the curve more accurately. The calculated thresholds for some of the possible reactions are shown in Table I. These thresholds include the mass difference between the reactants and the products and the energy required for passage over the potential barrier. The Coulombic requirements were calculated assuming that the nucleus and the emitted fragments are spherical and tangent at the nuclear radii (taken as $1.48\times10^{-13}A^{\frac{1}{3}}$ cm) and that the particles come out consecutively. Because of uncertainties inherent in calculating Coulombic barrier requirements for charged particle reactions, it is not possible to determine from energy considerations what the actual mechanism is for the formation of Na^{22} near the threshold. Undoubtedly, there is some contribution from each of the following reactions: $Al^{27}(p, p\alpha n)Na^{22}$, $Al^{27}(p, \alpha d)Na^{22}$, and $Al^{27}(p, Li^6)Na^{22}$.

Na^{22} from Magnesium

Natural magnesium is an isotopic mixture of 78.6 percent Mg^{24}, 10.1 percent Mg^{25}, and 11.3 percent Mg^{26}. From the threshold considerations shown in Table I, the excitation function can be divided into two parts. The portion of the curve below 15 Mev represent the contribution to the excitation function due entirely to the $Mg^{25}(p, \alpha)Na^{22}$ reaction. Above this energy the mechanism of the reaction leading to the formation of Na^{22} is uncertain, and undoubtedly, is a combination of the possible reactions shown in Table I.

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Angular Distributions of 22-Mev Protons Elastically Scattered by Various Elements

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Angular distributions of 22-Mev protons elastically scattered by fifteen elements from beryllium to thorium were measured with the internal, circulating beam of the Oak Ridge National Laboratory 86-inch cyclotron. The experimental methods, which are somewhat unconventional, are described. The results show all the characteristics of diffraction scattering including at least two maxima and minima for each element. The angles at which these occur can be traced from element to element through the periodic table, following a $\lambda\!\!\!^{-}/R$ dependence with fairly good accuracy.

I. INTRODUCTION

THE theory of the scattering or absorption of a wave by matter with which it interacts is a well-known differential equation-boundary value problem of classical physics. While the mathematical techniques are devious and varied, basically the solution is obtained by assuming the appropriate wave equation (in the case of particle scattering, the Schrödinger equation) valid throughout the region external to the scattering or absorbing material, and inserting the appropriate boundary conditions at its surface.[1] Computations of this type for the elastic scattering of neutrons have been made by Feld *et al.*,[2] assuming spherical nuclei and the Feshbach-Weisskopf[3] boundary condition. For incident neutrons with wavelength of the order of the dimensions of the scattering nucleus, they find angular distributions characterized by several maxima and minima, as is expected in analogy with the familiar diffraction patterns in optical and acoustical scattering. As is also expected from these analogies, the angles at which the maxima and minima occur are determined only by the radius of the scattering nucleus, being essentially independent of the boundary conditions. On the other hand, the relative heights of the various maxima and minima are extremely sensitive to the boundary condition, varying by as much as a factor of 100 for various not unreasonable assumptions. Computations by Le Levier and Saxon[4] confirm that these features are valid for a more general type of boundary condition, and for charged particles as well as neutrons. It is thus apparent that reasonably accurate measurements of elastic scattering angular distributions should provide:

(a) Accurate determinations of nuclear radii, since the positions at which maxima and minima occur can be determined with good precision.

(b) A very sensitive determination of the nuclear boundary condition. This boundary condition is also of great importance in nuclear reaction theory since the theoretical treatments of absorption and scattering are essentially two parts of the same problem.

(c) Some estimate of deviations from sphericity of the nuclear surface. This would provide a very important check on the determinations through quadrupole moments which do not have a completely satisfactory status.

[1] J. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).
[2] Feld, Feshbach, Goldberger, Goldstein, and Weisskopf, NYO-636 (unpublished).
[3] H. Feshbach and V. F. Weisskopf, Phys. Rev. 76, 1550 (1949).
[4] R. E. Le Levier and D. S. Saxon, Phys. Rev. 87, 40 (1952).

Amaldi *et al.*[5] have made measurements for 14-Mev neutrons on lead, but only with sufficient accuracy to check the qualitative features of the theory. Two experiments[6,7] using 90-Mev neutrons and 340-Mev protons have been reported, but at these energies the nuclear radius loses meaning because of the high nuclear transparencies, and the boundary condition is less significant for the same reason and also because at these energies it is of little importance in connection with nuclear reaction theory. Neither of these experiments was able to provide accurate information on nuclear radii. A few measurements with low-energy protons have been reported,[8,9] but in these, the low energy severely reduces the diffraction region. In none of the measurements thus far mentioned has more than one minimum and maximum been observed.

Burkig and Wright[10] have made measurements with 18-Mev protons, and some of their results have been interpreted by Le Levier and Saxon.[4] Their data were taken at 10° intervals, however, and the details of the angular distributions are missed. Gugelot[11] has made measurements at 30° intervals for a few elements, but these suffer from the same deficiency.

In this paper, we present angular distributions of 22-Mev protons elastically scattered by fifteen elements as measured with the internal, circulating beam of the Oak Ridge National Laboratory 86-in. cyclotron by somewhat unconventional experimental methods. Data were taken at intervals of 2° to 4° which were found adequate to resolve the structure of the curves; the selection of scattering targets is such that these structural details can be traced from element to element through the periodic table. In practically every feature the accuracy of the measurements is limited by the characteristics of the internal cyclotron beam which include secular variations in mean energy of about 8 percent, energy inhomogeneity of about 10 percent, secular variations in angle of incidence on the target of about $\frac{1}{2}°$, and inhomogeneity in this angle of a little less than 1°. It is believed that a suitable theoretical analysis of these data would provide items (a) and (b) above; it is doubtful whether, in its present form, it could provide information on nuclear eccentricities [item (c)]; however, if theoretical developments of the general theory can be extended to show exactly what type of data is necessary, it is believed that the techniques herein described might be used to provide such information.

[5] Amaldi, Bacciarelli, Cacciapuoti, and Trabacchi, Nuovo cimento **15**, 203 (1946).
[6] Bratenahl, Farnbach, Hildebrand, Leith and Moyer, Phys. Rev. **77**, 597 (1950).
[7] Richardson, Ball, Leith, and Moyer, Phys. Rev. **86**, 29 (1952).
[8] Baker, Dodd, and Simmons, Phys. Rev. **85**, 1051 (1952).
[9] L. M. Goldman, Phys. Rev. **89**, 349 (1953).
[10] J. W. Burkig and B. T. Wright, Phys. Rev. **82**, 451 (1951).
[11] P. C. Gugelot, Phys. Rev. **87**, 525 (1952).

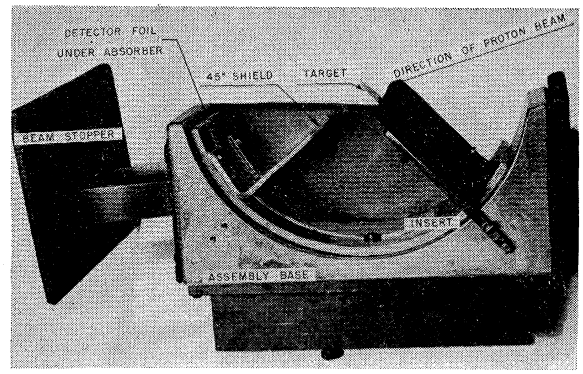

FIG. 1. Target assembly for measurements of angular distributions of elastically-scattered protons. One side shield is removed.

II. EXPERIMENTAL METHOD

A detailed account of the methods being used for measuring angular distributions of nuclear reaction products with the internal beam of the Oak Ridge National Laboratory 86-in. cyclotron is available elsewhere,[12] so that only the most pertinent details will be outlined here.

The target assembly, one variation of which is pictured in Fig. 1, consists essentially of a thin target at the center of a $3\frac{1}{2}$-in. radius assembly base, an exchangeable insert fitting on the assembly base for mounting detectors and absorbers, a large plate to stop the beam after it has passed the assembly, and various pieces to shield against background.

The elastically-scattered protons are detected by the 38-minute beta activity they induce in copper foils by the (p,n) reaction. The copper must be covered with an absorber to eliminate activities induced by inelastically-scattered protons, deuterons, etc. The absorber thickness was chosen so as to slow the elastically-scattered protons to the energy at which the (p,n) cross section[13] is at its maximum. For example, for 21.5-Mev protons the detectors are covered with about 400 mg/cm^2 of aluminum which reduces 21.5-Mev protons to 13 Mev (the maximum of the cross section) and reduces protons of less than 18 Mev to below the 4.1-Mev threshold of the reaction. For light elements where the energy of the scattered protons varies with angle due to center-of-mass effects, the absorbers are stepped to equalize the energy and, therefore, the detection efficiency at each angle.

By far the most difficult experimental problem was the elimination of background. This was accomplished only after a long trial and error procedure in which a number of adjustments and changes were made to various parts of the assembly. The process was made especially difficult by the fact that the background was extremely variable from run to run, and seemed to

[12] B. L. Cohen and R. V. Neidigh, Rev. Sci. Instr. (to be published).
[13] S. N. Ghoshal, Phys. Rev. **80**, 939 (1950).

come from the most unexpected directions. Some of the most important steps toward background reduction were: using a large beam stopper, beveling all edges so that protons striking any exposed part of the target cannot reach the detectors without several scatterings, enlarging the carbon shield on the target holder, employing side shields, etc. The point was finally reached where the only important background was in the vicinity of 90°. It was found to be coming from the general direction of the beam stopper, although particles coming from the beam stopper itself would require several scatterings before they could reach the detectors. The best way found for eliminating them was to place a thick plate at about 45°, as shown in Fig. 1. This, of course, distorts the data in the vicinity of 45°, so that another run is needed with the 45° shield removed. A single measurement of an elastic proton angular distribution thus consists of three equal and consecutive bombardments: the first, with the 45° shield, the

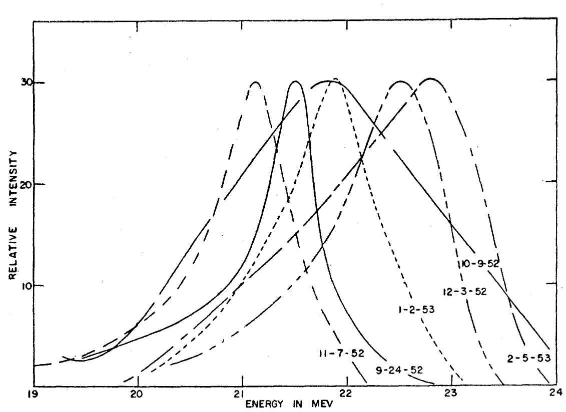

Fig. 2. Energy distribution of protons incident on the target for various dates. The great majority of the data was accumulated between 10–15–52 and 1–15–53. The method of making these measurements is described in reference 15.

second without it, and the third with the target removed to determine the background.

Up to about 90°, the background was seldom more than a few percent. At the largest angles, however, it occasionally was as high as 30 percent so that, even though it was corrected for by a separate run, its variability from run to run was sufficient to cause appreciable errors in a few cases (see caption for Fig. 7).

In order to find the absolute cross sections, it is necessary to measure the current on the target. This is done by removing and counting the target some time after the run, and calibrating the activity in the target with an auxiliary experiment in which the current is measured by using the published copper (p,n) cross section[13] near its maximum. Counting efficiencies and uncertainties in the absolute cross section of copper do not affect the result because the same activity is used in detecting the elastically-scattered protons. The largest source of error is probably in differences in energy and intensity between the angular distribution and calibration runs. In cases where the activity of the target is due to impurities, these were sometimes distilled off; and in some cases there was evidence that foreign materials were deposited on the target during the bombardment. Errors of $\sim$30 percent were found to be common, and in some cases, they were much larger. In the final plotting of the data, a considerable weight was given to the facts that the Rutherford law must be valid at small angles

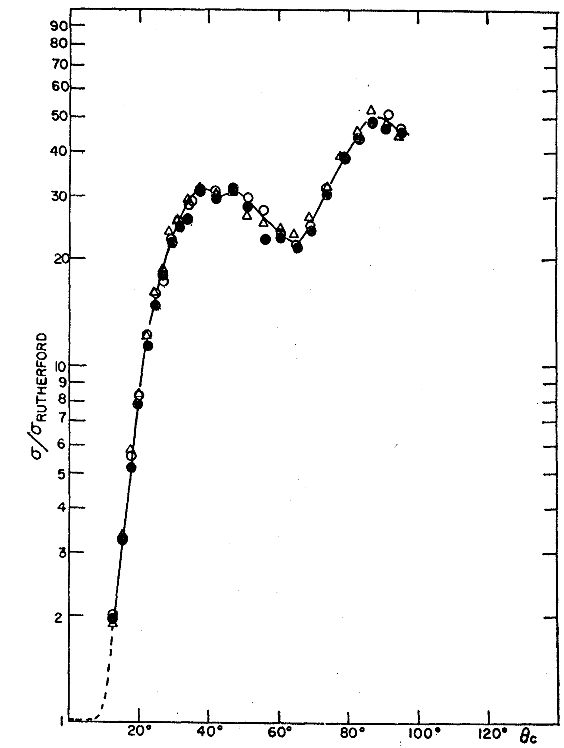

Fig. 3. Angular distribution of 22-Mev protons elastically scattered by beryllium. Data are plotted as the ratio of observed to Rutherford cross section to bring out the details. The curve at angles less than 12° is obtained from the absolute cross section with the inference that the Rutherford law is valid at small angles. Each experimental point represents a series of three runs as described in the text. Errors due to counting statistics are in all cases smaller than the size of the symbols representing the experimental points. Other sources of error are illustrated in Figs. 4 to 9; their magnitude may best be judged from the lack of agreement between the various runs. For beryllium, the energy in the center-of-mass system is 19.8 Mev.

for heavy elements, and that the ratio of the observed to the Rutherford cross section should vary slowly and continuously from element to element. For example, since the reproducibility of the absolute calibration for carbon was found to be poor, the Rutherford portion of the angular distribution was plotted by interpolating between beryllium and magnesium. The measured shapes of the angular distributions are not, of course, affected by these considerations.

The energy and energy distribution of the incident

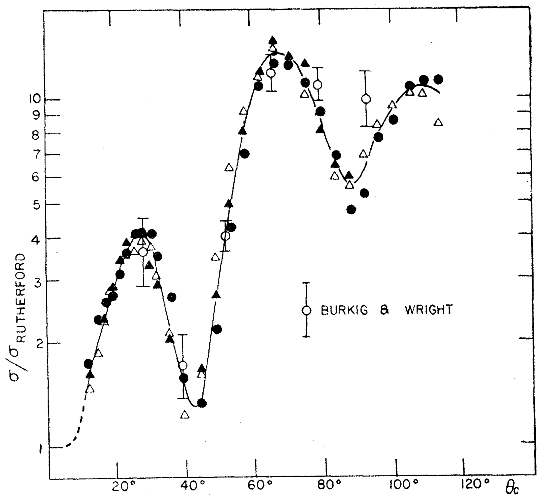

FIG. 4. Angular distribution of 22-Mev protons elastically scattered by aluminum. See caption under Fig. 3. Data of Burkig and Wright (see reference 10) was obtained with 18-Mev protons. For aluminum, the energy in the center-of-mass system is about 21.2 Mev. The run represented by the solid triangles was not considered valid at angles greater than 90° because the activity in the detector was not sufficiently greater than background.

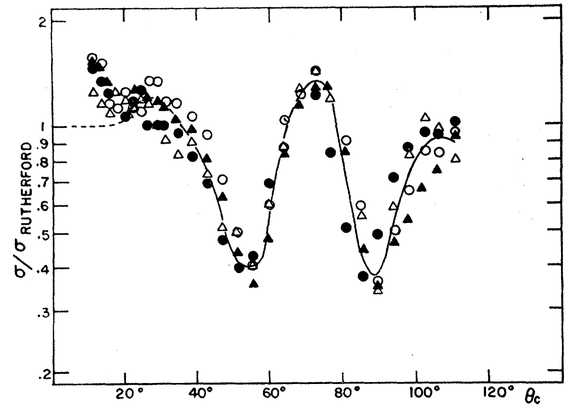

FIG. 6. Angular distribution of 22-Mev protons elastically scattered by niobium. See captions under Figs. 3 and 5. The general agreement between various runs was poorer for niobium than for any of the 14 other target elements used. The progressive shift of the open circles and solid triangles to the right of the solid circles indicates that the energy was higher for the latter run.

beam were determined from time to time by measuring the copper excitation function in the vicinity of its threshold and comparing it with the very accurate published curve.[14] A method for rapid calculation of the energy spectrum from this data was developed.[15] Figure 2 shows the energy spectrum at various times while data were being taken.

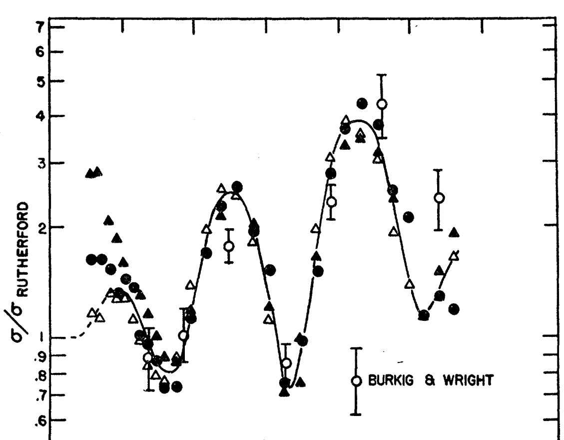

FIG. 5. Angular distribution of 22-Mev protons elastically scattered by nickel. See captions under Figs. 3 and 4. The poor data at small angles can be explained by a shift in the angle of incidence of the protons on the target, or in its homogeneity. Although this effect was found quite common, in no case were the discrepancies as large as in the case of nickel, which might also be partly the result of an error in target positioning.

The average value of the angle of incidence, and its uniformity, are determined essentially by the distribution of orbit centers in the cyclotron and the radial width of the beam on the target. These were determined from time to time by auxiliary experiments which are described elsewhere.[12,16] A check is obtained by assuming the Rutherford law to be valid at small angles for heavy elements.

Radiation exposure of personnel incurred while changing detector inserts presented a very serious difficulty which was finally alleviated by arranging for the operation to be carried out very rapidly (~15 seconds), by the use of about 40 pounds of leaded protective clothing, and by waiting a short while after the end of bombardment.

Sparking from the dees melted parts of the target on several runs. This trouble was decreased (but never

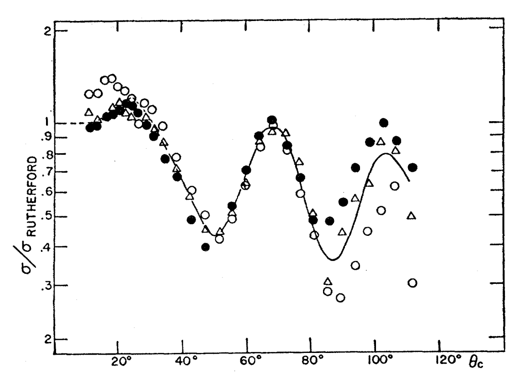

FIG. 7. Angular distribution of 22-Mev protons elastically scattered by palladium. See captions under Figs. 3 and 5. In several cases bad discrepancies between different runs were found at angles greater than 90°, but in no case were they as pronounced as in this; they may be due in part to background difficulties.

[14] J. P. Blaser *et al.*, Helv. Phys. Acta **24**, 3 (1951).
[15] B. L. Cohen, Oak Ridge National Laboratory Report ORNL-1347 (unpublished).
[16] B. L. Cohen, Oak Ridge National Laboratory Report ORNL-1348 (unpublished).

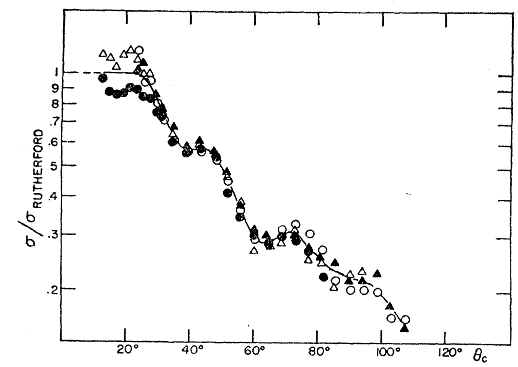

FIG. 8. Angular distribution of 22-Mev protons elastically scattered by thorium. See captions under Figs. 3 and 5. The runs represented by the solid triangles and open circles were not considered valid at angles less than 20° because of multiple scattering considerations. The other two runs were made with specially prepared thin targets.

eliminated) by setting the target at a larger than normal radius, by rounding all edges of the target assembly, and by canceling runs when the cyclotron vacuum was poor. The very considerable secular variation in the proton energy was greatly reduced, before data taking was initiated, by operating at a constant oscillator plate voltage. A complete enumeration of all the minor experimental difficulties encountered in an experiment

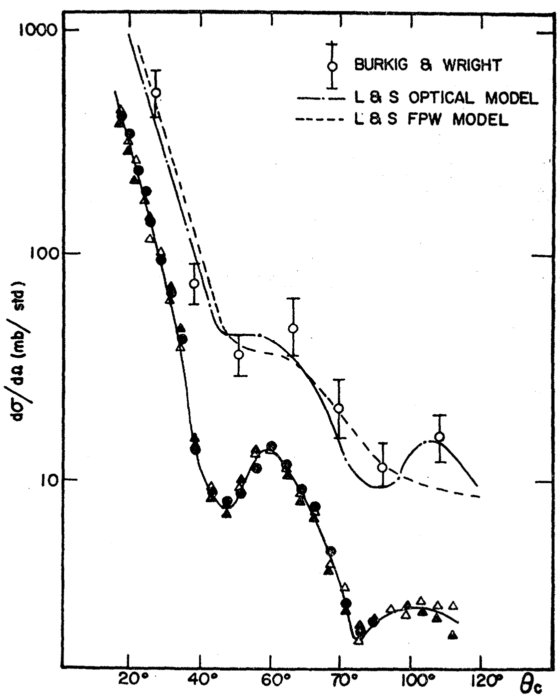

FIG. 9. Angular distribution of 22-Mev protons elastically scattered by aluminum. The data is the same as that shown in Fig. 4, except that it is plotted directly rather than as the ratio of observed to Rutherford cross section. The upper curves show the 18-Mev data of Burkig and Wright (see reference 10), and the calculations (for 18-Mev protons) of Le Levier and Saxon (see reference 4). The data from the experiment has been divided by five to avoid overlapping.

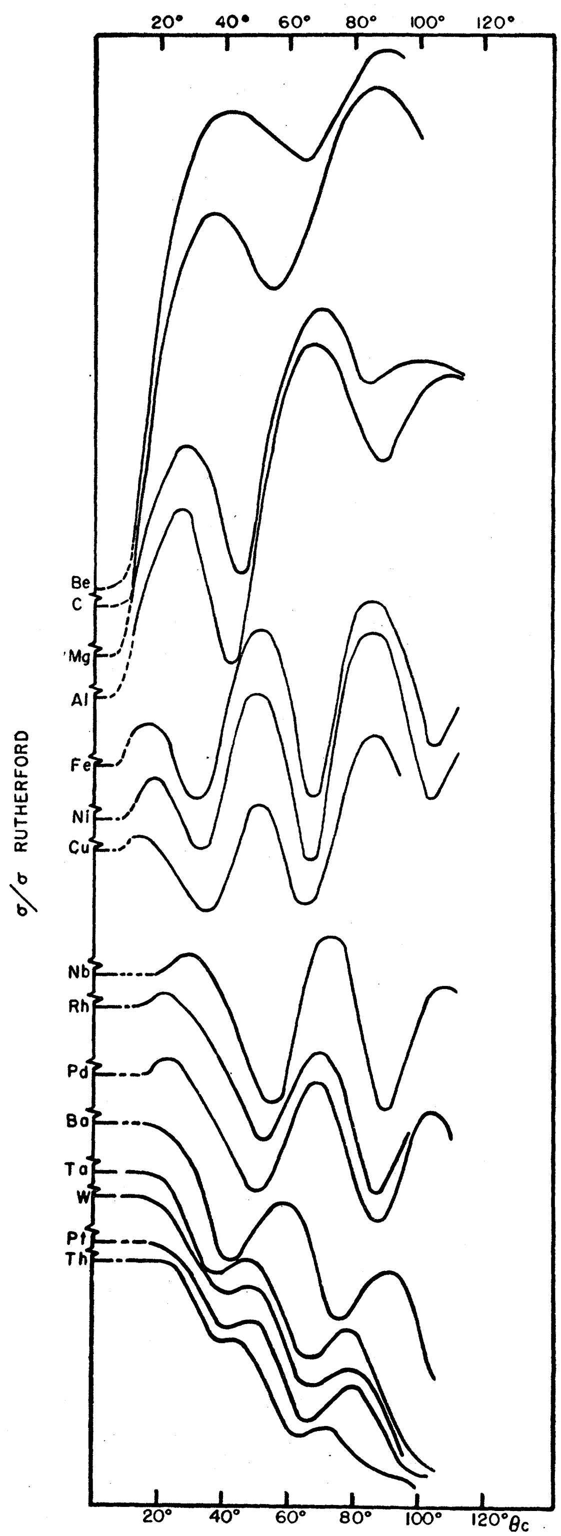

FIG. 10. Angular distribution of 22-Mev protons elastically scattered by various elements. The curves are based on the data given in Figs. 3–8 and on comparable data for nine other elements. There is no significance in the spacing between the various curves.

of this type would, of course, be very lengthy but, in general, the solutions were straightforward.

III. RESULTS AND DISCUSSION

The angular distributions of protons scattered from several elements are shown in Figs. 3 to 8. The data are plotted as the ratios of observed to Rutherford cross sections since this method brings out the details.

The raw data for aluminum are shown in Fig. 9 to illustrate the difference. Each type of pointer represents a series of three consecutive runs as described in the experimental procedure. It is quite apparent that the scattering of data on a single series of runs is much smaller than the variation between data taken at different times. In some cases, the angle between consecutive maxima varies; this can only be explained by a change in the incident proton energy. There are a few cases where the depth of minima vary, probably due to variations in the homogeneity of the energy and angle of the incident protons. The data at small angles are somewhat unreliable because small changes in the angle of incidence or its homogeneity, such as are known to occur from run to run, produce relatively large errors. The data are generally reproducible, however, and leave little doubt as to the pattern of the angular distributions.

TABLE I. Angles (in degrees) of maxima and minima in Fig. 10.

	Max	Min	Max	Min	Max	Min	Max
Be	42	65	90				
C	36	55	88				
Mg	28	45	70	85	102		
Al	27	42	67	90	111		
Fe	17	32	52	69	86	106	
Ni	19	33	51	68	87	105	
Cu	13	35	52	66	88		
Nb			~30	56	75	90	110
Rh			~21	53	71	88	~106
Pd			~23	51	70	89	105
Ba			~18	43	60	76	92
Ta				38	48	68	79
W				41	49	67	79
Pt				41	50	67	80
Th				40	44	63	72

The best curves through the data for each of the fifteen elements measured are combined in Fig. 10. Each curve represents at least three series of three runs; in many cases extra runs were made in order to increase the reliability of the data. The most striking feature of Fig. 10 is the way in which each maximum and minimum can be followed from element to element through the periodic table. From the usual theory for diffraction effects, one might expect the angle at which any particular feature occurs to be approximately proportional to λ/R, where λ is the wavelength of the incident protons and R is the nuclear radius. The angle at which each maximum and minimum occurs, multiplied by R/λ, (where R is taken to be $1.45A^{\frac{1}{3}}\times10^{-13}$ cm) is shown in Fig. 11. The $\theta \propto \lambda/R$ law[17] is borne out with an accuracy that would seem to be beyond expectation. It includes cases spanning almost the entire periodic table, as for example, the minimum which occurs at 85° for magnesium and 40° for thorium, following the λ/R law to within 10 percent for every element.

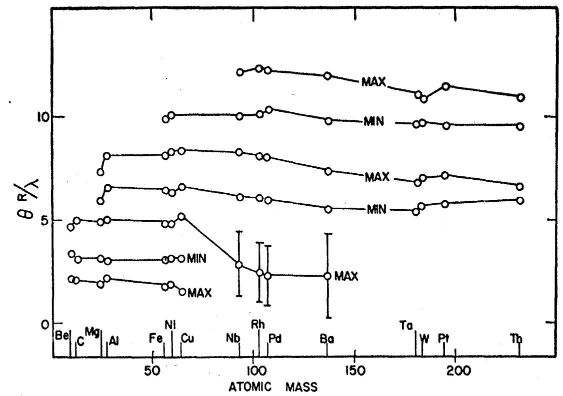

FIG. 11. Positions of maxima and minima from Fig. 10. The angle at which each maximum and minimum occurs is multiplied by R/λ, as is explained in the text. R was arbitrarily assumed to be $1.45\times10^{-13}A^{\frac{1}{3}}$.

Table I shows the angles at which the various maxima and minima occur.

Since detailed calculations would involve long phase-shift computations with Coulomb wave functions, they were considered beyond the scope of this work. The 18-Mev calculations of Le Levier and Saxon[4] for aluminum seem to agree qualitatively with these measurements, as shown in Fig. 9, but their minimum at 45° does not seem to be deep enough.

More precise measurements of these angular distributions will be possible when the beam of the 86-in. cyclotron can be deflected to an external target; new measurements will then be made if theoretical developments warrant.

The authors wish to acknowledge the interest and encouragement of R. S. Livingston and J. L. Fowler, and the various contributions of A. L. Boch, M. B. Marshall, E. Newman, B. L. Ferrell, and the operating staff of the Oak Ridge National Laboratory 86-in. cyclotron in carrying out this work.

[17] Actually, Fig. 11 verifies only that $\theta \propto 1/A^{\frac{1}{3}}$. The constant 1.45×10^{-13} was arbitrarily chosen as the ratio of R to $A^{\frac{1}{3}}$ since at the time these calculations were carried out, no other value had been proposed.

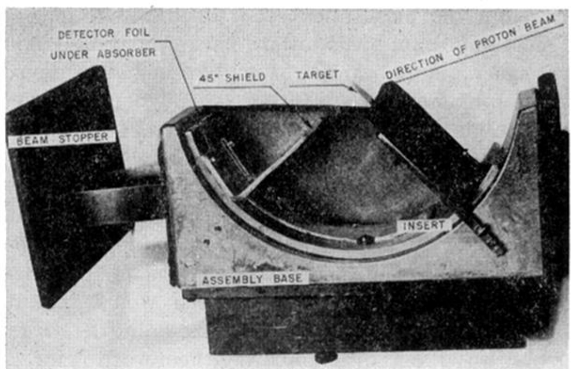

FIG. 1. Target assembly for measurements of angular distributions of elastically-scattered protons. One side shield is removed.