Meson Production in High-Energy Nucleon-Nucleon Collisions*

William L. Kraushaar and Lawrence J. Marks
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received August 21, 1953)

A model for meson production in high-energy nucleon-nucleon collisions similar to a model suggested by Takagi is discussed. It is assumed that the primary collisions result in the formation of two highly excited nucleons which decay as free particles. Forward and backward center-of-mass collimation is a natural consequence. Angular momentum is discussed and treated quantitatively by means of thermodynamic arguments similar to those used by Fermi. The model implies a tendency for the mesons to be emitted in a plane. The multiplicities predicted are larger than those predicted from the Fermi model and depend upon an additional parameter related to angular momentum.

I. INTRODUCTION

Some general characteristics of meson production in very high-energy nucleon-nucleus collisions seem to be established despite the small number of experiments.[1–3] It is by no means certain that these collisions can be correctly interpreted in terms of nucleon-nucleon encounters,[4] but if they are so interpreted, the following general features seem evident: (*a*) the existence, in at least some cases, of two cones of fast particles, one forward and one backward as viewed from the collision center-of-mass system (referred to as frame 1 hereafter); (*b*) a meson multiplicity which is small compared to the maximum multiplicity kinematically possible; and (*c*) a meson multiplicity which probably increases less rapidly than the energy available in the center-of-mass system.

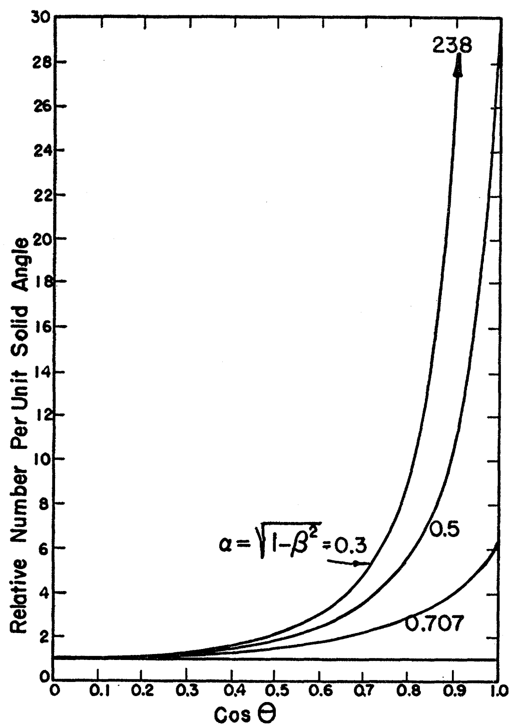

Fig. 1. Collision center-of-mass angular distributions for various values of the parameter $\alpha=(1-\beta^2)^{\frac{1}{2}}$, where βc is the velocity of the excited nucleons in the collision center-of-mass system.

We wish to discuss here a model, certain aspects of which have already been suggested by Takagi.[5] We suppose that the effect of a nucleon-nucleon collision is to excite normal modes of the nucleons, or to change each of the nucleons into greatly excited, heavy isobaric states. Kinematically this amounts to assuming that the nucleons, as viewed from frame 1, exchange a portion of their momentum at the instant of collision, and then travel away from the collision center, each keeping its original total energy but having a *smaller* momentum and hence larger mass than before. We further suppose that these excited nucleons have lifetimes sufficiently long so that by the time they decay into mesons they are free particles. If we neglect for the present any angular momentum considerations, the angular distribution of the emitted mesons is isotropic in the frame of reference (frame 2) in which an excited nucleon is at rest. In other words, we are assuming that there exist high-energy large-multiplicity counterparts of the established heavier-than-nucleon V particles, and that these highly excited nucleons are produced in pairs in very high-energy nucleon-nucleon collisions. The angular distribution of the emitted mesons in frame 1 has then the characteristic two cones of fast particles, one forward and one backward, if the excited nucleons maintain the direction of the incident nucleons.[6] Angular distributions computed according to these assumptions are shown in Fig. 1 for various values of the velocity βc of the excited nucleons relative to frame 1. We have taken here the extreme relativistic limit $\beta_\pi=1$ for the mesons in frame 2. In this approximation the

* This work was supported in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950).
[2] M. F. Kaplan and D. M. Ritson, Phys. Rev. **88**, 386 (1952).
[3] Lal, Pal, Peters, and Swami, Proc. Indian Acad. Sci. **36**, 75 (1952).
[4] F. C. Roesler and C. B. A. McCusker, Phys. Rev. **91**, 690 (1953).
[5] S. Takagi, Progr. Theoret. Phys. (Japan) **1**, 123 (1952).
[6] If the excited nucleons do not maintain the directions of the incident nucleons, the two cones of mesons would no longer be coaxial in the laboratory system.

angular distribution has the simple form given by the equation

$$N(\theta)=\frac{(1-\beta^2)}{8\pi}\left[\frac{1}{(1-\beta\cos\theta)^2}+\frac{1}{(1+\beta\cos\theta)^2}\right], \quad (1)$$

where $N(\theta)$ is the fraction of particles emitted per unit solid angle at the polar angle θ.

It is of interest to apply the above ideas to the "S" star.[1] If we take 20 000 Bev as the energy of the incident nucleon, the observed angular distribution is consistent with the angular distribution which would result from the decay of two excited nucleons traveling in the same direction as the incident nucleons, but with velocities only $0.92c$, as viewed from frame 1. The corresponding value for the rest mass of each of the excited nucleons is about $40M$, where M is the nucleon mass. Upper and lower bounds for the lifetimes of the excited nucleons may be estimated from (a) the fact that the track of the primary colliding particle and the origin of both the wide and diffuse cones of fast particles apparently coincide in the nuclear emulsion, and (b) the fact that, if these excited states do exist, and do live long enough to become free particles, there must have been enough time available for several light-signal traversals of the volume, taken here to be $\sim$ (meson Compton wavelength).[3] These speculative and crude estimates set $10^{-16}>\tau>10^{-21}$ sec.

The inclusion of angular momentum considerations has an interesting consequence. Suppose the total energy in frame 1 is W. Then in the extreme relativistic limit, the angular momentum of the initial state is $W\mathcal{R}/c$, where $\mathcal{R}$ is the impact parameter for each of the colliding nucleons. This angular momentum could be carried off entirely via translatory motion, or partly via translatory motion and partly via intrinsic angular momentum of the excited nucleons. It is the latter possibility which we wish to explore further here.

If we characterize the collision by the velocity βc of the excited nucleons in frame 1, the rest mass of the excited nucleons is $U/c^2=\alpha W/2c^2$, where $\alpha=(1-\beta^2)^{\frac{1}{2}}$. Then the angular momentum of the final state is only $W\beta\mathcal{R}/c$, if the impact parameter of the excited nucleons upon leaving the center of the interaction is the same as that for the initial collision. Angular momentum can then be conserved by assigning a spin-like angular momentum $W\mathcal{R}(1-\beta)/2c$ to each of the excited nucleons. Conservation of total energy and angular momentum are more conveniently considered in frame 2, in which an excited nucleon is at rest. In this frame the transformed total energy is just U, and the transformed angular momentum is $M_z=U\mathcal{R}/(1+\beta)c$. Since UR/c, where R is the radius of an excited nucleon, is the maximum angular momentum that can be carried off in the decay process, angular-momentum conservation has the possibility of being a significant restraint. In this case there would be a tendency for the mesons to be emitted in a plane perpendicular to the z direction.

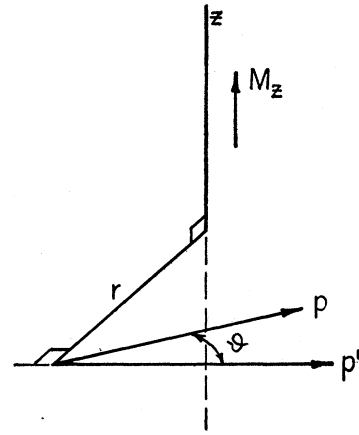

FIG. 2. Coordinate system in the frame of reference in which an excited nucleon is at rest.

If high intrinsic angular-momentum particles are themselves among the decay products, this tendency would be less pronounced.

II. THERMODYNAMIC APPROXIMATION

Some of the above notions can be treated from the standpoint of thermodynamic equilibrium in a manner analogous to Fermi's treatment of high-energy multiple meson production.[7,8] We have already assumed that the excited nucleons are free particles by the time they decay, and have lost all memory of the original collision. Hence there should be sufficient time for the establishment of equilibrium. As the volume of the excited nucleon we take a sphere of radius R. R is taken to be of the order of a meson Compton wavelength. (In the high-energy limit assumed here, the zero-point energy is not an important consideration.) We take cylindrical shells with their axes along z as the volume elements for integration, $4\pi r(R^2-r^2)^{\frac{1}{2}}dr$. Then the number of mesons in a volume element in phase space is given by the equation:

$$dN=\frac{g2\pi p^2dpd(\cos\vartheta)4\pi r(R^2-r^2)^{\frac{1}{2}}dr}{(2\pi\hbar)^3[\exp(Apc-Bpr\cos\vartheta)-1]}. \quad (2)$$

Here we are assuming Einstein-Bose statistics and π-meson production only with a statistical weight $g=3$. A and B are constants to be determined from energy and angular-momentum conservation. (In the absence of angular momentum considerations, A is essentially the reciprocal temperature.) The angle ϑ is the angle between the meson direction p and the direction p' which the meson would have if it were to carry away the maximum possible angular momentum (see Fig. 2). We integrate Eq. (2) over p from 0 to ∞, over $\cos\vartheta$ from -1 to 1, and over r from 0 to R. The result is the total number of mesons emitted from one of the two excited nucleons, and depends explicitly on A and B. Then we multiply Eq. (2) by pc and integrate over the variables,

[7] E. Fermi, Phys. Rev. **81**, 683 (1951); Progr. Theoret. Phys. (Japan) **5**, 570 (1950).

[8] Hazen, Heineman, and Lennox, Phys. Rev. **86**, 198 (1952).

as above. The result is the total energy U. Equation (2) is then multiplied by $rp\cos\vartheta$ and integrated. The result is the total angular momentum carried off by the mesons and is equated to M_z. The parameters A and B can be eliminated from the three equations, and the result for the total number of mesons emitted from both excited nucleons is given by the equation:

$$N=\frac{2^{7/8}}{\pi^{\frac{1}{4}}}\left(\frac{R}{\lambdabar}\right)^{\frac{3}{4}}\left(\frac{g}{3}\right)^{\frac{1}{4}}\frac{a}{b^{\frac{3}{4}}}\alpha^{\frac{3}{4}}\left(\frac{W'}{Mc^2}\right)^{3/8}\frac{F_1(\rho)}{[F_2(\rho)]^{\frac{3}{4}}}. \qquad (3)$$

Here R = radius of volume of excited nucleon.

W'/Mc^2 = laboratory energy of incident nucleon measured in units of the nucleon rest energy.

$2\pi\lambdabar$ = nucleon Compton wavelength. (This appears only because we have measured energy in units of Mc^2.)

$$a=2\sum_{n=1}^{\infty}\frac{1}{n^3}=2.413,$$

$$b=6\sum_{n=1}^{\infty}\frac{1}{n^4}=6.494,$$

$$\alpha=(1-\beta^2)^{\frac{1}{2}},$$

$$F_1(\rho)=\frac{3}{2\rho^2}\left\{\frac{\tan^{-1}[\rho/(1-\rho^2)^{\frac{1}{2}}]}{\rho(1-\rho^2)^{\frac{1}{2}}}-1\right\},$$

$$F_2(\rho)=1+\frac{\rho^2}{1-\rho^2}\left[1+\frac{F_1(\rho)}{3}\right].$$

The parameter ρ affords a measure of the extent to which angular-momentum conservation is an important restraint on the decay process. Its value is given implicitly by the relation:

$$\frac{M_z}{UR/c}=\frac{F_2(\rho)-F_1(\rho)}{\rho F_2(\rho)}\equiv G(\rho). \qquad (4)$$

$G(\rho)$ and $F_1(\rho)/[F_2(\rho)]^{\frac{3}{4}}$ are plotted as functions of ρ in Fig. 3.

For comparison, the expression similar to our Eq. (3) from the Fermi model is

$$N=\frac{3}{2^{\frac{3}{4}}\pi^{\frac{1}{4}}}\left(\frac{R}{\lambdabar}\right)^{\frac{3}{4}}\left(\frac{g}{3}\right)^{\frac{1}{4}}\frac{a}{b^{\frac{3}{4}}}\left(\frac{W'}{Mc^2}\right)^{\frac{1}{4}}K(\mathcal{R}/R), \qquad (5)$$

where the symbols have the same meanings as before, and $K(\mathcal{R}/R)$ is a function of the impact parameter $\mathcal{R}$. The similarity of the expressions reflects the thermodynamic treatment used in both models. The difference in the energy dependence arises from the Lorentz contraction of the excited volume assumed only in the Fermi model.

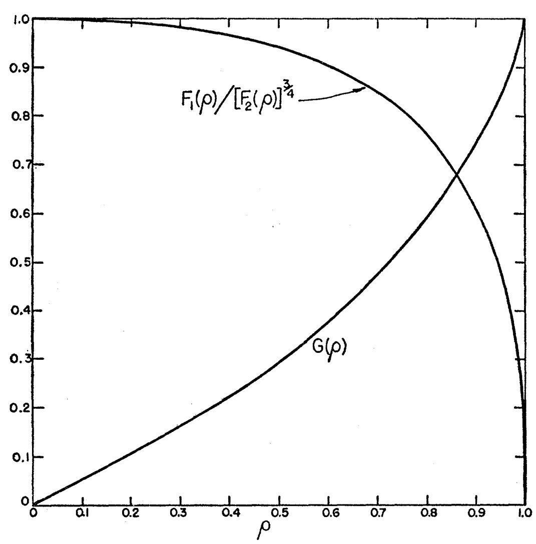

FIG. 3. The functions $G(\rho)$ and $F_1(\rho)/[F_2(\rho)]^{3/4}$ *vs* ρ. $G(\rho)$ serves to define the value of the angular momentum parameter ρ, and $F_1(\rho)/[F_2(\rho)]^{3/4}$ determines the multiplicity.

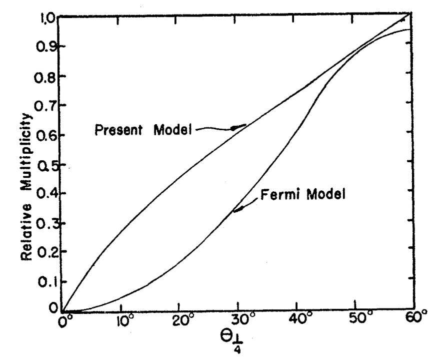

FIG. 4. Relative multiplicity for the present model and for the Fermi model plotted against the collision center-of-mass angle, $\theta_{1/4}$, which contains one-fourth of the emitted particles.

Let us characterize the angular distribution for both models by considering the center-of-mass polar angle $\theta_{\frac{1}{4}}$, which contains one-fourth of the particles. In the Fermi model a small value of $\theta_{\frac{1}{4}}$ means a large angular-momentum-type collision ($\mathcal{R}/R\sim 1$) with a resultant small multiplicity, as reflected by the behavior of the function $K(\mathcal{R}/R)$. [$K(\mathcal{R}/R)$ approaches zero as $\mathcal{R}/R$ approaches 1, and approaches $2\sqrt{2}/3=0.943$ as $\mathcal{R}/R$ approaches zero.] In the present model, a small value of $\theta_{\frac{1}{4}}$ means a small momentum transfer between the colliding nucleons, and hence only a small increase in nucleon mass. The excitation energy of the nucleons is therefore small, and few mesons are emitted. In Fig. 4 we have shown $\alpha^{\frac{3}{4}}$ and $K(\mathcal{R}/R)$ plotted against $\theta_{\frac{1}{4}}$ to show how the angular distribution and meson multiplicity are related in the two models. The numerical factors in the two expressions for N are almost identical, so that, except for the factors $\alpha^{\frac{3}{4}}$ and $K(\mathcal{R}/R)$ discussed above, the multiplicities from the present model exceed those from the Fermi model by a factor of $(W'/Mc^2)^{1/8}F_1(\rho)/[F_2(\rho)]^{\frac{3}{4}}$, provided the radius of the excited volume in the present model and the radius of the uncontracted excited volume in the Fermi model are set equal to one another. $[(W'/Mc^2)^{1/8}\approx 3.5$ for $W'=20\,000$ Bev.$]$ The factor $F_1/F_2^{\frac{3}{4}}$ is equal to, or less than 1, and has a value determined by the angular-momentum parameter ρ. This is discussed further in the following paragraphs.

According to Eq. (2), there is predicted an angular distribution of the mesons in frame 2 involving the angle ϑ. This angular distribution is given by the equation

$$F_3(\eta)d(\cos\vartheta) = \left[\frac{2-\eta^2}{2\eta^2(1-\eta^2)}+\frac{\pi}{2\eta^2}-\frac{(\frac{1}{2}\pi+\sin^{-1}\eta)(2-3\eta^2)}{2\eta^3(1-\eta^2)^{\frac{3}{2}}}\right]d(\cos\vartheta), \quad (6)$$

where $\eta=\rho\cos\vartheta$. $F_3(\rho\cos\vartheta)$ is not an observable angular distribution, because ϑ measures the angle between a meson's direction and the direction it would have if it were to carry off its maximum possible amount of angular momentum. It is therefore convenient to consider the angular distribution obtained by projecting the meson directions on a plane passing through the z axis in frame 2. This angular distribution $F_4(\phi)d\phi$, where ϕ is measured from the z axis, is given in terms of the function F_3 by the relation

$$F_4(\phi)=\frac{1}{2\pi}\int_0^\pi\int_0^\pi F_3(\rho\cos\beta(1-\sin^2\gamma\cos^2\phi)^{\frac{1}{2}}) \times\sin\gamma d\beta d\gamma. \quad (7)$$

If the plane in which ϕ is measured is considered to be perpendicular to the direction of flight of the excited nucleon, $F_4(\phi)$ is invariant to a coordinate transformation and so corresponds to the angular distribution in target diagrams such as those given by Kalpon and Ritson.[2] Figure 5 shows $F_4(\phi)$ for various assumed values of the parameter ρ.

As mentioned before, the parameter ρ affords a measure of the extent to which angular momentum is an important restraint on the decay process. In Eq. (4), UR/z is really just the maximum possible angular momentum that can be carried off from an excited nucleon of total energy U and radius R. If M_z is small compared to UR/c, $G(\rho)$ and hence ρ are small. This in turn means that $F_1(\rho)/[F_2(\rho)]^{\frac{3}{4}}$, and hence the meson multiplicity tend towards zero. Such mesons as are emitted tend to be emitted in a plane, and so $F_4(\phi)$ has a strong maxima at 90 and 270 degrees. The details of these considerations depend, of course, upon the thermodynamic approximation that has been used. The more general characteristics are essentially kinematical, and suggest that the low multiplicity limit arising either from low excitation energy or high angular momentum would correspond to the established V_1^0-type particle. This is, of course, not a "prediction" from the model but rather is a return to one of the phenomena which sug-

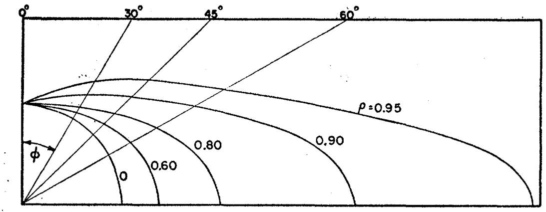

FIG. 5. Polar plot of the target diagram angular distribution, $F_4(\phi)$ *vs* ϕ for various values of the angular momentum parameter ρ. ϕ is measured from the z axis.

gested the model in the first place. In part I it was mentioned that the angular distribution for the "S" star was consistent with the present model for excited-nucleon masses of about $40M$. This corresponds to a value of 0.40 for our parameter α and was obtained by taking 20 000 Bev as the incident nucleon energy in the laboratory system and $\theta_{\frac{1}{2}}=20°$ in the collision center-of-mass system. The predicted number of charged particles $[\frac{2}{3}$ of N from Eq. (3)$]$ is then $43\,F_1(\rho)/[F_2(\rho)]^{\frac{3}{4}}$, if R is taken as $1/2\pi$ times the meson Compton wavelength. The predicted number from the Fermi model under the same assumptions as above is 4.6, and the observed number was 15.

A comparison of the group of high-energy interactions observed by Kaplon and Ritson with our predicted charged meson multiplicities shows the same general tendency as shown by the above comparison with the "S" star. That is, if we ignore the factor $F_1(\rho)/[F_2(\rho)]^{\frac{3}{4}}$ by setting it equal to one, some of the predicted multiplicities are in close agreement with those observed, while many of them are a factor of two or more higher. In evaluating this comparison, it should be emphasized that none of these interactions are known to be nucleon-nucleon interactions, and the effects of the other nucleons present in the target nucleus are unknown. In both the Fermi model and in the present model, the radius R is to be considered a free parameter (within reasonable limits). We have ignored the factor $F_1(\rho)/[F_1(\rho)]^{\frac{3}{4}}$ by setting it equal to one, because we have no *a priori* method of evaluating ρ. In pure nucleon-nucleon interactions a study of the target diagram angular distributions would at least permit one to put an approximate upper limit on ρ. This, of course, is a crucial point in evaluating the worth, if any, of the model.

III. CONCLUSIONS

Basic to all the aforementioned considerations are the assumptions that the nucleons in very high-energy nucleon-nucleon collisions are converted into heavy, highly-excited states travelling essentially in the direction of the incident nucleons, and that these states live long enough to become essentially free particles before decaying into mesons or other particles. These assumptions lead to the characteristic center-of-mass forward and backward cones of particles. Essentially independent of the thermodynamic approximation is the suggestion of asymmetric target diagrams or a tendency for the mesons in a cone to be coplanar. The meson multiplicity and target diagram angular distribution are directly dependent upon the thermodynamic approximation.

There is no logical necessity in our assumption that the excited nucleons take on the directions of the colliding nucleons. On the other hand, excitation energies appropriate to the observed angular distributions result from comparatively small momentum transfers, so that the angles between the colliding and excited nucleons would in general be expected to be small. Similarly, there is no logical necessity in our assumption that (other than on the average) the masses, or excitation energy of the excited nucleons, be equal. That is, while the excited nucleon momenta must be equal and opposite, their excitation energies could be different. This would require energy as well as momentum transfer between the nucleons and would lead to dissimilar center-of-mass forward and backward cones.

Lal, Pal, Peters, and Swami have observed several meson-induced interactions, and remark that most of the mesons seem to be emitted in the backward direction in the center-of-mass system.[3] If the general features of the model are to apply to this type of interaction, it seems necessary to assume either of two alternatives: (a) energy as well as momentum is transferred, and in general the nucleon gets the higher excitation energy; or (b) only momentum is transferred, but the meson and nucleon excited volumes are inherently different, so that the excited nucleon decays into more particles.[9]

It is a pleasure to acknowledge a helpful discussion of these matters with Professor F. Villars.

APPENDIX

The spherical shape used for the excited volume in the thermodynamic approximation is perhaps the most natural choice. Equation (3) may be written more generally so as to be valid for other volumes. This expression is

$$N=\left(\frac{6\sqrt{2}}{\pi^2}\right)^{\frac{1}{4}}\left(\frac{V^{\frac{1}{3}}}{\lambda}\right)^{\frac{3}{4}}\frac{a}{b^{\frac{3}{4}}}\left(\frac{g}{3}\right)^{\frac{1}{4}}\alpha^{\frac{3}{4}}\left(\frac{W'}{Mc^2}\right)^{3/8}\frac{f_1(\rho)}{[f_2(\rho)]^{\frac{3}{4}}}, \qquad (3')$$

where V is the volume assumed. The functions f are given as follows:

$$f_1(\rho)=\frac{1}{V}\int\frac{dV}{(1-\rho^2r^2/R^2)^2},$$

$$f_2(\rho)=\frac{1}{V}\int\frac{(3+\rho^2r^2/R^2)}{(1-\rho^2r^2/R^2)^3}dV,$$

$$f_3(\rho\cos\vartheta)=\frac{1}{V}\int\frac{dV}{[1-\rho(r/R)\cos\vartheta]^3}.$$

Here r is the distance from the z axis to the volume element dV; and R is the maximum radius of the volume. (Symmetry about the z axis has been assumed.)

[9] If taken seriously, our assumption of momentum transfer without energy transfer would imply that V_1^0 particles, if made in neutron-neutron collisions, would be made in pairs, but that V_1^0 particles made in π^- free-proton collisions would be accompanied by neutral particles of mass 1210 electron masses. Similar reasoning applied to π^- free-proton collisions in which particles of mass 975 electron masses are created, gives a mass of 2060 electron masses to the excited nucleon.