

The Theory of the Deuteron Stripping Reactions*

N. C. FRANCIS, *Department of Physics, Indiana University, Bloomington, Indiana*

AND

K. M. WATSON, *Department of Physics, University of Wisconsin, Madison, Wisconsin*

(Received October 7, 1953)

The deuteron stripping reactions are analyzed from the point of view of the many-body problem. A rigorous formal solution is given. From this it is shown how assumptions less restricted than Butler's can be employed to reduce the many-body interactions to those of a deuteron moving in a potential well. At this point, the relation to previous theories is readily established. The present theory might be termed "the optical model" of the stripping reactions and represents a considerable generalization of current theories.

I. INTRODUCTION

BUTLER'S analysis¹ of the deuteron stripping reactions at intermediate energies has provided a useful tool for analyzing experiments pertaining to these processes. Alternate methods of calculation^{2,3}—that is, the so-called Born-approximation theories—have led to rather similar results. One purpose of the present note is to derive both types of theories from a single model as somewhat limiting cases. It will be seen that the alternate points of view are closely related and that the distinction between them involves questions of nuclear structure as much as of mathematics.

For the sake of being specific, we shall speak of the (d, p) reaction, although the method of analysis applies equally to the (d, n) reaction. Our results are applicable also to the (p, d) and (n, d) processes (the pickup reactions) either directly or by means of the principle of detailed reversibility.

Butler¹ makes three significant assumptions in order to simplify the development of his theory. These are [we are now referring to the (d, p) reaction]:

- (1) The proton does not interact at all with the nucleus which is struck.
- (2) Once the neutron enters the nucleus, it is captured and does not interact with the proton which was originally in the deuteron.
- (3) The incoming deuteron wave function is undistorted by the reaction.

Proceeding from these assumptions, Butler encloses the nucleus within a sphere of radius R_0 . Within this sphere the neutron is bound to the nucleus. The outside region contains the incoming deuteron and the outgoing proton. Matching these wave functions and their derivatives on the sphere R_0 determines the cross section.

In the present discussion it will prove convenient to relax somewhat each of the above three assumptions. In so doing we shall cast the theory into a new form

* Supported in part by a grant from the National Science Foundation.

¹ S. T. Butler, Proc. Roy. Soc. (London) **208**, 559 (1951).

² Bhatia, Huang, Huby, and Newns, Phil. Mag. **43**, 485 (1952).

³ P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).

which, it is hoped, will add to the understanding of the physical basis of the Butler and "Born-approximation" theories, as well as to provide a more general theoretical basis for analyzing the stripping reactions. Our starting point will be a rigorous formulation of the process as a many-body problem. An exact formal solution will be given. By use of a somewhat modified form of Butler's assumptions, it is then shown how the many-body aspects of the problem can be reduced to those of a deuteron moving in a potential well. By imposing the appropriate conditions on the nature of the potential well, the Butler and "Born-approximation" theories can be obtained directly.

We shall neglect any specific discussion (as have previous analyses) of the role played by the Coulomb field on the stripping reaction. This involves no conceptual difficulties, as we can suppose the (screened) Coulomb interaction to be included in the definitions of the other interactions introduced in the next section. On the other hand, a numerical discussion of effects of the Coulomb forces would be quite involved and presumably would not add greatly to our understanding of the mechanism of the reaction.

II. DEVELOPMENT OF THE BUTLER AND BORN-APPROXIMATION THEORIES

A. General Discussion

We wish to calculate the transition rate for the reaction in which a deuteron strikes a nucleus with emission of a proton, the residual nucleus having "captured" the extra neutron. We suppose that the wave function for the initial nucleus is

$$g_A(\xi),$$

where ξ is a complete set of nuclear coordinates, and that the deuteron is moving with a momentum \mathbf{K} with respect to the nucleus. Then, if $D(r)$ is the deuteron wave function, the initial state is

$$\chi_a = \frac{e^{i\mathbf{K}\cdot\mathbf{x}}}{(2\pi)^{\frac{3}{2}}} D(r) g_A(\xi), \quad (1)$$

where $\mathbf{x} = \frac{1}{2}(\mathbf{r}_P + \mathbf{r}_N)$, $\mathbf{r} = \mathbf{r}_P - \mathbf{r}_N$, and \mathbf{r}_P and \mathbf{r}_N are the respective coordinates of the proton and neutron with respect to the center of mass of the nucleus.

The state χ_a is evidently not antisymmetrized in the coordinates of identical particles as required by the Pauli principle, even though we suppose the nuclear states g_A (and g_B) to be so antisymmetrized. The antisymmetrization is most easily effected by first solving the Schrödinger equation with the *unsymmetrized* initial state χ_a and then antisymmetrizing this solution by means of an antisymmetrization operator. In principle this is simple to do, but to avoid complicating the argument, we shall for the moment (apparently) treat the initial neutron and proton as distinguishable from the nucleons in the nucleus (except that the final nuclear state is considered to be properly antisymmetrized). We shall return to this question in Sec. V.

After the reaction is completed, we suppose the system to be in a final state in which the neutron is captured and the proton is moving as a free particle of momentum \mathbf{p} (with spin wave function s_P).

This state we write as

$$\chi_b = \frac{e^{i\mathbf{p}\cdot\mathbf{r}_P}}{(2\pi)^{\frac{3}{2}}} s_P g_B(\xi, r_N), \quad (2)$$

where g_B is the wave function of the final nucleus including the captured neutron (which may or may not be bound into a stable state).

We next give the explicit Schrödinger equations satisfied by the states χ_a and χ_b . If V is the deuteron binding potential, then

$$(H_0 + V)\chi_a = E_a\chi_a, \quad (3)$$

where $H_0(\xi, r_N, r_P)$ is the nuclear Hamiltonian plus the kinetic energy operators of the neutron and proton in the deuteron. If further $v(r_N, \xi)$ represents the interaction of the neutron with the nucleus, then

$$[H_0 + v]\chi_b = E_b\chi_b. \quad (4)$$

The final states χ_b , which are of most interest to us, are those for which energy is conserved, or $E_a = E_b$. We finally suppose that the interaction of the proton with the nucleus is given by the "potential" $u(r_P, \xi)$. Then the entire process is described by the appropriate solution to the Schrödinger equation

$$[H_0 + V + v + u]\Psi = E_a\Psi. \quad (5)$$

B. The Butler Theory

In accordance with Butler's¹ assumptions, we shall in the remainder of the present section set $u = 0$. That is, we suppose that the proton does not interact with the nucleus. The solution to Eq. (5) with $u = 0$ will be designated by ψ to distinguish it from the exact solution Ψ (which will be given in Sec. III).

The desired solution to Eq. (5) is then easily obtained as⁴

$$\begin{aligned} \psi &= \left[1 + \frac{1}{a - V - v} \right] \chi_a \\ &= \chi_a + \frac{1}{a - v} \Lambda \chi_a, \end{aligned} \quad (6)$$

where $a = E_a + i\eta - H_0$ (η is a small positive parameter, such as occurs in the Lippmann-Schwinger⁵ formulation of scattering problems). The quantity Λ is

$$\Lambda = v + V \frac{1}{a - V - v} v. \quad (7)$$

The proof of Eq. (6) is trivial. The Schrödinger equation to be satisfied by ψ is

$$[a - V - v]\psi = 0.$$

Applying the operator $[a - V - v]$ to the right-hand side of Eq. (6), we have by Eq. (3)

$$-v\chi_a + (a - V - v) \frac{1}{a - V - v} v\chi_a = 0.$$

The last step in Eq. (6) is just an algebraic identity.

On introducing a complete set of final states,⁶ $g_B^{(-)}\lambda_k$, where

$$\lambda_k = e^{i\mathbf{k}\cdot\mathbf{r}_P} / (2\pi)^{\frac{3}{2}} s_P, \quad (8)$$

Eq. (6) can be expressed as

$$\psi = \chi_a + \sum_{B,k} \frac{1}{a - v} g_B^{(-)}\lambda_k T(B, \mathbf{k}_N; a). \quad (9)$$

Here,

$$T(B, \mathbf{k}; a) = (g_B^{(-)}\lambda_k, \Lambda\chi_a). \quad (10)$$

Now, because of Eq. (4),

$$\begin{aligned} \frac{1}{a - v} g_B^{(-)}\lambda_k &= \frac{1}{E_a + i\eta - E_b} g_B^{(-)}\lambda_k \\ &= \frac{1}{\epsilon_{pB} + i\eta - \epsilon_k} g_B^{(-)}\lambda_k, \end{aligned} \quad (11)$$

where ϵ_{pB} equals the initial kinetic energy of the deuteron plus the difference in binding energies between initial and final nuclear states (or just the energy of the outgoing proton). Also

$$\epsilon_k = k^2/2M, \quad (12)$$

⁴ The notation is that introduced by G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952).

⁵ B. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950).

⁶ The state $g_B^{(-)}$ is equal to g_B if the state is bound. Otherwise $g_B^{(-)}$ contains *incoming* scattered waves whereas g_B has *outgoing* scattered waves. The necessity for using $g_B^{(-)}$ in Eq. (9) was demonstrated by K. Watson, Phys. Rev. **88**, 1163 (1952).

where M is the nucleonic mass. When r_P is large, Eq. (9) becomes⁷

$$\begin{aligned} \psi &= \chi_a + \sum_B (2\pi)^{-\frac{3}{2}} \int d^3k \frac{e^{ik \cdot r_P}}{\epsilon_{p_B} + i\eta - \epsilon_k} \\ &\quad \times s_P g_B^{(-)} T(B, \mathbf{k}; a) \\ &= \chi_a - \sum_B (2\pi)^{-\frac{3}{2}} \left\{ (2\pi)^2 M \frac{e^{i p_B r_P}}{r_P} \right. \\ &\quad \left. \times s_P g_B^{(-)} T(B, \mathbf{p}_B; a) \right\}, \end{aligned} \quad (13)$$

where $p_B^2/2M = \epsilon_{p_B}$ and $\mathbf{p}_B = p_B(\mathbf{r}_P/r_P)$.

From Eq. (13) we see that the cross section for stripping is (for a final nuclear state B)

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 M^2 \frac{2p_B}{K} S |T(B, \mathbf{p}_B; a)|^2, \quad (14)$$

where S is an appropriate sum and average over final and initial spin substates, respectively.

Our next problem is to reduce $T(B, \mathbf{p}_B; a)$ to a more manageable form. Since in Eq. (14) we need $T(B, \mathbf{p}_B; a)$ only on the energy shell, we may suppose $E_b = E_a$ [Eq. (4)] and drop the subscript "B" on p_B . Then from (7) and (10),

$$\begin{aligned} T(B, \mathbf{p}_B; a) &= (g_B^{(-)} \lambda_p, \Lambda \chi_a) \\ &= \left(g_B^{(-)} \lambda_p, V \left[1 + \frac{1}{a - V - v} \right] \chi_a \right). \end{aligned} \quad (15)$$

The last step follows since

$$(g_B^{(-)} \lambda_p, v \chi_a) = (g_B^{(-)} \lambda_p, V \chi_a).$$

[That is, $v g_B^{(-)} \lambda_p = a g_B^{(-)} \lambda_p$ and $V \chi_a = a \chi_a$ are just Eqs. (3) and (4)]. Because of Eq. (6), we may also write Eq. (15) as⁸

$$T(B, \mathbf{p}; a) = (g_B^{(-)} \lambda_p, V \psi) = (\chi_b^{(-)}, V \psi), \quad (15')$$

which is a formally exact solution to our problem once ψ is known.

To interpret Eq. (15), we note that

$$V \left[1 + \frac{1}{a - V - v} \right] \chi_a$$

describes the entering of the nucleus by the neutron (as part of the deuteron) and its subsequent interaction with the nucleus. The V on the left means that the last step is a scattering of the neutron and proton. This last

⁷ See for instance, P. A. M. Dirac, *Quantum Mechanics* (Oxford University Press, London, 1947), third edition, p. 193.

⁸ Equations similar to (15') have recently been derived independently by E. Gerjuoy, Phys. Rev. **91**, 645 (1953), and by M. Gell-Mann and M. Goldberger, Phys. Rev. **91**, 398 (1953) [who considered the pickup process—see Sec. IV]. These authors neglected the many-body aspects of the problem, however, considering the deuteron to interact with a fixed force center only.

scattering leaves the system in the final state χ_b . According to the second of Butler's assumptions mentioned in the Introduction, this final scattering cannot occur when the neutron has entered the nucleus. Thus, according to Butler, the contribution to the stripping reaction occurs from neutron coordinates which lie *outside* the nucleus in Eq. (15). We shall return to this point in a moment.

Instead of Butler's drastic second assumption that the neutron does not interact with the proton once within the nucleus, we shall make the weaker assumption that the neutron does not interact with the proton once it has *excited* the nucleus to a state above its ground state. This seems quite plausible since an interaction which raises the nucleus to an excited state is expected to react violently on the more weakly bound deuteron, breaking it up and separating the neutron and proton of the deuteron. If this happens, we can imagine the chance that the neutron and proton will "find" each other to scatter again via the potential V to be small.

To incorporate this approximation into Eq. (15), we note that ψ contains in general a large number of excited states of the initial nucleus. Our assumption implies that we replace ψ by ψ_e in Eq. (15), where ψ_e is just that part of ψ which describes the nucleus in its ground state.⁹ Then ψ_e describes the "elastic scattering" of the deuteron by the nucleus (that is, "elastic" in the sense that the nuclear state is not changed).

It has been shown⁹ that such a "wave function" as ψ_e satisfies a Schrödinger equation with an "optical model" potential. If we write ψ_e as $\psi_e = \Omega_{CN} \chi_a$, where Ω_{CN} is the appropriate Møller wave matrix, then Ω_{CN} satisfies the Lippmann-Schwinger integral equation:

$$\Omega_{CN} = 1 + \frac{1}{a - V} \mathcal{U}_{CN} \Omega_{CN}, \quad (16)$$

where \mathcal{U}_{CN} is the "optical model potential." The important feature of Eq. (16) is that we no longer have to deal with a many-body problem, since \mathcal{U}_{CN} describes the motion of the deuteron in a "potential well" [although \mathcal{U}_{CN} may also depend upon the spins of both the nucleus and the deuteron]. We finally obtain

$$T(B, \mathbf{p}; a) = (g_B^{(-)} \lambda_p, V \Omega_{CN} \chi_a). \quad (17)$$

It is quite tempting to relate \mathcal{U}_{CN} to the "optical potential" deduced by Feshbach, Porter, and Weisskopf¹⁰ from Barschall's¹¹ neutron scattering experiments.

⁹ N. C. Francis and K. M. Watson, Phys. Rev. **92**, 291 (1953). The replacement of the many-body interaction v by the "potential well," \mathcal{U}_{CN} has been discussed in detail in this reference and here forms the basis of the present model of the stripping reaction.

¹⁰ Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953). We evidently must not take the numerical values of Eq. (18) too literally. For instance, \mathcal{U}_{CN} is expected in general to be energy-dependent.

¹¹ H. H. Barschall, Phys. Rev. **86**, 431 (1952).

If this were true, we would write

$$\mathcal{U}_{CN} = -(19+i)\text{Mev} \quad (18)$$

within the nucleus and set $\mathcal{U}_{CN}=0$ outside [that is, when the neutron is outside].

In general, we may expect the interaction described by \mathcal{U}_{CN} to polarize the deuteron (that is, distort its wave function). It is this modified wave function on which V operates in Eq. (17). We have, however, made estimates of the importance of this distortion for a potential such as that given by Eq. (18) and found that the effect is small, especially for the innermost parts of the deuteron wave function on which V operates. [The wave function might be distorted more if \mathcal{U}_{CN} contained spin-orbit interactions or a strong absorption coefficient.] For the evaluation of Eq. (17), it is especially convenient to be able to neglect any distortion of the deuteron, since V can then be eliminated from this expression by means of the Schrödinger equation satisfied by the deuteron wave function.

If we suppose that \mathcal{U}_{CN} is such an interaction which does not appreciably excite the deuteron, it is not an unreasonable approximation to set $\Omega_{CN}=1$ in Eq. (17). Then

$$T(B, \mathbf{p}; a) = (g_B^{(-)\lambda_p} V \chi_a) \quad (19)$$

is just the "Born approximation" as proposed by Daitch and French.³ For a potential \mathcal{U}_{CN} such as that given by Eq. (18), this expression may not be an unsatisfactory approximation.

On the other hand, if one were to suppose that the neutron interacts very strongly with the nucleus so that immediately upon entering it a compound state is formed in which the deuteron is broken up, the effective absorption coefficient¹² in \mathcal{U}_{CN} is expected to be large [this is just the situation supposed by Butler in his assumption (2)]. Then we could expect to write

$$\begin{aligned} \Omega_{CN}\chi_a &= \chi_a & \text{when } r_N > R \\ &= 0 & \text{when } r_N < R, \end{aligned} \quad (20)$$

where R is the radius of the nucleus. Daitch and French have shown that with this modification of the Born approximation one obtains just Butler's result, although they did not present any justification for doing this nor did they give a derivation of their "Born approximation" expression. We thus see that if we make the assumption implied by Eq. (20), which is equivalent to Butler's assumptions (2) and (3), we obtain just Butler's result from our theory, which is formulated quite differently from his.

It is evident that Eqs. (19) and (20) represent somewhat extreme limits on the nature of the physical

¹² It should be noted that the "effective absorption coefficient" for the stripping process is not necessarily identical with that of the potential \mathcal{U}_{CN} . The former measures the rate at which the deuteron is completely "broken up," whereas the latter measures the rate at which the deuteron excites the nucleus. A precise definition of the former is not necessary for our purposes, since the problem is specified by the wave function $\Omega_{CN}\chi_a$.

problem. Equation (19) assumes a weak nuclear interaction with the incoming neutron, whereas Eq. (20) assumes that this interaction is very strong. The analysis of Daitch and French³ suggests that the deduced cross sections are quite similar for the two models. From this one might hope to conclude that the gross features of the stripping reaction tend to be somewhat insensitive to the exact nature of the potential \mathcal{U}_{CN} and so to that of Ω_{CN} .

III. THE EFFECT OF THE INTERACTION OF THE PROTON WITH THE NUCLEUS

We now no longer set $u=0$. A formally exact solution to Eq. (5) is

$$\Psi = \left[1 + \frac{1}{a - V - v - u} u \right] \left[1 + \frac{1}{a - V - v} v \right] \chi_a. \quad (21)$$

This differs from Eq. (6) only by the factor on the left. The first factor describes the interaction of the neutron with the nucleus. The second describes the interaction of the proton with the nucleus in a representation which considers the neutron now to be a part of the nucleus [i.e., in which $(a-v)$ is diagonal]. This expression is a little unsymmetrical since the first interaction is just through the potential u while subsequent interactions are through the potential $(u+V)$. Expressed somewhat differently, for the first scattering of the proton, the initial neutron does not contribute. For the subsequent scattering it does, however. Since this neutron is just one of many nuclear particles, its omission for the first scattering is probably¹³ not of much importance for our interpretation of the factor on the left in Eq. (21) as a scattering of the proton by the nucleus.

The argument is now similar to that which we used in Sec. II. The proton may or may not excite the final nucleus (containing the extra neutron). If the proton excites the nucleus, we may suppose it to form a compound state from which it may be later emitted. We shall further suppose that this type of process is not of the sort in which we are interested and that it is experimentally distinguishable from the type of stripping reaction considered by Butler.¹⁴ Then, if we consider only the elastic scattering of the proton to contribute, we may make the substitution⁹

$$\left[1 + \frac{1}{a - V - v - u} u \right] = \Omega_{CP}, \quad (22)$$

where

$$\Omega_{CP} = 1 + \frac{1}{a - v} \mathcal{U}_{CP} \Omega_{CP}. \quad (23)$$

¹³ It will become clear in the next paragraph that it is only elastic scattering of the proton which is of interest to us. For elastic scattering there seems to be considerable evidence that one additional nucleon in the nucleus will not be of importance.

¹⁴ For instance, one might construct a wave packet of incident deuterons to measure the lag in time of protons emitted by compound state formation.

\mathcal{U}_{CP} is the "optical model" potential for the elastic scattering of the proton by the nucleus. Defining the elastically scattered proton wave function by f_P , we have

$$f_P = \Omega_{CP} \lambda_P. \quad (24)$$

Following the argument of the previous section, we readily find that Eq. (17) is modified as follows:⁶

$$T(B, \mathbf{p}; a) = (g_B^{(-)} f_P^{(-)}, V \Omega_{CN} \chi_a), \quad (25)$$

or that the plane wave λ_P is replaced by the elastically scattered wave $f_P^{(-)}$. $f_P^{(-)}$ contains the "shadow" cast by the nucleus, and as such contains a "bundle" of plane waves centered about the direction \mathbf{p} with an angular spread whose width is that of the diffraction pattern. In a momentum representation, we write $f_P^{(-)} = f_P^{(-)}(k)$, so that Eq. (25) becomes

$$T(B, \mathbf{p}; a) = \int d^3k f_P^{(-)}(k) T^0(B, \mathbf{k}; a), \quad (26)$$

where T^0 is the expression given by Eq. (17). The modification arising from the use of $f_P^{(-)}$ is then seen to be a "smearing" of Butler's angular distribution over angles of the order of the width of the nuclear diffraction pattern. For instance, the zeros in Butler's angular distribution will tend to be filled in somewhat. This correction does not seem to be of much practical importance, since the finite angular resolution of the experimental detecting equipment will also "smear out" the angular distribution. Furthermore, compound nucleus formation, which we neglected when approximation (22) was made, is probably equally important in obscuring the finer details of the angular distribution of Butler. An effect similar to this is the diffraction of the incoming deuteron wave by \mathcal{U}_{CN} . The wave function $\Omega_{CN} \chi_a$ includes this automatically, but the effect was neglected, for instance, in Eq. (20).

We conclude from this section, then, that the role played by the proton in the (d, p) reaction can be simply expressed but that it is not of great importance except for the contribution through compound state formation.

IV. THE PICKUP PROCESS

The inverse of the stripping reaction is the deuteron pickup process [i.e., the (p, d) and (n, d) reactions]. The cross section for pickup can be obtained either directly as in the preceding sections or by applying the time reversal operator to Eq. (25). The result is

$$T(a; B, \mathbf{p}) = (\Omega_{CN}^{(-)} \chi_a, V f_P g_B) \quad (27)$$

in the notation of Eq. (25). The matrix $\Omega_{CN}^{(-)}$ is the solution to⁹ [see Eq. (16)]:

$$\Omega_{CN}^{(-)} = 1 + \frac{1}{a^\dagger - V} \mathcal{U}_{CN} \Omega_{CN}^{(-)}. \quad (28)$$

Equation (27) describes the reaction in which an incident proton of momentum \mathbf{p} collides with a nucleus and picks up a neutron to emerge as a deuteron with momentum \mathbf{K} .

V. FINAL COMMENTS

Except for the need of some additional comment concerning the Pauli principle, a rigorous formulation of the Butler theory has been given by Eqs. (15') and (21). We may however, antisymmetrize our wave function merely by applying the antisymmetrization operator to it. The antisymmetrization of the proton coordinates will introduce terms corresponding to the exchange of the incident proton with one of those in the nucleus. Because of the fact that rather large impact parameters for the proton seem to be involved,¹ it appears to be consistent with the spirit of the calculation to neglect the proton exchange terms. On the other hand, if we expand the scattered waves in a set of antisymmetrized nuclear states $g_B(\xi, \eta_N)$ —as we have done, for instance, in Eq. (9)—then (except for the incident wave) our wave functions are automatically antisymmetrized in the neutron coordinates. We have thus neglected only the antisymmetrization with respect to the incident proton, and this seems to be a reasonable approximation.

The crucial points in our analysis have been those which replaced the many-body interactions u and v by the "potential well" interactions \mathcal{U}_{CP} and \mathcal{U}_{CN} , which do not depend at all upon the nucleon coordinates. These approximations have been explicitly defined mathematically above. Physically, they imply that no large momentum transfers between the proton and the nucleus are permitted.

If the potentials \mathcal{U}_{CN} and \mathcal{U}_{CP} contain spin-orbit interactions, we may expect that in general the spin of the proton will be polarized (this is not expected on the basis of the Butler theory). Such spin-orbit interactions would not be surprising, since it is not unlikely that our \mathcal{U}_{CN} and \mathcal{U}_{CP} are related to the "single particle potential" frequently presupposed in the nuclear shell model. Another mechanism to obtain polarization of the proton in the stripping reaction has been proposed by Newns.¹⁵ In terms of the present theory, his model is equivalent to assuming that \mathcal{U}_{CP} has a large absorption coefficient.

¹⁵ H. C. Newns, Proc. Phys. Soc. (London) **A66**, 477 (1953).