positrons with a maximum energy of 0.9 Mev in agreement with the reported characteristics of Zr^{89}. The Zr^{89} is most probably produced by the reaction $Nb^{93}(p,\alpha n)Zr^{89}$. The excitation curve for this reaction is also shown in Fig. 3. The exact masses required for the calculation of the expected threshold are not known.

The author is indebted to Professor J. R. Richardson for his interest and advice, to Mr. S. Plunkett and the operating crew of the cyclotron for their cooperation in obtaining the bombardments, and to Dr. Harold Ticho and Mr. David Green for performing the gamma-ray spectrometer measurements.

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Angular Distribution of Protons from the Reaction $Na^{23}(d,p)Na^{24}$ †

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The angular distribution of several of the proton groups from the $Na^{23}(d,p)Na^{24}$ reaction was studied using 3-Mev deuterons. The results were compared with the Butler theory of the angular distributions from (d,p) reactions.

A value of $l_n=2$ is assigned to the orbital angular momentum of the neutron captured in the formation of the ground state of Na^{24} from Na^{23}. Since the available evidence indicates that the parity of the ground state of Na^{24} is even, a value of $l_n=2$ implies that the ground state of Na^{23} also has even parity.

The two proton groups resulting when Na^{24} is produced in the 0.472-Mev and 0.564-Mev excited states could not be resolved. The angular distribution of the sum of these two groups could not be uniquely interpreted. The author favors the interpretation that one of these levels is formed by capture of a neutron with $l_n=0$ and the other by capture of a neutron with $l_n=2$. Then both levels would have even parity, the possible spin values being 1 or 2 for one of the levels and 1, 2, 3, or 4 for the other level.

The data on the 1.341-Mev level in Na^{24} indicate that it is formed by capture of a neutron with $l_n=0$. Hence this level has even parity and a spin of either 1 or 2.

I. INTRODUCTION

THE study of the angular distribution of several of the proton groups from the $Na^{23}(d,p)Na^{24}$ reaction has been undertaken here. The results are compared with the theory of Butler[1] in order to obtain information on the spins and parities of several of the lower excited states of Na^{24} and on the parity of the ground state of Na^{23}. The results may also be used to check the accuracy of the shell model[2] in the assignment of orbital angular momentum states to individual nucleons in the nucleus.[3]

The energy of the proton groups resulting from the deuteron bombardment of Na^{23} has been studied by Sperduto and Beuchner[4] using magnetic analysis. Their results which are pertinent to this experiment are given in Table I.

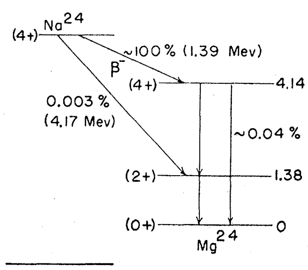

FIG. 1. Decay scheme of Na^{24}.

The spin[5] of the ground state of Na^{23} is $\frac{3}{2}$ and the spin[6] of the ground state of Na^{24} is 4.

The information available on the decay scheme[7] of Na^{24} is shown in Fig. 1. This information has to be analyzed so that the parity of the ground state of Na^{24} can be established. Mg^{24} is even-even so that the ground state has spin 0 and even parity. The angular correlation[8] of the cascade gamma rays from the 4.14-Mev level of Mg^{24} and the measurement of the internal pair conversion coefficients[9] of these gamma rays establish the spins and parities of the 1.38-Mev level and the 4.14-Mev level as shown in Fig. 1.

One can use this information on the energy levels of Mg^{24} and the data[10] available on the β decay of Na^{24} to determine the parity of the ground state of Na^{24}. The selection rules[11] for β decay indicate that if the parity of Na^{24} is odd, then both the 1.39-Mev and the 4.17-Mev β transitions are first forbidden. This is not in agreement with the ratio of the ft values for the two transitions; the ratio of the ft values for the 4.17-Mev

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[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).
[2] See for instance, P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).
[3] H. A. Bethe and S. T. Butler, Phys. Rev. **85**, 1045 (1952); S. T. Butler, Phys. Rev. **88**, 685 (1952).
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[9] S. D. Bloom, Phys. Rev. **88**, 312 (1952).
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[11] E. J. Konopinski and L. M. Langer, Ann. Rev. Nuc. Sci. **2**, 261 (1953).

β transition to that of the 1.39-Mev transition is 4×10^6. If the parity of Na^{24} is even, then the 1.39-Mev transition is allowed and the 4.17-Mev transition is second forbidden, which is in reasonable agreement with the ratio of the *ft* values. Furthermore, the 4.17-Mev β spectrum has an allowed shape. If this β transition were first forbidden, it would have the unique forbidden shape[11] which deviates from the allowed shape. Hence the 4.17-Mev β transition cannot be first forbidden. From this one can conclude that the parity of the ground state of Na^{24} is even. Furthermore the 1.39-Mev β spectrum has an allowed shape[12] and the β-γ angular correlation[13] from Na^{24} is isotropic. This is in agreement with the conclusion that the 1.39-Mev transition is allowed.

II. APPARATUS

The statitron at the State University of Iowa was used as a source of 3-Mev deuterons. The energy spread of the deuteron beam was less than 0.5 percent and the energy of the deuteron beam could be repeated from day to day to better than 0.5 percent. The absolute value of the energy is only known to 5 percent. The

TABLE I. Proton groups from $Na^{23}(d,p)Na^{24}$ reaction.

Proton group	Q (Mev)	Resultant energy level in Na^{24} (Mev)
1	4.731	0 (ground state)
2	4.259	0.472
3	4.167	0.564
4	3.390	1.341

above limits are conservative; actually spread in energy, reproducibility of energy setting and absolute value of energy are believed correct to a much higher accuracy.

Figure 2 shows a schematic diagram of the target chamber and counter. The target chamber is insulated from the statitron and can be used as a Faraday cage to collect the beam. The beam enters the target chamber through a slit (shown in Fig. 2) which is $\frac{1}{16}$ in. wide and $\frac{1}{8}$ in. high. This slit, which is insulated from the target chamber, determines the maximum size of the beam at the entrance to the target chamber.

A furnace was incorporated into the target chamber so that sodium targets could be evaporated in a vacuum to prevent oxidation of the sodium. The targets were evaporated onto a 1-mil silver foil,[14] which was used as a target backing. The silver target backing foils were checked for background by bombarding them with 3-Mev deuterons. Negligible background was found on all of the foils checked. The target was used at two

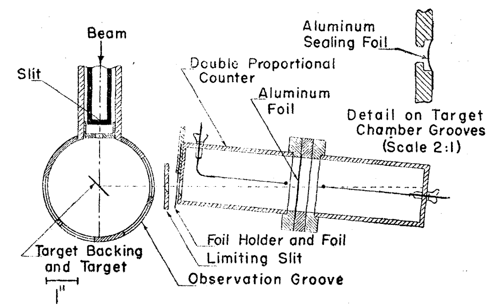

FIG. 2. Target chamber and counter.

orientations with respect to the beam, the target making an angle of 45° with respect to the beam in both orientations.

The protons to be counted at various angles come out of the target chamber through the target chamber grooves. These grooves were designed so that the protons could be observed at any angle with respect to the beam between 10° and 108° if part of the data were taken on one side of the beam and part of the data on the other side of the beam. For angles of 75° or less, the protons were observed going through the backing foil, and for angles greater than 85°, the protons were observed without going through the target backing. The shape of the target chamber grooves is shown in detail in Fig. 2. The grooves are $\frac{1}{8}$ in. high on the inside of the target chamber. The spread in azimuthal angle of the protons reaching the movable counter is limited to 3.5° by the limiting slit. With this arrangement protons within a solid angle of 1/2700 of a sphere are counted by the movable counter. The front half of a double proportional counter designed by Wm. E. Nickell was used as the movable counter. The movable counter has a large enough aperture so that all protons going through the observation grooves and limiting slit will be counted.

The central wire of the movable counter is a 5-mil tungsten wire maintained at a positive voltage of 1225 v. This counter and the monitor counter were filled with a mixture of 95-percent argon and 5-percent carbon dioxide at a pressure of 36 cm of mercury. Gas continually flowed through the counters in order to keep the counter gas as pure as possible. The pressure was kept at 36 cm by means of a Cartesian Manostat.[15]

The reaction was monitored by counting protons at a fixed angle with respect to the beam. The protons to be counted by the monitor counter come out through the window of one of two monitor tubes (not shown). These monitor tubes are on opposite sides of the beam. They make an azimuthal angle of 45° with respect to the beam and an angle of 30° with respect to the horizontal plane, thus making an angle of 52° with respect to the beam. The monitor counter could be placed over

[12] K. Seigbahn, Phys. Rev. **70**, 127 (1946).
[13] D. T. Stevenson and M. Deutsch, Phys. Rev. **83**, 1202 (1951).
[14] The silver foil was obtained from Baker and Company, Inc., 20 Church Street, New York 7, New York.

[15] Cartesian Manostat Model No. 5 was purchased from the Emil Greiner Company, 161 Sixth Avenue, New York 13, New York.

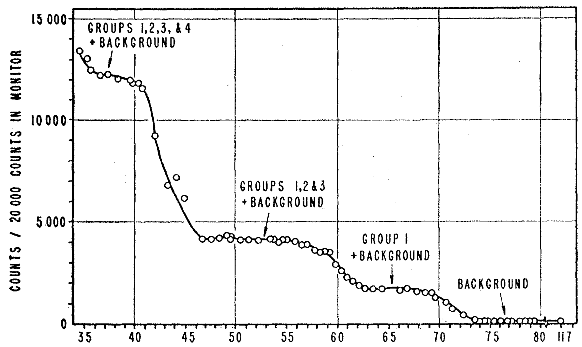

FIG. 3. Counts in movable counter *vs* foil thickness (taken at 90°).

either of the monitor tubes and was always used so that it made an azimuthal angle of 90° with respect to the target. The monitor counter has sufficiently large aperture so that it counts all protons getting through the monitor tube. Protons within a solid angle of 1/1750 of a sphere were counted by the monitor counter. The central wire of 5-mil tungsten was maintained at a positive voltage of 1150 volts. The inside diameter of this counter was $\frac{7}{8}$ in., and it had an effective length of approximately 2 in.

The pulses from the proportional counters went to cathode followers which fed them to conventional linear amplifiers. The output of the amplifiers went to integral pulse-height discriminators and then to scaling circuits.

Aluminum foils of various thicknesses were used as absorbers in order to sort out the protons of different energies. These aluminum foils were carefully weighed and mounted on foil holders so that they could be placed in front of the movable counter. The thickness of these aluminum foils is known to ± 0.1 mg/cm².

III. PROCEDURE

A thin sodium target was evaporated onto the silver backing foil. No attempt was made to measure the target thickness. A target was considered to be of the right thickness if it was thin enough to permit resolution of the proton groups being studied and at the same time was thick enough to give a reasonable counting rate.

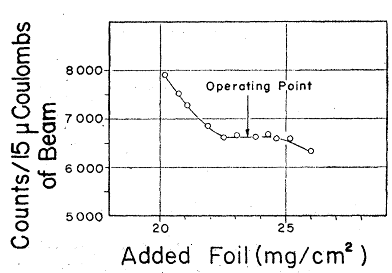

FIG. 4. Calibration of monitor counter.

The proton groups were separated by measurement of their range. The set of aluminum foils which could be placed in front of the movable counter was used as a variable absorber. The aluminum sealing foil on the target chamber and, in some cases, the target backing, were also between the target and the movable counter. Thus the movable counter could be used to count protons whose range is greater than a given value. The counts in the movable counter were all taken for a fixed number of counts in the monitor counter. A typical set of data is shown in Fig. 3.

The monitor counter counted all four groups of protons studied in this experiment. In order to determine the operating point of the monitor, the counting rate in the monitor was taken with various foils interposed between it and the monitor tube. These data and the operating point selected are shown in Fig. 4.

The counting rate in both the monitor counter and the movable counter are reasonably independent of discriminator setting. The monitor counting rate changes about $1\frac{1}{2}$ percent for a 5-v change in discriminator setting, and the counting rate in the movable counter does not change more than $1\frac{1}{2}$ percent for a 3-v change in the discriminator setting.

A set of data similar to that in Fig. 3 was taken at several angles. The abscissa in Fig. 3 only shows the aluminum foil added in front of the movable counter. It does not show any of the fixed absorbers that are between the target and the counter. The values obtained from these data for the range of the proton groups agrees reasonably well with the work of Sperduto and Buechner.[6] Groups 2 and 3 could not be resolved with this equipment. Therefore, the angular distribution of the sum of these two groups will be reported.

It was not necessary to run through a complete set of data, such as is shown in Fig. 3, at every angle. Since the energy of the protons varies slowly with angle, one can calculate the location of the flat regions of the curve and take counts only in these regions. This procedure does not introduce any error because data were taken over a sufficient range of oil thicknesses to insure that the right region was being covered.

In order to determine the intensity of any one of the proton groups, the background and the intensity of all proton groups of higher energy must be subtracted from the data. The background is due to neutrons counted in the movable counter and may be seen in Fig. 3. It is the small counting rate beyond the range of group 1.

As a check against systematic errors, the counting rate for the sum of all four proton groups was repeated on both sides of the beam for two angles. These counting rates taken on opposite sides of the beam did not agree. A thorough check made on the dimensions of all of the apparatus and on the alignment of the target chamber to the statitron showed that the beam spot was off-center with respect to the target chamber and that the limiting aperture of one of the monitor tubes was farther away from the target than the other one. Corrections were made to the data to compensate for these errors. These corrections never exceeded 3 percent and could not introduce an error of more than 0.5 percent in the final results.

IV. RESULTS AND CONCLUSIONS

The experimental results, corrected and converted to the center-of-mass coordinate system, are shown as circles in Figs. 5, 6, and 7. The probable error in any point does not exceed the radius of the circle used in plotting the point. The curves are calculated from the Butler[1] theory of the angular distributions from (d,p) reactions using a value of $r_0=1.47\times(A^{\frac{1}{3}}+1)\times10^{-13}$ cm $=5.65\times10^{-13}$ cm. Although arbitrary units are used for the ordinates in the figures, the same units are used for all of the figures. Thus one can read the relative yields of the proton groups from the plotted results.

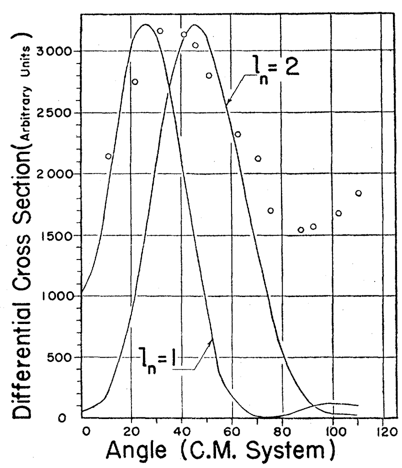

FIG. 5. Angular distribution of protons (ground state).

Angular Distribution of Protons (Ground State)

The theoretical curves for $l_n=1$ and $l_n=2$ are shown in Fig. 5 for comparison with the experimental results. If there were no further evidence available, one could not decide on this basis alone whether the experimental results are in agreement with the curve for $l_n=1$ or for $l_n=2$. The peak of the experimental curve lies between the peaks of the two theoretical curves and the width of the experimental curve is larger than that of the theoretical curves.

Values of l_n between 2 and 6 are the only values consistent with the known spins of the ground states of Na^{23} and Na^{24}. Hence a value of $l_n=2$ is assigned to the orbital angular momentum of the captured neutron in this reaction.

This result is in agreement with the shell model as one would expect that the ground state of Na^{24} is formed from Na^{23} by capture of a neutron into a d orbit. A value of $l_n=2$ implies no change of parity in the reaction.

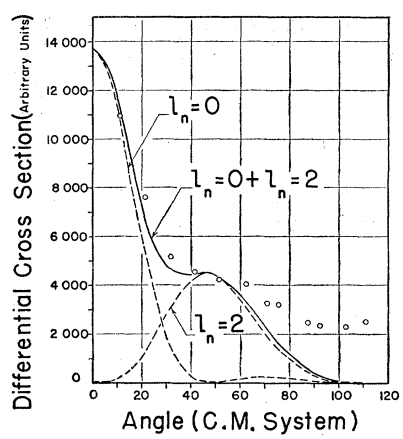

FIG. 6. Angular distribution of protons (0.472- and 0.564-Mev levels).

Since the parity of the ground state of Na^{24} is even, then the ground state of Na^{23} also must have even parity.

Angular Distribution of Protons (0.472-Mev and 0.564-Mev Levels)

Proton groups 2 and 3 could not be resolved with the apparatus used for this experiment. Figure 6 shows the angular distribution for the sum of these groups.

If one tries to fit the experimental results using a sum of theoretical curves involving only two values of l_n, then the only possible fit is for $l_n=0$ plus $l_n=2$. This is shown as a solid curve in the figure, the dashed curves being the individual curves for $l_n=0$ and for $l_n=2$. This result can be interpreted in four ways:

(1) One of the levels in Na^{24} is formed by capture of a neutron with $l_n=0$, and the other level is formed by capture of a neutron with $l_n=2$.

(2) Combinations of $l_n=0$ and $l_n=2$ may be involved in the formation of both of these levels.

(3) Both $l_n=0$ and $l_n=2$ are involved in the formation of one level while the other level is formed by capture of neutrons with either $l_n=0$ or $l_n=2$.

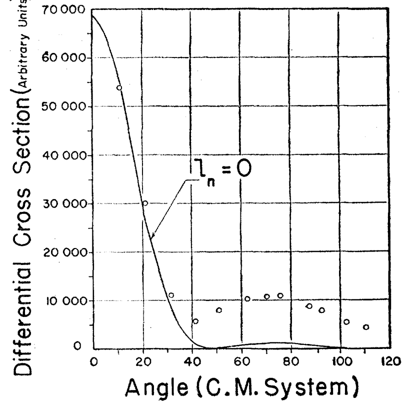

FIG. 7. Angular distribution of protons (1.341-Mev level).

(4) One of the levels is formed by capture of neutrons with both $l_n=0$ and $l_n=2$, whereas the other level is formed mostly by compound nucleus formation with the stripping process contributing through large values of l_n which could not be detected.

The first of these alternative interpretations is preferred by the author since very few cases of mixing of two values of l_n have been observed experimentally. Then both levels would have even parity, the possible spin values being 1 or 2 for one of the levels and 1, 2, 3, or 4 for the other level. The alternative interpretations (2) and (3) would also indicate that both levels have even parity. These interpretations are in agreement with the shell model. The fourth alternative seems quite unlikely but cannot be ruled out completely.

Other interpretations may be made if one uses mixtures of three or four values of l_n. Since the experimental results rise sharply in the forward direction, any mixture of various values of l_n must contain $l_n=0$. In addition to this, one could fit these results by adding to this two or three of the values $l_n=1, 2,$ or 3. Then there would be enough parameters available so that one could be sure to fit the experimental results. This would imply that the two levels in Na^{24} are of opposite paraty. The author does not favor these interpretations as they would be in complete disagreement with the shell model. Previous studies[16] of (d,p) and (d,n) angular distributions have in general shown agreement with the shell model. It is reasonable to assume that the shell model is correct unless positive evidence to the contrary is found.

It is hoped that future work on this reaction with better resolution will clarify the identification of these levels in Na^{24}.

Angular Distribution of Protons (1.341-Mev Level)

The experimental results for this group of protons, shown in Fig. 7, clearly fit the theoretical curve for $l_n=0$. Hence this level has even parity and a spin of either 1 or 2. This result is in agreement with the shell model.

Discussion

The experimental results are in reasonable agreement with the theoretical curves except for the angular distribution of group 1. These results suggest that although the Butler theory predicts the general shape of the angular distribution, some improvement in the theory is needed to account for all of the details of the angular distribution.

It may be necessary to take account of the Coulomb interaction of the incoming deuteron with the initial nucleus. The Coulomb barrier at a distance equal to r_0 from the center of the Na^{23} nucleus is 2.80 Mev as compared to the deuteron energy of 2.76 Mev in the center-of-mass system. On the other hand, Butler[17] has stated that for (d,p) reactions, the theory[3] should be quite good even for deuteron energies lower than the Coulomb barrier. He states that it is not so much the deuteron energy but the final proton energy which is of importance since the Coulomb field only enters in the way it affects the outgoing protons. In this experiment, the energy of the outgoing protons in the center-of-mass system was between 6.1 Mev and 7.5 Mev which was well above the Coulomb barrier.

All of the experimental angular distributions are higher than the theoretical curves at backward angles. This is probably due to a contribution from that part of the reaction which goes by compound nucleus formation. In addition to this, if the contribution from compound nucleus formation is very anisotropic, it might account for the poor fit of the experimental data for group 1 to the theoretical curves.

It is also possible that there may be a mixture of higher values of l_n in the stripping process which cannot be detected. This would also cause the experimental angular distributions to be higher in the backward direction than the theoretical curves shown.

V. ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Professor James A. Jacobs who directed this research and made many valuable suggestions during the course of the work. I am indebted to Professor F. Coester for several valuable discussions concerning the theory related to this experiment. Thanks are also due to Philip R. Malmberg and William E. Nickell for their help and suggestions and to Dr. W. W. Pratt who assisted in taking the data during long runs on the statitron. The cooperation of Mr. J. G. Sentinella and the members of the instrument shop who constructed the apparatus is very much appreciated.

[16] See for example Parkinson, Beach, and King, Phys. Rev. **87**, 387 (1952); J. S. King and W. C. Parkinson, Phys. Rev. **88**, 141 (1952).

[17] S. T. Butler (private letter dated March 28, 1952).