processes occurring in rarefied gases, it is desirable to have a method of solution which deals with the entire range of pressures in a unified manner and which can satisfy general microscopic boundary conditions. The present communication sketches such a method for the special case of Maxwellian molecules, which repel each other with an inverse fifth-power law of force.

The Boltzmann equation governing the distribution function $f(\mathbf{v},\mathbf{x},t)$, in the absence of external forces, is

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla_x f=\int f(\mathbf{v}',\mathbf{x},t)f(\mathbf{v}_1',\mathbf{x},t)dAd\mathbf{v}_1 -f(\mathbf{v},\mathbf{x},t)\int f(\mathbf{v}_1,\mathbf{x},t)dAd\mathbf{v}_1, \tag{1}$$

where dA contains the geometrical variables specifying a collision and is independent of velocity; $\mathbf{v}'$ and $\mathbf{v}_1'$ are velocities after collision and are expressed in terms of $\mathbf{v},\mathbf{v}_1$ and the geometrical variables.

We introduce a cutoff in the form of a maximum impact parameter so that the integrals converge separately. We give the names "emission term" and "absorption term" to the first and second terms, respectively, on the right-hand side of (1). Our procedure is then the following: at each step the absorption term is left intact. For the first step we insert a locally Maxwellian distribution f_0 in the emission term. Here,

$$f_0\equiv\rho(\mathbf{x},t)\left(\frac{m}{2\pi kT(\mathbf{x},t)}\right)^{\frac{3}{2}}\exp\left\{-\frac{m}{2kT}[\mathbf{v}-\mathbf{q}(\mathbf{x},t)]^2\right\}, \tag{2a}$$

with

$$\rho(\mathbf{x},t)\equiv\int fd\mathbf{v}, \tag{2b}$$

$$\mathbf{q}(\mathbf{x},t)\equiv\frac{1}{\rho}\int\mathbf{v}fd\mathbf{v}, \tag{2c}$$

$$\frac{3kT(\mathbf{x},t)}{m}\equiv\frac{1}{\rho}\int\{\mathbf{v}-\mathbf{q}(\mathbf{x},t)\}^2fd\mathbf{v}, \tag{2d}$$

where k is Boltzmann's constant and m is the mass of the molecule. Using the conservation laws $v^2+v_1^2=v'^2+v_1'^2$ and $\mathbf{v}+\mathbf{v}_1=\mathbf{v}'+\mathbf{v}_1'$, we obtain

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla_x f=(f_0-f)\frac{\rho(\mathbf{x},t)}{\sigma}, \tag{3}$$

where σ is a constant depending on the cutoff; to insure consistency of the procedure it has a definite value for a given force constant. In the second step we insert the solution f_1 of (3) into the emission term. The equation satisfied at the nth step is

$$\frac{\partial f_n}{\partial t}+\mathbf{v}\cdot\nabla_x f_n+\frac{f_n\rho_n}{\sigma}=\int f_{n-1}(\mathbf{v}_1',\mathbf{x},t)f_{n-1}(\mathbf{v}',\mathbf{x},t)dAd\mathbf{v}_1. \tag{4}$$

Equation (3) is the kinetic equation proposed by Bhatnagar, Gross, and Krook[3] as a model for collision processes. It has the feature that particle number, momentum, and energy are instantaneously conserved by collisions. While (3) is a nonlinear integro-differential equation, it is of a simpler mathematical nature than the Boltzmann equation since only the first few moments of the distribution function occur in the collision terms. Indeed, for small-amplitude sound oscillations (translational dispersion of sound) the exact solution can be exhibited as a contour integral[3] and investigated as a function of mean free path and boundary conditions. While (3) leads to sensible physical results for a wide variety of situations, the solutions are not quantitatively exact. The sequence (4) is proposed as a systematic improvement. A general study of the convergence of (4) to the solution of the Boltzmann equation can be made using the methods of Wild.[4] We discuss here the results of (3) for three different types of problem and indicate the improvement brought about by (4).

a. A calculation of transport properties using (3) yields a ratio of heat conductivity to viscosity, which is $\frac{2}{3}$ the correct value. Equation (4) gives the series $\frac{2}{3}[1+\frac{1}{3}+\cdots+(\frac{1}{3})^{n-1}]$, which converges rapidly to unity.

b. According to (3), a spatially homogeneous velocity distribution which is non-Maxwellian initially, decays to Maxwellian form exponentially with a time constant σ/ρ. The nature of the decay is independent of the initial form and amplitude of the distribution. Equation (4) yields the distribution function as a power series in $e^{-t\rho/\sigma}$, the coefficients depending on the initial distribution. The results for the decay of the lowest moments agree with the exact results of Maxwell.[5]

c. Equation (3) permits a clear survey of the problem of translational dispersion. However, in the limit where the Enskog-Chapman procedure is valid there is a discrepancy of 20 percent in the dispersion; this is removed by the iteration (4).

The chief physical reason for the success of the iteration is that we successively improve the emission term by inserting more exact distribution functions; the behavior of Eq. (1) for most problems is not sensitive to this term. The physically reasonable solutions of (3) imply that we start with good over-all behavior. For some problems the iteration (4) may not be the most practical procedure. Nevertheless, methods involving a successive set of kinetic equations which are solved exactly appear promising for cases where the expansion of the distribution function in some orthogonal set does not lead to definite results.

A more extensive treatment, together with applications, will be published later.

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Further Correlation between Magnetoresistance Variations and the de Haas-van Alphen Effect

Ted G. Berlincourt and John K. Logan
U. S. Naval Research Laboratory, Washington, D. C.
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MAGNETORESISTANCE anomalies have been observed in a single crystal of natural graphite at low temperatures, further verifying and extending the correlation already illustrated in bismuth[1,2] between the magnetoresistance variations and the de Haas-van Alphen effect (oscillatory dependence of magnetic susceptibility upon magnetic field). Furthermore, application of the same type of analysis used in bismuth and graphite to the zinc magnetoresistance data of Nachimovich[3] and the de Haas-van Alphen data of Marcus[4] and Mackinnon[5] reveals the existence of a like correlation. The existence of such a correlation in zinc, which is a relatively good conductor, implies that the electrons causing the susceptibility oscillations, i.e., the de Haas-van Alphen electrons (while as few as 10^{-5} per atom) may play a greater role in determining the electronic properties of metal single crystals than was heretofore believed. Indeed, such magneto-oscillatory behavior must be expected in still other electronic properties measured at low temperatures. Because de Haas-van Alphen effect investigations yield relatively precise values for electronic effective masses and chemical potentials, they should prove valuable in interpreting such behavior.

The magnetoresistance measurements on graphite were carried out at 4.22°K on a sample cleaved from a large single crystal of natural graphite (Catalog No. 48789, Essex County, New York) kindly loaned by Dr. George Switzer of the Smithsonian Institution. The magnetic field was parallel to the hexagonal or c axis, and the direction of the measuring current was along a binary or a axis. Electrical contact was obtained by copper plating the ends of the crystal which were then clamped to an insulating member with small copper bars to which current and potential

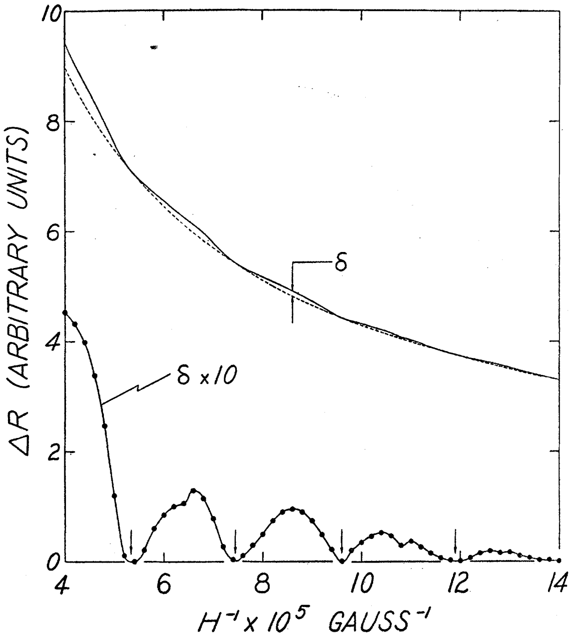

FIG. 1. Change in resistance ΔR in a graphite single crystal as a function of H^{-1} at 4.22°K (solid line). δ is the difference between the experimental values and the dashed curve.

leads were soldered. Since an unknown amount of contact resistance is present in such a configuration, the change in resistance (rather than the usual ratio of change in resistance to zero field resistance) is reported. In Fig. 1, the change of resistance ΔR of the sample in a magnetic field H is plotted as a function of H^{-1}. Since the probable error was estimated as less than the thickness of the plotted curve, the experimental points could not be indicated. Consequently, in order to illustrate the nature of the anomalies better, the differences δ (as read from a larger graph) between the experimental points and the dashed curve (which is accurately represented by $\Delta R\propto H^{0.811}$ between 12 and 25 kilogauss) are plotted on an expanded scale at the bottom of Fig. 1. Values of H^{-1} for which δ exhibits minima are indicated by arrows in Fig. 1 and are plotted against integers in Fig. 2 (solid line). For comparison, Shoenberg's de Haas-van Alphen data[6] on graphite were extrapolated to the same orientation (H parallel to the hexagonal axis), and values of H^{-1} for which graphite exhibits susceptibility minima are also plotted on Fig. 2 (dashed line). Since these lines are parallel, a one-to-one correspondence in period $\beta/E_0=2.2\times10^{-5}$ gauss^{-1} is indicated.[7] This point should be emphasized in view of the fact that the more complicated nature of the electronic constant energy surfaces for bismuth had made it impossible to state unambiguously from the experimental evidence alone whether the period of the magnetoresistance oscillations was one half that of the long-period susceptibility oscillations or was just equal to the period of the short-period susceptibility oscillations. (The one-to-one correspondence in period is further borne out by the zinc data.) Although the separation between the lines in Fig. 2 indicates a phase difference of approximately $\pi/2$ between the resistance variations along a binary axis and the susceptibility oscillations along the hexagonal axis in graphite, no great accuracy is claimed for this parameter since the susceptibility and resistance measurements were not carried out on the same crystal. Furthermore, the analysis is complicated somewhat by the fact that δ is not a simple monatonically modulated sinusoid (see Fig. 1). However, this is to be expected in view of the existence of a subordinate term in the susceptibility[6] having a period $\frac{3}{4}$ that of the dominant term.

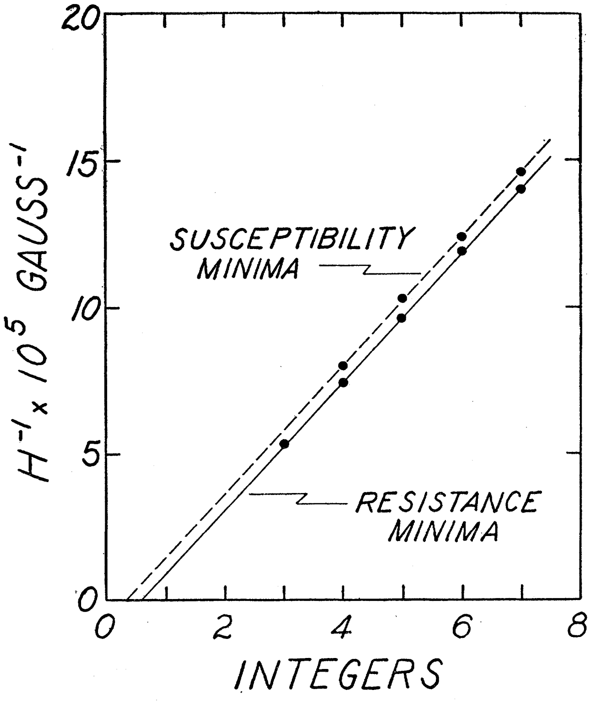

FIG. 2. Values of H^{-1} for which magnetoresistance and susceptibility oscillations in graphite exhibit minima are plotted against integers. The susceptibility data are those of Shoenberg (reference 6).

More detailed investigations of the correlative effects are in progress and will be reported at a later date.

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Electroluminescence in Thin Films of ZnS:Mn

R. E. HALSTED AND L. R. KOLLER
General Electric Research Laboratory, Schenectady, New York

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TRANSPARENT films of sulfide phosphors of the order of 10 microns in thickness have been prepared by the vapor reaction method of Cusano and Studer.[1] Such films offer a crystalline form of these materials particularly suitable for study under high voltage gradients. Preliminary data on films of ZnS:Mn (approximately 0.5 percent Mn) reveal some interesting variations from previously reported electroluminescent behavior.

Samples were prepared in the form of condensers with one transparent conducting electrode on a glass substrate, a transparent phosphor dielectric, and a second electrode of evaporated metal. ZnS:Mn films so prepared respond with a Mn emission peaked at 5900A to either cathode-ray or electroluminescent excitation. They have a specific resistance in the dark of the order of 10^{13} ohm cm measured at an average gradient of 10^4 volts/cm. Thermoluminescent measurements between $-196°$ and $+200°$C yield one "glow peak" corresponding to a 0.3-ev trap (frequency factor of 10^8/sec assumed). This trap is observed also in ZnS films with no Mn.

Except for a transient flash on application and removal of dc fields, no dc luminescence is produced by fields below breakdown (approximately 5×10^5 volts/cm). Two light pulses per cycle are observed on application of an ac potential. As can be seen in Fig. 1, these light pulses are very closely in phase with the applied