The Tendency for Positive Nuclear Quadrupole Moments*

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An examination of the signs of nuclear quadrupole moments and of the nuclear states which produce them affords fairly striking evidence that a positive quadrupole moment tends to lower the energy of a nuclear state.

For nuclei containing several particles outside of closed shells the shell model often does not uniquely specify the sign of the quadrupole moment. The ground states of such nuclei are, however, known to have positive quadrupole moments in the great majority of cases. Additional evidence for this tendency is provided by a discussion of:

(a) The ground states of Cu, Ga, As, Br, and Rb.
(b) The exceptions to the single-particle version of the shell model, Na^{23}, Mn^{55}, Se^{79}, Eu^{153}, and Yb^{173}.
(c) The spins of ground states of odd-odd $N=Z$ nuclei.

The favoring of positive quadrupole moments appears to be accounted for by electrostatic and other known forces in conjunction with large core deformations. It may, perhaps, also be influenced by tensor forces.

INTRODUCTION

It has been known for some time that quadrupole moments of odd-A nuclei vary in a more or less regular fashion with the number of odd protons or neutrons.[1,2] Some features of these variations can be accounted for on the basis of the shell model.[1,3] According to the independent-particle j-j coupling shell model, one would expect that quadrupole moments (Q) of nuclei containing one proton outside a closed shell would be negative and that nuclei lacking one from a closed shell would have positive Q. This expectation agrees with experimental observations for all major shells. However, for nuclei containing several particles (or holes) outside closed shells, the sign and magnitude of Q cannot be so uniquely predicted.

The size of some measured quadrupole moments (particularly of rare earth nuclei) is much larger than such a model would predict. Also, a number of odd-neutron nuclei are known to have a quadrupole moment in spite of the absence of electric charge on a neutron. The latter observation can be accounted for on the basis of polarization of the proton distribution by the neutrons.[1] If this type of polarization is allowed to deform the proton distribution including those in closed shells or the nuclear core, qualitative agreement can be obtained with experiment as far as magnitudes are concerned.[4] In fact, simple calculations of this polarization effect tend to overestimate quadrupole moments somewhat. The assumption of a deformable core does not prevent a definite prediction of the sign of Q for nuclei containing one proton outside of, or missing from a closed shell, and it allows prediction of the sign of Q to be made for odd-neutron nuclei.

An examination of the signs of nuclear quadrupole moments and of the nuclear states which produce them affords fairly striking evidence that a positive quadrupole moment tends to lower the energy of a nuclear state.

In Sec. I, evidence for a predominance of positive Q for odd-A nuclei is discussed. Those odd-A nuclei which do not correspond to a closed major shell plus or minus one particle and for which Q has been measured, have a positive quadrupole moment in the large majority of cases. For a number of other nuclei for which the quadrupole moment has not been measured, an interpretation of the measured spins and magnetic moments can be taken as further evidence for a predominance of positive quadrupole moments. In addition, certain exceptions to the expected spins of ground states can be interpreted as due to a favoring of positive Q.

In Sec. II, evidence for the predominance of positive Q in odd-odd $N=Z$ nuclei is discussed. As in the case of odd-A nuclei, support for this tendency comes both from measured Q's and from an interpretation of some spins.

In Sec. III and in the appendix, some theoretical discussion of this tendency for favoring positive Q's is given. It is pointed out that electrostatic and surface forces in conjunction with large core deformations can produce such an effect. Second-order effects of tensor forces might also contribute to it.

I. EVIDENCE FOR FAVORING OF POSITIVE QUADRUPOLE MOMENTS (ODD-A NUCLEI)

Consider a nucleus made up of closed shells of nucleons plus an additional even number of neutrons and an odd number of protons. It is normally expected

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[1] Townes, Foley, and Low, Phys. Rev. **76**, 1415 (1949).
[2] W. Gordy, Phys. Rev. **76**, 139 (1949).
[3] M. G. Mayer, Phys. Rev. **75**, 1969 (1949); **78**, 16 (1950); Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949); Z. Physik **128**, 295 (1950).
[4] J. Rainwater, Phys. Rev. **79**, 432 (1950); A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **26**, No. 14, (1952); R. van Wageningen and J. De Boer, Physica **18**, 369 (1952); D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953); K. W. Ford, Phys. Rev. **90**, 29 (1953).

that for the ground state of such a nucleus, the neutrons will all be paired[5] and all but the last odd proton will be paired; i.e., the state has seniority 1. If all of these protons are in the same shell and subshell, it can be shown that the nuclear quadrupole moment is a simple linear function of the number of protons, having a maximum negative value when there is only one proton in the subshell, a maximum positive value of equal magnitude when the subshell is complete except for one proton and a value zero when the subshell is half-filled. Hence if this subshell fills regularly as protons are added (or if it fills in a completely random way), there will be an equal number of nuclei with positive and with negative quadrupole moments. A similar argument holds for the signs of the quadrupole moments of odd-neutron nuclei if it is remembered that the neutrons can create an electric quadrupole moment by polarizing the core of the paired protons.

All odd-A nuclei whose quadrupole moments have so far been measured are listed in Table I with the values of their spins.[6–10] Except where noted, the data are taken from Klinkenberg's recent comprehensive table.[6] Klinkenberg's configuration assignments for these nuclei are given in Table I except for some cases discussed below.

TABLE I. Signs of measured quadrupole moments of odd-A nuclei.

Nucleus	Number of odd nucleons	Spin	Configuration of odd nucleons		Sign of Q	Footnotes
B^{11}	5	3/2	$(p_{3/2})^3$		$+$	
O^{17}	9	5/2	$d_{5/2}$	$+1$	$-$	
Na^{23}	11	3/2	$(d_{5/2})^2 s_{1/2}$		$+$	a
Al^{27}	13	5/2	$(d_{5/2})^5$		$+$	
S^{33}	17	3/2	$d_{3/2}$		$-$	
Cl^{35,37}	17	3/2	$d_{3/2}$		$-$	
S^{35}	19	3/2	$(d_{3/2})^3$	-1	$+$	
Mn^{55}	25	5/2	$(f_{7/2})^5$		$+$	b
Cu^{63,65}	29	3/2	$(p_{3/2})$	$+1$	$-$	
Ga^{69,71}	31	3/2	$(p_{3/2})^3$		$+$	
As^{75}	33	3/2	$(p_{3/2})^3(f_{5/2})^2$		$+$	
Br^{79,81}	35	3/2	$(p_{3/2})^3(f_{5/2})^4$		$+$	
Ge^{73}	41	9/2	$(p_{1/2})^2(g_{9/2})$		$-$	c
Se^{79}	45	7/2	$(g_{9/2})^7$		$+$	d
Kr^{83}	47	9/2	$(p_{1/2})^2(g_{9/2})^7$		$+$	e
In^{113,115}	49	9/2	$(p_{1/2})^2(g_{9/2})^9$	-1	$+$	
Sb^{121}	51	5/2	$d_{5/2}$	$+1$	$-$	
Sb^{123}	51	7/2	$g_{7/2}$	$+1$	$-$	
I^{127}	53	5/2	$(g_{7/2})^2 d_{5/2}$		$-$	
I^{129}	53	7/2	$(g_{7/2})^3$		$-$	
Eu^{151}	63	5/2	$(g_{7/2})^8(d_{5/2})^5$		$+$	
Eu^{153}	63	5/2	$(g_{7/2})^5(d_{5/2})^6(h_{11/2})^2$		$+$	f
Lu^{175}	71	7/2	$(g_{7/2})^7$		$+$	
Ta^{181}	73	7/2	$(g_{7/2})^7$		$+$	
Re^{185,187}	75	5/2	$(d_{5/2})^5$		$+$	
Xe^{131}	77	3/2	$(d_{3/2})$		$-$	
Ir^{191,193}	77	3/2	$(d_{3/2})^3$		$+$	g
Bi^{209}	83	9/2	$h_{9/2}$	$+1$	$-$	
Yb^{173}	103	5/2	$(f_{7/2})^5$		$+$	h
Os^{189}	113	3/2	$(p_{3/2})^3$		$+$	i
Hg^{201}	121	3/2	$(p_{3/2})^3$		$+$	j

[a] Na^{23}. Q measured by Sagalyn (reference 7). $(d_{5/2})^3$ would give $Q=0$.
[b] Mn^{55}. Q measured by Javan, Silvey, Townes, and Grosse (reference 8).
[c] Ge^{73}. $(g_{9/2})^3$ is an alternative configuration, which also has negative Q.
[d] Se^{79}. Q measured by Hardy, Silvey, and Townes (reference 9). The configuration $(p_{1/2})^2(g_{9/2})^5$ would not be expected to have the large positive quadrupole moment which is observed.
[e] Kr^{83}. $(g_{9/2})^9$ is an alternative configuration which also yields positive Q.
[f] Eu^{153}. This configuration assignment is suggested by both the large deviation of measured μ from the $d_{5/2}$ Schmidt line, and by the large positive measured Q. (In contrast to values of μ and Q for Eu^{151}.)
[g] Ir^{191,193}. Q measured by Murakawa and Suwa (reference 10). The assignment[6] $d_{3/2}$ would give $Q<0$.
[h] Yb^{173}. This assignment is suggested by the large deviation of the measured μ from the $f_{5/2}$ Schmidt line and the large positive measured Q. The assignment $(z_{5/2})^3$ would give $Q=0$.
[i] Os^{189}. I and Q measured by Murakawa and Suwa (reference 10).
[j] Hg^{201}. The assignment (reference 6) $p_{3/2}$ would give $Q<0$.

A number of nuclei can reliably be considered to have exactly one particle outside of, or missing from, a closed major shell. Such nuclei are labeled by the symbols $+1$ and -1, respectively. For nuclei of this type, the configurations are uniquely specified, and so are the signs of the expected quadrupole moment. In all these cases, the measured signs of the measured quadrupole moments are in agreement with the expected ones. For the other nuclei, the configurations are not necessarily specified uniquely, and in some cases, it is possible to construct reasonable configurations having positive Q and others having negative Q. Table II shows the relative occurrence of positive and negative quadrupole moments. It is seen that positive quadrupole moments clearly predominate, particularly for those nuclei which have several particles outside of, or missing from, a closed shell. It should be noted also that positive quadrupole moments are in general two or three times larger than negative ones, as may be seen from inspection of the plot of quadrupole moments given by Townes, Foley, and Low.[1]

Quadrupole moments for many nuclei may be predicted from the configurations given by Klinkenberg.[6] Of those for which configurations are given and quadrupole moments have not yet been measured, 26 are predicted to be positive and 22 negative. This indicates that for the configurations assigned by Klinkenberg on reasonable bases to those nuclei whose quadrupole moments are unknown, there is no strong preference for positive or negative quadrupole moments. However, there are alternative reasonable configurations for many of these nuclei, and we would expect that measurements will show that there are more positive quadrupole moments in this group of nuclei than are expected from Klinkenberg's assignments.

An examination of Klinkenberg's configuration assignments shows that types of deviations from a simple regular filling of shells and subshells occur to produce a predominance of positive quadrupole moments. There is a pronounced tendency to fill a subshell

TABLE II. Number of odd-A nuclei having measured positive (or negative) Q.

Type of nucleus	$Q>0$	$Q<0$	Total
Closed shell ±1	3	6	9
Other	22	7	29
Total	25	13	38

[5] M. G. Mayer, Phys. Rev. **78**, 22 (1950).
[6] P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).
[7] P. Sagalyn (to be published).
[8] Javan, Silvey, Townes, and Grosse, Phys. Rev. **91**, 222 (1953).
[9] Hardy, Silvey, and Townes, Phys. Rev. **86**, 608 (1952).
[10] K. Murakawa and S. Suwa, Phys. Rev. **87**, 1048 (1952).

almost, but not quite full. This of course produces positive quadrupole moments. The usual ideas of shell structure and additional stability inherent in a closed shell[5] would lead one to expect a more or less regular filling of a shell, with some tendency to complete the shell. However, experimental results show that some subshells are filled except for the last particle, after which subsequent particles go in pairs into new subshells, hence giving the nucleus a positive quadrupole moment. The tendency to leave shells in an almost filled condition accounts for the large majority of irregularities in the filling of shells.

Consider, for example, the nuclei ${}_{29}Cu^{63}$, ${}_{31}Ga^{69}$, ${}_{33}As^{75}$, ${}_{35}Br^{79}$, and ${}_{37}Rb^{85}$—odd-proton nuclei following immediately after a closed major shell of 28 particles. The first particle after 28 occupies a $p_{3/2}$ level, necessarily giving Cu a negative quadrupole moment. In Ga, two more particles have been added to give the filled shell minus one configuration $(p_{3/2})^3$. Pairs of additional particles go into the $f_{5/2}$ shell, so that As and Br also have positive quadrupole moments. Such behavior might possibly be attributed to a large pairing energy in the $f_{5/2}$ state, which requires always an even number of particles in this state, rather than a tendency to leave the $p_{3/2}$ shell almost filled. However, with the addition of two more particles, a configuration $(p_{3/2})^4(f_{5/2})^5$ is obtained in Rb^{85}. Here the $p_{3/2}$ shell has been able to fill in spite of the greater pairing energy in $f_{5/2}$, because the $f_{5/2}$ shell can be left almost filled, thus giving a large positive quadrupole moment. A similar anomaly occurs in the neutron levels at this same number of particles, and in several other subshells.

Another interesting case is the odd-neutron nucleus Hg^{201} for which Klinkenberg[6] gives the reasonable configuration $(f_{5/2})^6(i_{13/2})^{14}(p_{3/2})$. If this configuration were correct, Hg^{201} would have a negative Q instead of the measured positive value.[11] The alternative configuration which gives the proper positive quadrupole moment can be fairly uniquely assigned as $(f_{5/2})^6(i_{13/2})^{12}(p_{3/2})^3$. A similar situation exists for the pair of odd proton nuclei $Ir^{191,193}$, for which Klinkenberg[6] made the assignments $d_{3/2}$. Recently the Q of both nuclei was measured and found to be positive,[10] which requires instead a $(d_{3/2})^3$ assignment. In these three cases, the tendency for positive quadrupole moments appears to dominate over any tendency to complete subshells (e.g., $i_{13/2}$). Measurement of quadrupole moments for some of the other heavy nuclei may reveal other similar cases where Klinkenberg's assignments need modification.

There are five well-known nuclei which represent exceptions to Mayer's[5] scheme for nuclear spins. These are ${}_{11}Na^{23}$, ${}_{25}Mn^{55}$, ${}_{34}Se^{79}$, ${}_{63}Eu^{153}$, and Yb^{173}.[11] The quadrupole moments of all five of these nuclei have now been measured, and all are large and positive, again indicating that some interaction which tends to lower the energy of nuclei with positive quadrupole moments is strong enough to produce exceptions to the normal nuclear ground states. A more detailed discussion of these five well-established exceptions is given below.

${}_{11}Na^{23}$ has recently been found to have a rather large positive quadrupole moment.[7] This is inconsistent with the configuration $(d_{5/2})^3$ which is usually assumed for Na^{23}, since this configuration would give zero quadrupole moment in accordance with Table III. A more likely configuration is $(d_{5/2})^2(s_{1/2})$. With a spin of 3/2 this gives a rather large positive quadrupole moment as observed, and furthermore gives a magnetic moment of 1.77 nuclear magnetons. This is closer to the observed value 2.22 nm than the 2.87 nm to be expected from a $(d_{5/2})^3$ configuration with spin 3/2. There appears to be no good reason why $(d_{5/2})^2(s_{1/2})$ should be less favored energetically as the ground state of Na^{23} than $(d_{5/2})^3$, and it is the only configuration of three protons in $d_{5/2}$ and $s_{1/2}$ orbits which gives the observed positive quadrupole moment.

The alternative configuration $(d_{5/2})^7$ (protons *and* neutrons outside of closed shells), with spin $I=3/2$, "charge" $T_z=-1/2$, isotopic spin $T=1/2$, "seniority" $v=3$, and "reduced isotopic spin" 3/2 which was suggested by Umezawa,[12] gives a theoretical μ of 1.98 nm in good agreement with experiment.[13] It has, in fact, been shown[12] that magnetic moments of light odd-A nuclei calculated by considering both protons *and* neutrons outside of closed shells are in better agreement with observed values than those calculated by considering only the odd nucleons (e.g., the Schmidt lines for only one odd nucleon or hole). However the quadrupole moment of the above configuration was calculated and found to vanish, in disagreement with the experimental result.

In both V^{51} and Mn^{55} with 23 and 25 protons, respectively, all the protons above 20 are believed to go into the $f_{7/2}$ orbit. The only proton assignments consistent with the spins of these nuclei are $(f_{7/2})^3{}_{7/2-}$ for V^{51} and $(f_{7/2})^5{}_{5/2-}$ for Mn^{55}. At present it is not too well understood why these two nuclei have different spins. Two-body velocity-independent interactions between any pair of 7/2 protons would, in first order, give the same level spacing for the configuration $(7/2)^5$ as for the complimentary configuration $(7/2)^3$ provided the radial wave functions are the same.[14]

The quadrupole moments expected for both con. figurations can be calculated by the method of traces- Relative values of quadrupole moments for some configurations of interest are given in Table III. Suppose that for both V^{51} and Mn^{55} there is competition for the

[11] W. G. Holliday and R. G. Sachs have also pointed out in a private communication the necessity for modifying Klinkenberg's assignments for this nucleus and for Yb^{173} (see following).

[12] M. Umezawa, Prog. Theor. Phys. 8, 509 (1952). See also B. H. Flowers, Phil. Mag. 43, 1330 (1952).

[13] Seniority = number of unpaired particles. Reduced isotopic spin = isotopic spin of unpaired particles. Umezawa (reference 12) gives 2.38 nm for this configuration, but our calculations give the above value.

[14] G. Racah, Phys. Rev. 62, 438 (1942); A. M. L. Messiah, Atomic Energy Commission Report NYO-3212 (unpublished).

TABLE III. Relative values of quadrupole moments Q_p for some proton configurations of interest (in terms of the magnitude of quadrupole moment $|Q_{\mathrm{s.p.}}|$ produced by a single proton).[a] v = seniority number = number of unpaired particles.

Config. $(j)^n$	Spin I	v	Quadrupole moment of protons Q_p				
			$n=1$				
$(1/2)^n$	1/2	1	0				
			$n=1$	3 (or -1)			
$(3/2)^n$	3/2	1	-1	$+1$			
			$n=1$	3	5 (or -1)		
$(5/2)^n$	5/2	1	-1	0	$+1$		
	3/2	3		0			
			$n=1$	3	5 (or -3)	7 (or -1)	
$(7/2)^n$	7/2	1	-1	-0.33	$+0.33$	$+1$	
	5/2	3		-0.93	$+0.93$		
			$n=1$	3	5	7 (or -3)	9 (or -1)
$(9/2)^n$	9/2	1	-1	-0.5	0	$+0.5$	$+1$
	7/2	3 or 5		-1.34	degen. $-0.2<Q_p<+0.2$	$+1.34$	

[a] The quadrupole moment for a single proton in a state of spin j and $m=\pm j$ is

$$Q_{\mathrm{s.p.}}=-\frac{2j-1}{2j+2}e\langle\psi_j|r^2|\psi_j\rangle.$$

ground state between the states 7/2− and 5/2−. Then the suggestion that the state of these two which has the largest algebraic (most positive) value of Q is favored energetically would, according to Table III, account for the spins of both V^{51} and Mn^{55}.[15]

The $(g_{9/2})^5$ configuration has two states of spin 7/2. The nuclear quadrupole moment depends on what combination of the two possible states make up the ground state of the nucleus. Hence in Table III the moment is labeled by the word "degenerate," indicating that some, but not much variation is possible. Recently the quadrupole moment of ${}_{34}Se^{79}$, which has 45 neutrons and a spin of 7/2, has been found to be large and positive.[9] Its magnetic moment[17] has also been found to be consistent with that expected from a configuration of 9/2 neutrons. Hence its state is probably $(g_{9/2})^7$. In any case, this exception to Mayer's rules for nuclear spin has a large positive quadrupole moment.

${}_{63}Eu^{153}$ has been assigned the configuration $(g_{7/2})^8(d_{5/2})^5$ by Klinkenberg. However, its magnetic moment does not correspond to the value expected for a $d_{5/2}$ proton. The most reasonable configuration which would give the correct spin and magnetic moment would be $(g_{7/2})^5(d_{5/2})^6(h_{11/2})^2$. In accordance with Table III, a spin of 5/2 in this configuration would give a large positive quadrupole moment as observed.

Yb^{173} has been assigned the configuration $(h_{9/2})^{10}(f_{7/2})^8(f_{5/2})^3$ by Klinkenberg. If this assignment were valid, Yb^{173} would have a zero or very small quadrupole moment instead of the very large positive value which has been measured. Moreover, its magnetic moment does not correspond to the value expected for a $f_{5/2}$ neutron, as pointed out to us by M. G. Mayer. A reasonable configuration which gives both the correct magnetic moment and the observed positive sign of Q is $(h_{9/2})^{10}(f_{5/2})^6(f_{7/2})^5{}_{5/2}$.

II. EVIDENCE FOR FAVORING OF POSITIVE QUADRUPOLE MOMENTS (ODD-ODD NUCLEI WITH $N=Z$)

Quadrupole moments of four odd-odd $N=Z$ nuclei have been measured to date. Three of these (H^2, B^{10}, and N^{14}) are known to be positive. One, Li^6, has a very small value and its sign is not yet determined.

If it is assumed, in addition to j-j coupling, that the shells fill up in known order (*viz.*, $s_{1/2}$, $p_{3/2}$, $p_{1/2}$, $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, $f_{7/2}$, etc.), and if the spin of a given nucleus is also known, the assignment of the nuclear state can often be uniquely determined. From the latter, the expected sign of the quadrupole moment can be predicted.

The ground-state spins of five of these nuclei, H^2, Li^6, B^{10}, N^{14}, and Na^{22}, have been measured.[6] The ground state spins of F^{18}, P^{30}, and K^{38} can be deduced from beta-decay schemes.[18,19] In all the above cases it is likely that $T=0$ and I is odd.[20] However, a recent

[15] The spins of Ca^{43} and Ti^{47} recently measured by C. D. Jeffries (private communication) are 7/2 and 5/2, respectively. This provides another case of the same type of behavior.

[16] The spins of these nuclei may possibly also be accounted for in the following manner. (See the recent paper by A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. **27**, 16, (1952), pp. 34–37). For a nucleus containing three particles or holes in a subshell of spin j, the ground state is expected to have spin j if the interactions between particles are of short range and if other interactions, such as surface effects, are not significant. Such a situation is likely to occur in ${}_{20}Ca_{23}{}^{43}$ and ${}_{23}V_{28}{}^{51}$, for which the even particles form closed shells. On the other hand, if the nucleus under consideration is strongly deformed, one would expect that surface effects become important and perhaps even dominate over interparticle forces with resulting spin of $j-1$ for the ground state. This may be the case for the nuclei ${}_{22}Ti_{25}{}^{47}$ and ${}_{25}Mn_{30}{}^{55}$, in which the even particles do not form closed shells.

[17] W. Hardy (private communication).

[18] R. W. King, Ph.D. dissertation, Washington University, St. Louis, Missouri, 1952 (unpublished). See also R. W. King and D. C. Peaslee, Phys. Rev. **90**, 1001, 1953.

[19] L. W. Nordheim, Revs. Modern Phys. **23**, 322 (1951).

[20] This point is discussed in more detail in a forthcoming paper by D. C. Peaslee and one of the present authors (S. A. M.).

analysis of the Cl^{34} decay scheme suggests an assignment $T=1$, $I=0$ for the ground state of this nucleus.[21]

These data are summarized in Table IV, which also gives j-j coupling configuration assignments, as well as measured and expected signs of quadrupole moments (the latter deduced from Table V, below). In none of the cases listed in Table IV, is a negative Q found experimentally, or expected on the basis of a j-j coupling model with the known spin values. As is seen from Table V, one would expect that a favoring of positive Q would lead to exactly the trend of spins found in Table IV; *viz.*, low spins near the beginning of a subshell and high spins near the end. The analysis of spins of odd-odd $N=Z$ nuclei thus furnishes additional evidence of some mechanism which favors positive quadrupole moments.

TABLE IV. Spins, isotopic spins, configuration assignments, and signs of quadrupole moments for ground states of odd-odd $N=Z$ nuclei.

Nucleus	Spin I	Isotopic spin T	Configuration	Quadrupole moment Measured	Expected
H^2	1[a]	0	$(s_{1/2})^2$	$+$	0
Li^6	1[a]	0	$(p_{3/2})^2$	~ 0	$+$
B^{10}	3[a]	0	$(p_{3/2})^{-2}$	$+$	$+$
N^{14}	1[a]	0	$(p_{1/2})^2$	$+$	0
F^{18}	1	0	$(d_{5/2})^2$		$+$
Na^{22}	3[a]	0	$(d_{5/2})^6$		degen.
P^{30}	1	0	$(s_{1/2})^2$		0
Cl^{34}	0	1	$(d_{3/2})^2$		0
K^{38}	3	0	$(d_{3/2})^{-2}$		$+$

[a] Denotes measured spin.

TABLE V. Relative values of quadrupole moments calculated for various j-j coupling configurations of isotopic spin 0.

Config. $(j)^n$	Spin I	Quadrupole moment Q_p		
$(1/2)^n$		$n=2$		
	1	0		
$(3/2)^n$		$n=2$	6 (or -2)	
	1	$+0.4$	-0.4	
	3	-1	$+1$	
		$n=2$	6	10 (or -2)
$(5/2)^n$	1	$+0.32$	degen.	-0.32
	3	$+0.03$	degen.	-0.03
	5	-1	degen.	$+1$

III. CONCLUSIONS

The regularities pointed out above suggest that in nuclei there is a tendency for states with positive quadrupole moments to be lower in energy than those with negative quadrupole moments. Such a tendency appears to be accounted for by electrostatic and other known forces associated with large core deformations.[4] It may perhaps also be influenced by tensor forces. It is well known that the energy required to deform a spherical nucleus into a spheroid (either disk or cigar-shaped) is proportional to the square of the deformation, in lowest order. However, it can be shown that there is slight favoring of the elongated shape, which increases with the extent of the deformation. A rough estimate of this effect in nuclei (see Appendix) suggests that it is large enough to account for the observed predominance of positive quadrupole moments.

Because of the fact that the positive quadrupole moment of the deuteron can be attributed to tensor forces, it seems reasonable to speculate on whether tensor forces acting between any pair of particles will energetically favor states of positive quadrupole moment also in heavy nuclei. A first-order calculation of the energy matrix of tensor forces shows no such favoring.[22] In fact, it has been shown by several authors[14] that velocity-independent two-body forces which satisfy certain invariance requirements[23] are expected to give the same level spacing for a configuration of particles as for the corresponding configuration of holes, provided the radial wave functions are the same. However, in second order, a favoring of positive quadrupole moment is quite possible due to the distortion of the nucleus by "attractive" tensor forces;[24] [e.g., for a system of nucleons with $L=0$, $S\neq 0$, $V_{12}=S_{12}V(r_{12})$, $V(r_{12})<0$, a positive quadrupole moment results from the deformation of nucleon orbits]. Repulsive tensor forces, $V(r_{12})>0$, would lead to a negative quadrupole moment. Although the known positive quadrupole moment of the H^2 ground state can be accounted for on the basis of an attractive n-p tensor force in the $^3S_1$ state, the character of the p-p tensor forces is not yet known.

Thus it appears that the tendency for positive quadrupole moments in nuclei is consistent with the presently known information about nuclear forces.

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APPENDIX. FAVORING OF POSITIVE QUADRUPOLE MOMENTS AS A CONSEQUENCE OF LARGE CORE DEFORMATIONS

Let the nucleus be treated as a system of[4] (1) a core containing the "inner" nucleons in filled shells, the boundary of this core being assumed to change slowly with time compared with the motions of the individual nucleons; (2) "outer" nucleons in unfilled shells.

Let the core be deformed from a sphere of radius R_0 into an axially symmetric shape of radius $\rho(\theta)$ given by

$$\rho(\theta)=R_0[1+a_0+a_2P_2(\theta)+a_4P_4(\theta)+\cdots], \quad (1)$$

[21] W. Arber and P. Stähelin, Helv. Phys. Acta **26**, 433 (1953).

[22] I. Talmi, Phys. Rev. **89**, 1065 (1953).

[23] L. Eisenbud and E. P. Wigner, Proc. Nat. Acad. Sci. **27**, 281 (1941).

[24] G. M. Volkoff, Phys. Rev. **62**, 126 (1942).

where a_2 and a_4 define the sign and magnitude of the deformations, P_2, P_4 are Legendre polynomials, and a_0 is a coefficient whose value is adjusted so as to conserve volume. The magnitude of known quadrupole moments suggests that in many nuclei the core is deformed strongly and that at least a_2 is no more than one order of magnitude smaller than unity.

The change in surface and electrostatic energy of the core, relative to spherical shape, is, for uniform charge and mass density, given by:[25]

$$\begin{aligned}\Delta E_{S+E}=4\pi R_0^2\mathcal{O}[&(0.4-0.4x)a_2^2\\&-(0.03810+0.07619x)a_2^3\\&+(1-0.3704x)a_4^2-(0.1143+0.3429x)a_2^2a_4\\&-(0.2171-0.2563x)a_2^4].\end{aligned}\tag{2}$$

Here $\mathcal{O}$ is the surface tension, and $x=3Z^2e^2/40\pi R_0^3\mathcal{O}$, which is half the ratio of Coulomb energy to surface energy for a sphere. The first term of ΔE_{S+E} has been considered by many authors.[4] The second term, however, is of interest for large deformations and gives lower energies to positive quadrupole moments ($a_2>0$).

The interaction energy between the outer (identical) particles and the core is taken to be:[26]

$$\Delta E_{INT}=\langle\Psi|\Sigma_i[V(r_i)-V_0(r_i)]|\Psi\rangle,\tag{3}$$

with the potentials assumed to be:

$$\begin{aligned}V(r)&=-D \text{ for } r<\rho, & V_0(r)&=-D \text{ for } r<R_0,\\ &=0 \quad\text{ for } r>\rho, & &=0 \quad\text{ for } r>R_0,\end{aligned}$$

and where Ψ is the wave function of the outer particles, an antisymmetrized linear combination of products of single-particle wave functions

$$\psi_i=\mathcal{R}_{nl}(r_i)u_{lj}^{\Omega_i}(\theta_i,\phi_i,\sigma_i),\tag{4}$$

characterized by the quantum numbers Ω_i, the component of angular momentum along the axis of deformation. Then ΔE_{INT} is given approximately by

$$\begin{aligned}\Delta E_{INT}=-DR_0^3\mathcal{R}_{nl}^2(R_0)[&(a_2+2/7a_2^2)\langle P_2\rangle\\&+(a_4+18/35a_2^2)\langle P_4\rangle],\end{aligned}\tag{5}$$

where

$$\langle P_2\rangle=\sum_i\frac{j(j+1)-3\Omega_i^2}{4j(j+1)}$$

and, in general,

$$\langle P_n\rangle=\langle\Psi|\Sigma_iP_n(\theta)|\Psi\rangle.$$

The total energy change of the nucleus is

$$\Delta E_T=\Delta E_{S+E}+\Delta E_{INT},\tag{6}$$

if direct interactions between the outer particles are not taken into account.

To find the expected ground state for a given number of outer particles, we minimize ΔE_T with respect to a_2 and a_4. Although this minimum energy ΔE_M depends on both $\langle P_2\rangle$ and $\langle P_4\rangle$, the terms in $\langle P_4\rangle$ are ignored here for the sake of simplicity. A more detailed calculation shows that the inclusion of the terms in $\langle P_4\rangle$ leaves the following conclusions essentially unchanged.

The minimum energy to terms up to the third power in $\langle P_2\rangle$ is given by

$$\Delta E_M=-\epsilon_1\langle P_2\rangle^2-\epsilon_2\langle P_2\rangle^3,\tag{7}$$

where

$$\epsilon_1=\frac{[DR_0^3\mathcal{R}_{nl}^2(R_0)]^2}{16\pi R_0^2\mathcal{O}(0.4-0.4x)}$$

and

$$\epsilon_2=\frac{[DR_0^3\mathcal{R}_{nl}^2(R_0)]^3}{32\pi^2R_0^4\mathcal{O}^2(0.4-0.4x)^2}\left[\frac{1}{7}+\frac{0.03810+0.07619x}{4(0.4-0.4x)}\right].$$

The deformation a_{2M} giving lowest energy is

$$a_{2M}=\frac{2\epsilon_1\langle P_2\rangle+2\epsilon_2'\langle P_2\rangle^2}{DR_0^3\mathcal{R}_{nl}^2(R_0)},\tag{8}$$

where ϵ_2' is identical with ϵ_2 except for the replacement of the factor 4 by 8/3.

The quadrupole moment of the core[27] is:

$$Q_c=\frac{I}{I+1}\frac{2I-1}{2I+3}\frac{6}{5}ZeR_0^2\left[a_{2M}+\frac{4}{7}a_{2M}^2\right].\tag{9}$$

The value of ΔE_M depends on the occupation numbers of the various substates in the outer shell and hence on the fraction f to which a shell is filled. For example, for one particle in a state of $j>3/2$, the lowest energy, i.e., the most negative value of ΔE_M, is obtained if $\Omega_i=\pm j$, with P_2, a_{2M}, and Q_c all negative.

To the approximations made in this paper, the lowest state is always one of the following two:[28]

[25] R. D. Present and J. K. Knipp, Phys. Rev. **57**, 751 (1940).

[26] Second-order terms due to nondiagonal contributions of this first-order interaction may well be significant here, as was pointed out by Dr. B. R. Mottelson in a private communication. These terms are probably of at least the same order of magnitude as the other terms in a_2^2 from the first-order interaction. They act in such a way as to lower the energy of the ground state (a general property of all second-order perturbation terms) but do not in general give the same lowering for states of positive and negative quadrupole moments. However, a detailed calculation of their effect on the *relative* energies of states with positive and negative Q for a half-filled shell requires knowledge of the radial wave functions of excited states and has not yet been made.

[27] The quadrupole moment Q of the nucleus is, of course, the sum of the quadrupole moments of the core Q_c and of the outer particles Q_p. The latter was discussed in Secs. I and II. If the core deformation is not negligible; i.e., if $a_{2M}>1/Z$, one would expect that the main contribution to the nuclear quadrupole moment comes from the core. This is likely to be the case for the great majority of nuclei. The factor $[I/(I+1)][(2I-1)/(2I+3)]$ is the ratio of Q_c (for a substate with $M=I$) to the intrinsic quadrupole moment of a symmetric top (with $\Omega=I$), as was pointed out by Aage Bohr, Phys. Rev. **81**, 134 (1951).

[28] The spin values I of the ground state, calculated according to this model, are (see reference 4)

$$\begin{aligned}&\text{(a) for } Q_c<0, & I&=\Omega=\Sigma_i\Omega_i=N_0-N & &\text{if } N \text{ is odd}\\ & & &=0 & &\text{if } N \text{ is even}\\ &\text{(b) for } Q_c>0, & I&=\Omega=\Sigma_i\Omega_i=N & &\text{if } N \text{ is odd}\\ & & &=0 & &\text{if } N \text{ is even}\end{aligned}$$

where j is the spin of each particle. N is the number of nucleons in the subshell.

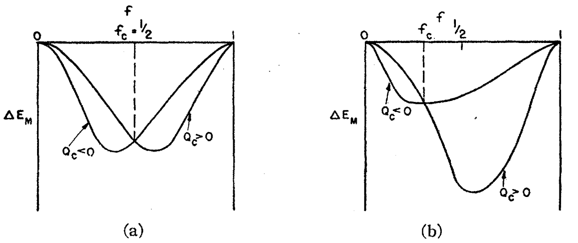

FIG. 1. Expected qualitative behavior of energy lowering as subshell is being filled. f = Fraction of subshell filled. E_M = Minimum energy for states with $Q_c<0$ and for states with $Q_c>0$. In (a) only terms in ϵ_1 are taken into account. In (b) both terms in ϵ_1 and in ϵ_2 are taken into account.

(a) substates filled from j downward

$$\Omega_i = \pm j,\ \pm j-1,\text{ etc.},\quad P_2,\ a_{2M},\ Q_c<0.$$

(b) substates filled from 1/2 upward

$$\Omega_i = \pm 1/2,\ \pm 3/2,\text{ etc.},\quad P_2,\ a_{2M},\ Q_c>0.$$

Approximate values of $\langle P_2\rangle$ for the fractional filling f of an outer shell of N_0 particles, for large N_0, are given by

$$\langle P_2\rangle_{Q_c<0} = -N_0 f(1-f)(2-f)/4, \tag{10a}$$

$$\langle P_2\rangle_{Q_c>0} = +N_0 f(1-f)(1+f)/4. \tag{10b}$$

In previous discussions,[4] only the term in ϵ_1 [Eq. (7)] has been considered. In this limit, the energy lowering is proportional to $\langle P_2\rangle^2$. As a consequence, for a less than half-filled shell ($f<1/2$), the disk-shaped form is favored, while if the shell is more than half-filled ($f>1/2$), the cigar-shaped form is lowest in energy. However, if the next order term in ϵ_2 is included in minimizing the energy, the "crossover" between these two forms occurs before the shell is half-filled. To the approximations employed here, the fraction f at crossover is given by

$$f_c = 0.5 - (9N_0\epsilon_2/128\epsilon_1). \tag{11}$$

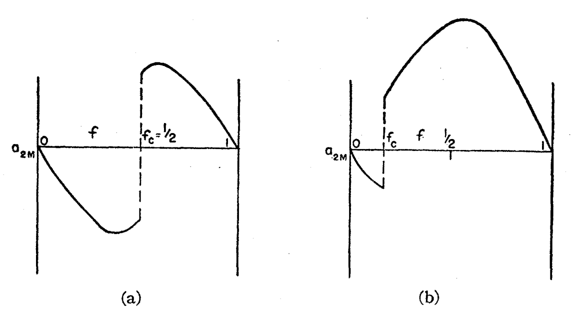

FIG. 2. Expected trend of equilibrium deformations as subshell is being filled. f = Fraction of subshell filled. a_{2M} = Equilibrium deformation. In (a) only terms in ϵ_1 are taken into account. In (b) both terms in ϵ_1 and in ϵ_2 are taken into account.

For a half-filled shell the state of positive quadrupole moment is lower than that of negative quadrupole moment by an energy

$$\Delta E_{M,\,Q_c<0} - \Delta E_{M,\,Q_c>0} = (27/16384)N_0^3\epsilon_2. \tag{12}$$

The expected qualitative behavior of energy lowering as function of fraction of subshell filled is sketched in Fig. 1. In Fig. 1(a) only terms in ϵ_1 are taken into account. In Fig. 1(b), both terms in ϵ_1 and in ϵ_2 are considered.

The dependence of energy lowering on the fraction of subshell filled has been calculated [Eqs. (11), (12)] for a special case to estimate the order of magnitude of this effect in a nucleus. The following assumptions were made:[4]

$$j=7/2,\quad Z=70,\quad A=170,\quad 4\pi R_0^2\theta = 14A^{2/3}\text{ Mev},$$

$$x=0.60,\quad\text{and}\quad DR_0^3\mathcal{R}_{nl}^2(R_0)=40\text{ Mev}.$$

Then from Eqs. (7) we have

$$\epsilon_1 = 5.7\text{ Mev.}\quad \epsilon_2 = 1.8\text{ Mev.} \tag{13}$$

According to Eq. (11), the calculated crossover between states of negative and positive quadrupole moments occurs when the subshell is about one-third filled ($f_c \sim 0.3$). For a half-filled shell, it follows from Eq. (12) that the lowest state of positive Q is below the state of negative Q_c by 1.5 Mev. The maximum deformation of the core can be fairly large. In the above example, $a_{2M}=0.22$ for a half-filled shell.

It should be kept in mind that this crude calculation may well over-estimate the expected core deformation[4] and therefore the energy lowering. Nevertheless, the surprisingly large result obtained here suggests that the energy lowering of states with positive Q relative to states with negative Q due to large core deformation is enough in many cases to overshadow other effects which might tend to act in the opposite direction, e.g., the known tendency of outer particles to couple to zero spin in pairs.

The qualitative trend of equilibrium deformations a_{2M} which can be deduced from measured quadrupole moments, is plotted as function of shell filling in Fig. 2. The trend of deformations shown in Fig. 2(b) is in qualitative agreement with that deduced from measured quadrupole moments for nuclei in the region $Z=50$ to 82.[1,4] Negative quadrupole moments mainly appear immediately after the closure of major shells. This is presumably connected with the fact that the subshells do not fill strictly one after the other. Thus the dependence of quadrupole moments on filling of subshells tends to be suppressed relative to their dependence on filling of major shells.