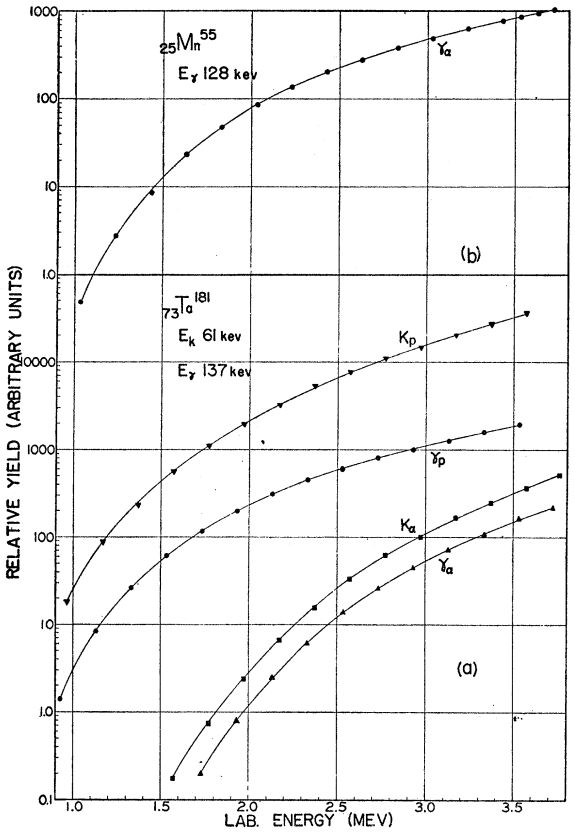

FIG. 2. (a) Excitation functions for K x-rays and gamma rays from $_{73}Ta^{181}$. $K_p = K$ x-ray yield (protons); γ_p = 137-kev gamma-ray yield (protons); $K_\alpha = K$ x-ray yield (alphas); γ_α = 137-kev gamma-ray yield (alphas). γ_p and γ_α curves are *theoretical* thick target yield curves, normalized at the highest energy points. (b) Coulomb excitation of the 128-kev gamma ray from $_{25}Mn^{55}$ by alpha particles. Solid curve is *theoretical* thick target yield curve, normalized at the highest energy. One-half the excitation energy has been subtracted before plotting experimental points in both (a) and (b).

The relative yields of K x-rays and gamma rays from tantalum under both proton and alpha-particle bombardment are shown in Fig. 2(a), illustrating the point mentioned above. The alpha-particle excitation function for one of the lighter nuclei ($_{25}Mn^{55}$) is shown in Fig. 2(b). Note that the curves for gamma excitation by protons and alphas in tantalum and by alphas in manganese are *theoretical* thick-target curves, calculated by numerically integrating the theoretical cross sections[6] over the particle ranges and normalized at the experimental points of highest energy. The agreement is indeed gratifying, completely confirming the

nature of the excitation process down to at least $Z=25$ for alphas and up to 3.6 Mev for protons on tantalum ($Z=73$). We postpone a detailed discussion of the results on the various nuclei we have investigated and restrict ourselves to a tabular presentation of the salient features in Table I. We list, in turn, the target nucleus, mass number, and relative abundance of the isotope believed responsible for the radiation, gamma-ray energy observed and energy levels known from other sources,[9] and a qualitative estimate of the relative yield of the observed radiation under 3-Mev alpha bombardment, *uncorrected* for internal conversion. The highest line excited was at 500 kev (In^{115}).

Elements with $Z<20$ are included here mainly for the purpose of demonstrating the main sources of radiation from possible light-element impurities. Li, F, and Na are found to be the only light elements yielding gamma rays below 500 kev under alpha bombardment. The excitations of the 478-kev state in Li^7 and the first two excited states in F^{19} at 108 kev and 196 kev (mainly by ordinary interlastic scattering of alphas involving compound nucleus formation with sharp resonances) have been studied and will be the subject of another publication.[10] In this connection, we have found striking evidence for Coulomb excitation of the 196-kev level in fluorine (second excited state) by alphas below 2.2 Mev before the onset of the resonances. This phenomenon is not observable with protons because of the complicated resonance structure which then extends down to very low energies.

We are extending our measurements into the rare-earth region where many nuclei have low-lying excited states. Our final results will include determinations of absolute yields and hence values of the reduced transition probabilities $B_\epsilon(2)$,[6] as well as a comparison of the experimental facts with the rotational interpretation[3,4] of these levels.

* A preliminary account of this work will be presented at the New York meeting of the American Physical Society, January 28–30, 1954.

[1] T. Huus and C. Zupančič, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 1 (1953).
[2] C. McClelland and C. Goodman, Phys. Rev. **91**, 760 (1953).
[3] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).
[4] D. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
[5] K. A. Ter-Martirosyan, J. Exptl. Theor. Phys. (U.S.S.R.) **22**, 284 (1952).
[6] K. Alder and A. Winther, Phys. Rev. **91**, 1578 (1953).
[7] We observe (capture) gamma radiation with energies greater than 4 Mev from most medium-heavy nuclei under proton bombardment.
[8] This level is known only from $Mn^{55}(p,p')Mn^{55*}$; see Hausman, Allen, Arthur, Bender, and McDole, Phys. Rev. **88**, 1296 (1951).
[9] Mostly as collected in Way, Fano, Scott, and Thew, *Nuclear Data*, National Bureau of Standards Circular 499 (U. S. Government Printing Office, Washington, D. C., 1950) and supplements; and Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).
[10] N. P. Heydenburg and G. M. Temmer, Phys. Rev. (to be published); a preliminary account will be presented at the New York meeting of the American Physical Society, January 28–30, 1954.

The Specific Ionization and Energy Loss of a Fast Charged Particle

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IT may be shown that the primary ionization and excitation density j_P (i.e., the average number of electrons excited or ejected per unit volume either directly or by absorption of Čerenkov radiation), because of the passage of a fast charged particle through a medium, is, at distances greater than ρ_0, the minimum impact parameter, given by

$$j_P=\frac{c}{h}\int\frac{\tau\mathfrak{E}^*\cdot\mathfrak{E}}{\omega}d\omega. \tag{1}$$

In this equation, the particle's field is treated as a perturbation in a semiclassical approximation; $\tau(\omega)$ is the photoelectric absorption coefficient; $\mathfrak{E}(\omega,\rho)$ is the Fourier time transform of the effective electric field.

If $\mathfrak{E}$ is calculated by the phenomenological approximation due to Fermi,[1] Eq. (1) and the similarly derived equation for the energy absorption density may be integrated over all impact parameters greater than ρ_0. We obtain for the total primary ionization and excitation ($\rho>\rho_0$)

$$P(\rho_0)=\int\Phi dE,\quad \tau\neq0;\quad P=0,\quad \tau=0 \tag{2}$$

and for the total energy absorption ($\rho>\rho_0$)

$$\Gamma_T(\rho_0)=W_0\int\Phi E dE,\quad \tau\neq0;\quad \Gamma_T=0,\quad \tau=0, \tag{3}$$

where

$$\Phi=\frac{\alpha^2Z^2}{\pi a_0\beta^2}\frac{g}{\kappa^*\kappa}\,\mathrm{Im}[\kappa\zeta^*K_0^*(\zeta)K_1(\zeta)];$$

$\kappa=\epsilon'+i\epsilon''$ is the complex dielectric constant; $g=(\eta^2+\xi^2)^{\frac{1}{2}}$; $\eta=1-\beta^2\epsilon'$, $\xi=\beta^2\epsilon''$; $\zeta=[\alpha\rho_0E/2\sqrt{2}a_0\beta][\sqrt{(g+\eta)}-i\sqrt{(g-\eta)}]$; W_0 is the Rydberg energy; a_0 is the Bohr radius; and E, the energy of the field components, is in rydbergs. Equation (3) is equivalent to Fermi's formula (22)[1] when $\tau\neq0$.

The absorption coefficient τ and the dielectric constant ϵ' were obtained for silver bromide by the methods described below and Eqs. (2) and (3) were integrated numerically for all transfers less than 5000 ev. The results relative to the plateau values are shown in Fig. 1 plotted against the kinetic energy of the particle

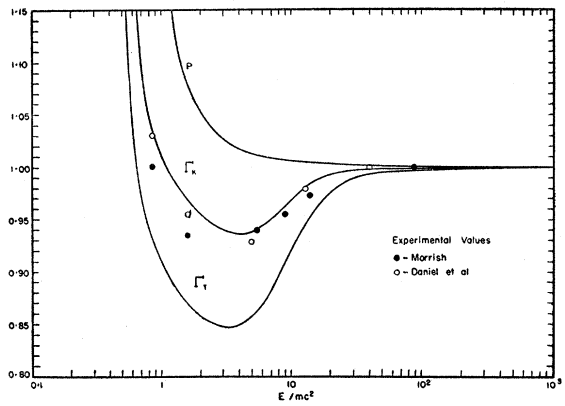

FIG. 1. P, the primary ionization and excitation; Γ_T, the energy absorption; and Γ_K, the kinetic energy of the primary electrons: computed for a singly charged particle (transfers <5000 ev, distances $>2a_0$) and shown relative to the plateau values $P(\beta=1)=50\,690$/cm, $\Gamma_T(\beta=1)=3.31$ Mev/cm, $\Gamma_K(\beta=1)=1.21$ Mev/cm as functions of E/mc^2 (E = kinetic energy). Experimental values of relative grain density are due to Daniel *et al.* (reference 8) (marked ○) and Morrish (reference 9) (marked ●).

in units of mc^2. A discussion of the absorption resulting from the valence band of silver bromide has already been published,[2] and it was shown that the large polarizability of this salt results in a considerable modification in the shape of the absorption bands. Also, partly because of the large polarizability and partly because of the effect of the exclusion principle, there is a shift in the oscillator strengths towards the lower frequency bands. These considerations have been extended to the calculation of the absorption from the next few bands lying below the valence levels. The shape of the absorption curves being assumed to be in accordance with these calculations, their magnitudes were checked by a comparison of the computed with the empirically observed dispersion in the visible and ultraviolet. The oscillator strengths and absorption coefficients for the far x-ray region were taken from experimental values,[3,4] whereas for intermediate frequencies the absorption was calculated by the methods of Stobbe[5] using appropriate screening constants and a correction for the polarization effect near the absorption edge. The values of the absorption edges (in Rydberg units) and the total oscillator strengths for the various bands are shown in Table I. The dielectric constant was computed from the absorption coefficient.

These detailed expressions for τ and ϵ' change considerably the conclusions obtained when a simple line absorption model is used, and the oscillator strengths are assumed proportional to the