Second Sound Propagation below 1°K*

D. DE KLERK,† R. P. HUDSON, AND J. R. PELLAM
Cryogenic Physics Laboratory, National Bureau of Standards, Washington, D. C.
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The pulse method has been employed to measure the velocity of second sound in liquid helium, u_2, in the temperature range 0.015°–1.0°K. The liquid helium was in thermal contact with a sample of chromic potassium alum and cooling was achieved by adiabatic demagnetization.

In the region of 0.8°K, u_2 rises quite abruptly, reaching the Landau velocity of $u_1/\sqrt{3}$ by 0.5°K. Thereafter it continues to increase, but much less rapidly, with decreasing temperature. A further sharp rise was observed for low-energy pulses at about 0.05°K, suggesting an approach to the velocity of sound u_1 at absolute zero.

In the phonon region below 0.5°K, the thermodynamic quantity ρ_n/ρ shows very nearly the fourth-power (actually 4.18) dependence on temperature predicted by Landau. Values for the roton contribution to the normal fluid concentration $(\rho_n)_{\rm rot}/\rho$ agree very well with Landau's predictions, with a small adjustment of the constants in the theory.

For the smallest pulses used, the distortion (or dispersion) is very small when the pulse temperature is smaller than the ambient value and large when the reverse is true. For larger pulses the spreading decreases very slowly as the temperature rises through the phonon region, then rapidly with the onset of excitation of rotons. Above 0.05°K, the velocity associated with the leading edge of the received pulse appears to be independent of pulse energy and the degree of distortion.

I. INTRODUCTION

THERE has been need for some time for a precise and reliable determination of the wave velocity of second sound below 1°K and its extension down to within a few hundredths degree above absolute zero. Important theoretical conclusions may be drawn from the general behavior of thermal waves in this temperature range, and basic thermodynamic quantities may be evaluated from the velocity measurements. This is particularly the case in view of new and unexpected behaviors of second sound in the region below 0.1°K or 0.2°K, suggesting need for rather more sophisticated theoretical treatment of the problem at these extreme low temperatures.

The original early stalemate between the Tisza[1] and Landau[2] viewpoints had already begun to be resolved in favor of Landau's predictions even prior to investigations at demagnetization temperatures. For example, Peshkov's[3] measurements down to 1.03°K revealed a suggestion of a minute upward trend with decreasing temperature. Shortly thereafter Maurer and Herlin[4] achieved a temperature of 0.9°K and observed the velocity to rise to 23 m/sec, compared with the minimum of roughly 18.7 m/sec near 1.05°K.

First observations of second-sound velocity behavior at temperatures attained by adiabatic demagnetization were made by Pellam and Scott,[5] who effectively eliminated Tisza's predictions for below 1°K by measuring velocities as high as 34 m/sec. The object of the experiment being an unambiguous decision between the opposing Tisza-Landau viewpoints, no effort was made to ascertain the temperatures reached. More recently, Atkins and Osborne[6] performed a similar experiment in which they cooled samples of liquid helium sufficiently to observe velocities in the range of 150 m/sec and from which they plotted wave velocity *versus* temperature below 1°K. This experiment contributed immensely to the final substantiation of Landau's predictions, but unfortunately their rapid warm-up time of about one minute effectively precluded meaningful temperature determinations. Although their plotted curve was roughly correct in general shape, it now appears that the measured temperatures were too low by about 2/10 deg throughout much of the range below 1°K, and in fact they evidently reached lowest temperatures of 0.3°K rather than their reported value of 0.1°K.

Present measurements[7] actually in the range from 0.3°K down to a few-hundredths deg K (apparent) reveal further, and quite unexpected, rises in velocity to values considerably exceeding the Landau prediction of $1/\sqrt{3}$ times the velocity of first sound which Atkins and Osborne had appeared to have fully substantiated. Furthermore, it now appears that the well-known second sound pulse broadening[5–7] may, under some circumstances—particularly at the lowest temperatures—constitute an "over-loading" phenomenon in which the liquid helium II is simply incapable of carrying sufficient heat current to transmit a well-bunched pulse.

Precise determination of second-sound velocity has now permitted an indirect evaluation of the normal fluid concentration ρ_n/ρ from 1°K down to 0.02°K. This

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[1] L. Tisza, J. phys. et radium 1, 165, 350 (1940); Phys. Rev. 72, 838 (1947); 75, 885 (1949).
[2] L. Landau, J. Phys. (U.S.S.R.) 5, 71 (1941); 8, 1 (1949); Phys. Rev. 75, 884 (1949).
[3] V. Peshkov, J. Phys. (U.S.S.R.) 10, 389 (1946); J. Exptl. Theor. Phys. (U.S.S.R.) 18, 951 (1948).
[4] R. Maurer and M. Herlin, Phys. Rev. 76, 948 (1949).
[5] J. R. Pellam and R. B. Scott, Phys. Rev. 76, 869 (1949).
[6] K. R. Atkins and D. V. Osborne, Phil. Mag. 41, 1078 (1950).
[7] de Klerk, Hudson, and Pellam, Phys. Rev. 89, 326 (1953).

is of particular interest in the 1.0°K–0.5°K range, where the effects of the roton contribution are most striking.

II. APPARATUS

A. Cryogenic

The low temperatures were produced by the adiabatic demagnetization of a mixture of paramagnetic salt and liquid helium from a starting temperature of 1.15°K and an initial field of about 20 000 oersteds. The "magnetic temperature" immediately after demagnetization was then of the order of 0.05°, corresponding to an absolute temperature of about 0.015°K.[8]

The cryogenic apparatus is very similar to that used by Kramers, Wasscher, and Gorter[9] for specific heat measurements and embodies the technique for maintaining liquid helium at very low temperatures developed by Hudson, Hunt, and Kurti.[10] The demagnetization unit A (Fig. 1) is enclosed in a vacuum chamber comprising a soft-glass envelope B which may be evacuated through the tube F. The unit is connected at a ring seal D to a second pumping line E via a capillary C, 8 cm long and a few tenths of a millimeter inside diameter. The salt (potassium chromic alum) was hand-packed into an ellipsoidal space G, 5 cm by 2.5 cm, small wads of glass wool H being inserted to prevent movement of the salt crystals during handling.

The second sound chamber is a cylindrical extension, J, of the salt container. The current leads from the heater elements (see below) pass upward through the salt to platinum-glass seals K, which are soldered to spirals of (No. 36) constantan wire L suspended from a second set of platinum-glass seals M. [From here the leads pass upward through the liquid helium bath via coaxial conductors (see below) and out of the cryostat through a simple rubber vacuum seal.] The constantan has the advantage of high thermal resistance at low temperatures and the disadvantage of a fairly high electrical resistivity, such that the consequent Joule heat developed in them could impair the thermal insulation of the liquid helium after cooling. The size of wire chosen effected a compromise between these conflicting considerations: the (uncoiled) wires were about 15 cm long and so had a resistance of about 4.5 ohms at 1°K which was quite small in comparison with the resistance of the transmitter and receiver (about 800 ohms and 2500 ohms, respectively). The heat influx through each wire was estimated, from the data of Berman,[11] to be about 2 ergs/min. (Spirals of fine lead wire were used at first, but their lack of strength and rigidity outweighed the advantage of the very small thermal conductivity of pure lead in the superconducting state.[12])

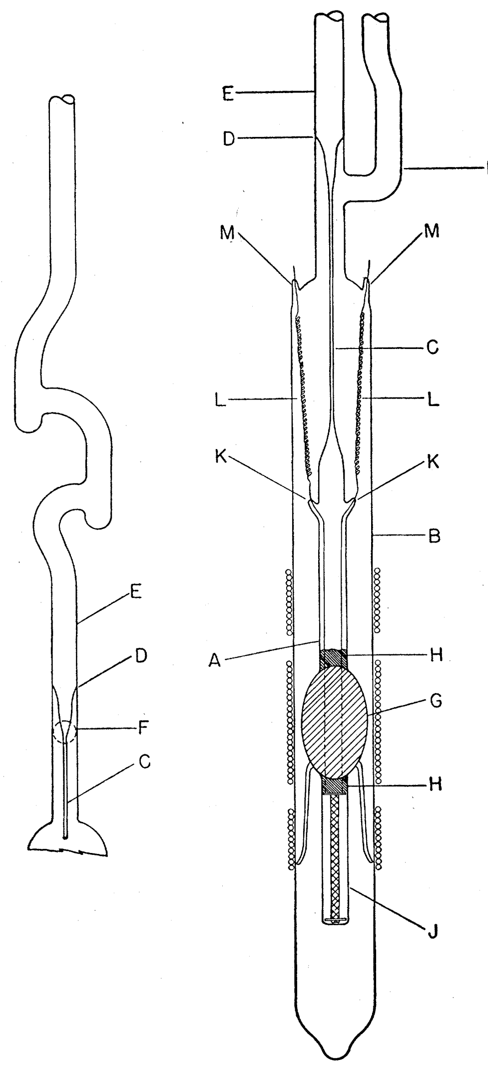

FIG. 1. Apparatus.

In this type of apparatus, there is an unavoidably large heat influx through the capillary due to the evaporation of the mobile helium film in the upper part of C (Fig. 1) and the subsequent recondensation of that part of the vapor which is not removed by the pumps connected to E. In our arrangement, this amounted to about 1800 ergs per min under the best conditions, corresponding to a "natural" warm-up time (i.e., with no injection of energy by pulsing) of well over an hour. In general, 20–30 velocity measurements were made per demagnetization, and the system returned to bath-temperature in about 30 minutes.

Radiation from the high-temperature parts of the cryostat can, however, contribute far larger heat-influxes, and for this reason the envelope B was silvered externally. The lines E and F were also provided with radiation traps; in Fig. 1 only one such trap is shown and, for the sake of clarity, it has been drawn at right

[8] de Klerk, Steenland, and Gorter, Physica **15**, 649 (1949).
[9] Kramers, Wasscher, and Gorter, Physica **18**, 329 (1952).
[10] Hudson, Hunt, and Kurti, Proc. Phys. Soc. **A62**, 392 (1949).
[11] R. Berman, Phil. Mag. **42**, 642 (1951).
[12] W. J. de Haas and A. Rademakers, Physica **7**, 992 (1940).

angles to its real position so as to lie in the plane of the paper.

"Magnetic temperatures" were derived from measurements of the susceptibility of the paramagnetic salt, made by means of a Hartshorn-type ac mutual inductance bridge.[13] A primary coil was wound on the liquid helium Dewar vessel so as to produce a uniform magnetic field over the salt. A secondary winding around G on the outside of the vacuum case, B (comprising 350 turns of No. 42 Formex), completed the mutual inductance of which G constituted the core.

B. Electronic

The usual well-known methods of heat pulsing[14] were employed. This involved feeding electrical impulses to the resistor comprising the transmitter element and detecting the resulting heat pulses upon their arrival at the opposite end of the second sound unit. The observed time in transit through the known separation between the inner faces of these elements provided the direct determination of second sound wave velocity.

The second sound unit was designed along more or less standard lines. This device, which is set vertically within the cylindrical enclosure J (Fig. 1) beneath the paramagnetic salt, consists of a thermal transmitter and receiver spaced with their effective inner surfaces 5.1 cm apart. Each element is composed of a thin layer of carbon supported on the inner surface of a nonconducting disk (commercial resistance strip); the outer edges and centers are coated with silver electrode paint to form electrodes. In each disk, current flows radially between the inner and outer electrode rings. These disks are spaced at the fixed distance of 5.1 cm and kept parallel to each other by means of a bakelite rod of diameter equal to the outside diameter of the inner conductor ring. Thus, in addition to providing support, the rod occupies the central cylindrical portion over which signals were not generated, thereby assuring plane-wave propagation. The diameter of the disks is about $\frac{1}{2}$ mm less than the inner diameter of tube J to provide thermal contact (via the liquid helium II) with the remainder of the system. The liquid helium within J, supporting the second sound propagation, is thus thermally linked with the paramagnetic salt for both cooling and temperature measurement. Above M (Fig. 1), one lead from each element is grounded to a small ($\frac{1}{32}$-in. copper-nickel) tubing providing coaxial shielding, the other constituting the coaxial center conductor.

III. PROCEDURE

A. Cryogenic

The method of calibrating the magnetic thermometer and measuring the magnetic temperature is well known enough to obviate the need for detailed description. Absolute temperatures were derived from the T^* values according to the measurements of de Klerk, Steenland, Gorter,[8] and Bleaney.[15]

We found it convenient to avoid the use of "exchange gas" and to remove the heat of magnetization by pumping off vapor from the capillary. The loss of liquid helium from the salt chamber per magnetization was only a small fraction of a cubic centimeter.

After demagnetization the warming-up of the system was observed by susceptibility measurements at intervals of a few seconds, with an interruption after every 5 or 6 readings while a velocity determination was being made.

B. Electronic

The data were observed oscillographically by triggering the horizontal time scale simultaneously with the start of the initial generating pulse, the received signal being presented upon the vertical scale at the position representing the intervening time delay. These voltage impulses were produced by the temperature dependence of the carbon detector surface carrying constant dc current (i.e., acting as a bolometer) and were amplified for presentation on the oscilloscope screen. Such data were recorded photographically for later analysis, the time delays being determined directly by counting the intervening scope delay time markers (for the fastest time sweep, employed at the lowest temperatures, the markers were 10 μsec apart; for the higher temperatures the markers occurred at 50-μsec intervals).

Whereas square wave dc pulses had been used in earlier investigations, the present experiments were conducted by applying square wave modulated ac electrical pulses to the thermal generator. These square wave pulses were of 22.5-kc/sec carrier-wave frequency, 80 μsec–100 μsec duration, and a repetition rate of 88 per second. The primary advantage of these "c-w pulses" was to reduce the effect of direct electrical pick-up between the transmitter and receiver leads along their unshielded portions within the extreme low temperature portion of the apparatus. (dc pulses shock the receiver unidirectionally and thus often leave it partly insensitive at the arrival-time of the heat pulse.) Actually these 22.5-kc/sec electrical pulses produced heat pulses of double this frequency, 45 kc/sec, superposed on a background dc square-wave pulse of equal amplitude (the two terms of $RI^2\cos^2 wt = \frac{1}{2}RI^2 + \frac{1}{2}RI^2\cos 2wt$). Since the receiver amplifier was not tuned to this second sound frequency of 45 kc/sec, but had instead essentially flat characteristics, the dc background component comprised the predominant received signal; the comparatively high thermal inertia of both the transmitter and receiver elements at 45 kc/sec apparently suppressed this signal relative to the steady

[13] L. Hartshorn, J. Sci. Instr. **2**, 145 (1925); Casimir, de Haas, and de Klerk, Physica **6**, 241 (1939).

[14] J. Pellam, Phys. Rev. **75**, 1183 (1949); D. V. Osborne, Proc. Phys. Soc. (London) **A64**, 114 (1951).

[15] B. Bleaney, Proc. Roy. Soc. (London) **A204**, 216 (1950).

background heat current accompanying each pulse. Thus, from the standpoint of interpreting the data, the results may be considered as representing simple dc pulsing.

IV. RESULTS

The majority of the results were obtained in a series of experiments conducted during July-August, 1952, and have been described briefly in an earlier publication.[7] Further work was carried out in February, 1953, by two of us (RPH and JRP) in order to determine whether the measured velocity was dependent on pulse-size and particularly to observe velocities for very small pulses at the lowest temperatures, where the problem of temperature equilibrium becomes acute. In these latter experiments we benefited from the kind cooperation of Dr. C. K. McLane, on leave from the University of Wisconsin.

A. Velocity

The velocity results are presented in two graphs, Fig. 2(a), showing the detailed variation of second sound velocity with temperature from 1.1°K down to 0.5°K, and Fig. 2(b), showing the over-all behavior from 1.1°K down to 0.015°K. Regarding the upper temperature range, we have already pointed out the large amount by which our measured values exceed those of Atkins and Osborne and have given the apparent reasons so that we need not dwell further on this discrepancy.

The primary purpose of an accurate determination of wave velocity u_2 in the 0.5°K–1°K range is to provide data for evaluating the normal fluid concentration ρ_n/ρ at these temperatures below the working range of the direct Andronikashvilli type of measurement. Accurate information on second sound velocity also permits the evaluation of the roton portion of normal fluid concentration $(\rho_n)_{\text{roton}}/\rho$.

For temperatures below 0.5°K, the second sound velocity behaves in a somewhat strange manner. Although the velocity shows some tendency toward leveling off at about the Landau value of $u_1/\sqrt{3}$ in the vicinity of 0.5°K, this effect is only partial. In fact the velocity continues to rise at a reduced but fairly uniform rate down to about 0.1°K and below this rises even faster. It appears that an ultimate value in the neighborhood of first sound velocity u_1 may occur at low enough temperatures.

From run to run the quality of the results obtained varied considerably, major difficulties being interference and instability in the electronic equipment and "cross-talk" between the transmitter and receiver circuits. The results here presented are derived only from such oscilloscope traces as could readily be "counted," and these were considerably fewer in the second series of experiments than in the first.

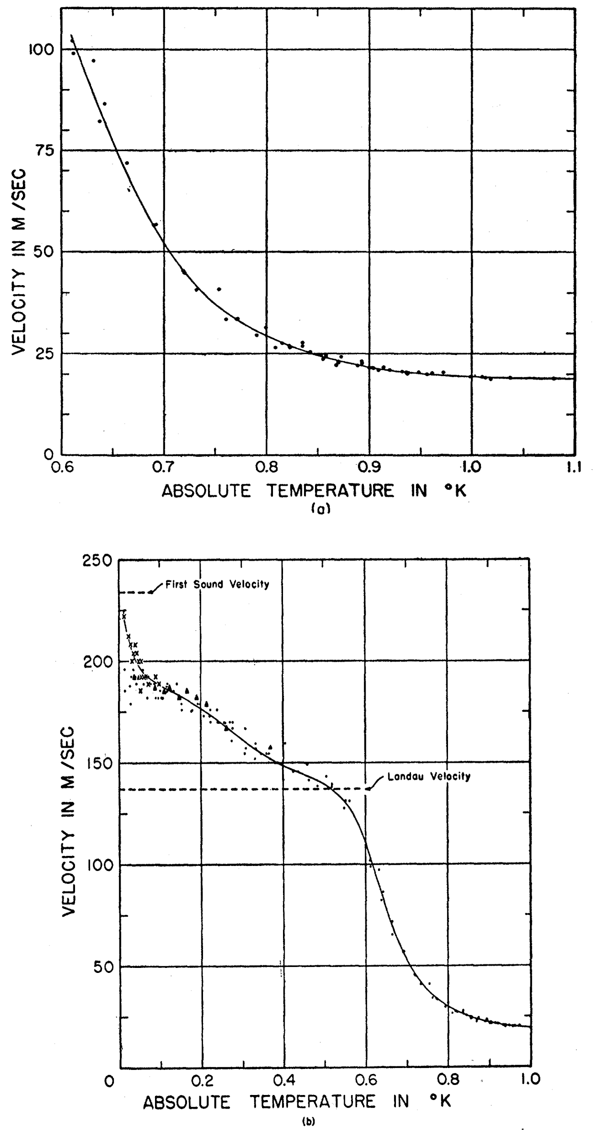

FIG. 2. (a) Second sound velocity between 0.5°K and 1.1°K. (b) Second sound velocity from 0.015°K to 1.1°K. ● First series: measurements (see reference 7) of July, 1952. ×, ▲ Second series: measurements of February, 1953, at low input energy levels.

It is believed now that little weight should be attached to the points below 0.1°K obtained in the first series. The pulses were of such magnitude as probably to cause a rise in temperature of the liquid helium above that of the salt in this region of low thermal conductivities. (See Sec. V.) Indeed, it was frequently observed—just as a pulsing was initiated and the trace inspected momentarily prior to photographing—that there was a suggestion of an initially high wave velocity which almost instantaneously diminished to a steady lower value, that "appearing" in the photograph. On occasions, a "ghost" trace was actually found in the photograph, slightly ahead of the main one (an example of which may be seen in the lowest picture of the sequence of Fig. 6).

The final curve through the cluster of experimental points has been drawn to be reasonably smooth and to represent closely the mean of the observed velocity values. At the lowest temperatures the low-energy-pulse points have been heavily weighted, and these show a very steep rise in wave velocity towards the (extrapolated) velocity of ordinary sound in liquid helium[16] at these temperatures.

Values of velocity read from this curve at regular temperature intervals are listed in Table I.

B. Normal Fluid Concentration

As pointed out earlier, our velocity measurements provide the means for determining the normal fluid concentration ρ_n/ρ in the temperature range below 1°K. At these temperatures the direct evaluation of ρ_n/ρ by means of the Andronikashvilli[17] rotating disks experiment is not feasible owing to the extremely low concentration of normal fluid (i.e., down to less than one percent even at 1°K). Accordingly the only available method at present for determining ρ_n/ρ between 0.01°K and 1°K appears to involve the evaluation of the Tisza-Landau thermodynamic expression

$$\rho_n/\rho=[1+Cu_2^2/S^2T]^{-1}. \quad (1)$$

This is a purely thermodynamic relationship based on the two-fluid model but valid for any two-fluid model as long as the thermal content of the mixture is associated solely with the normal fluid (i.e., excited) component. The values of specific heat C and entropy S obtained by Kramers *et al.*[9] were used for this evaluation.

TABLE I. Second sound velocity determinations (taken at regular intervals from smoothed curve of Fig. 2).

T degree K	u_2 m/sec	T degree K	u_2 m/sec	T degree K	u_2 m/sec
0.015	222	0.32	158_5	0.72	45.0
0.02	215	0.34	156	0.74	39.4
0.03	207	0.36	153	0.76	35.3
0.04	201_5	0.38	150_5	0.78	32.05
0.05	197	0.40	149	0.80	29.40
0.06	194_5	0.42	147	0.82	27.25
0.07	192_5	0.44	145_5	0.84	25.35
0.08	191	0.46	143_5	0.86	23.95
0.09	189_5	0.48	142	0.88	22.70
0.10	188_5	0.50	139_5	0.90	21.75
0.12	186	0.52	136_5	0.92	20.95
0.14	184	0.54	133	0.94	20.35
0.16	181_5	0.56	128	0.96	19.95
0.18	179_5	0.58	121	0.98	19.60
0.20	176_5	0.60	110	1.00	19.35
0.22	174	0.62	96.1	1.02	19.20
0.24	171	0.64	83.1	1.04	19.05
0.26	168	0.66	71.1	1.06	18.90
0.28	164_5	0.68	60.7	1.08	18.80
0.30	161_5	0.70	52.1		

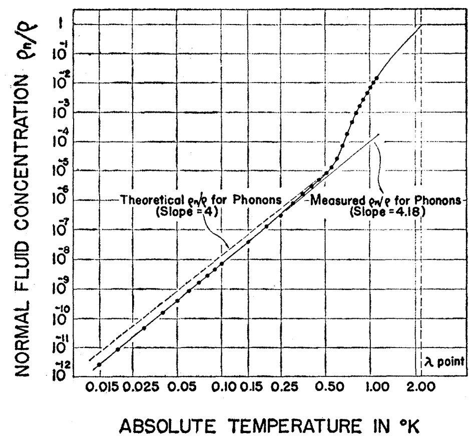

FIG. 3. Normal fluid concentration ρ_n/ρ *versus* temperature. Values below 1.1°K derived from present second sound velocity measurements; values between 1.1°K and 1.35°K derived from Maurer and Herlin's[4] data; values above 1.35°K are direct ρ_n/ρ measurements by Andronikashvilli (see reference 17).

Values of ρ_n/ρ obtained in this manner are plotted *versus* temperature in Fig. 3. A log-log scale has of necessity been used and shows the approach of this quantity to the fourth-power behavior at temperatures below about 0.5°K. It will be noted that the measured values (circles) drop to an effective concentration at 0.015°K of only about 2.6×10^{-12}. This value is about 40 thousand times lower than the normal helium 3 content of well helium (10^{-7}) so that it must be concluded from these measurements that all the initial helium 3 content was swept out of the sample during early magnetizations.

Considerable information of significance to theory is contained in the data of Fig. 3. The existence of the phonon region at the lower temperatures is clearly shown by the transition from the steep nonlinear portion of the curve above about 0.5°K to the nearly linear part below. The slope of this linear portion does not quite attain the value *four* predicted by Landau for $(\rho_n)_{\text{phonon}}/\rho$ on the basis of his phonon gas theory, reaching instead a lower limit of 4.18. This difference is the direct result of the continued increase in second sound velocity past the Landau prediction. The behavior predicted by Landau is represented by the dotted line, obtained by combining his $u_1/\sqrt{3}$ velocity with the above-mentioned values[9] of C and S. (In our earlier publication[18] we had regarded the velocity increase above about 160 m/sec as spurious and had extrapolated to this value in deducing our $(\rho_n)_{\text{ph}}/\rho$; accordingly these previous determinations appeared to have a more nearly fourth-power dependence.)

The difference between the total ρ_n/ρ and $(\rho_n)_{\text{ph}}/\rho$ may be interpreted in terms of Landau's roton gas concept. Thus it is apparent from Fig. 3 that the roton contribution sets in above 0.6°K, becoming very rapidly

[16] K. R. Atkins and C. E. Chase, Proc. Phys. Soc. (London) **A64**, 826 (1951).

[17] E. L. Andronikashvilli, J. Phys. (U.S.S.R.) **10**, 201 (1946).

[18] de Klerk, Hudson, and Pellam, Phys. Rev. **89**, 662 (1953).

predominant with increased temperature; by 1°K the roton mass already contributes nearly 99 percent of the total normal fluid density. Our determination of roton contribution is shown in the curve of Fig. 4 where $(\rho_n)_{\text{roton}}/\rho$ is plotted *versus* temperature. This was obtained as the difference between our measured values above 0.6°K and an extrapolation of the straight-line "phonon portion" of the curve, assuming that

$$(\rho_n)_{\text{total}} = (\rho_n)_{\text{phonon}} + (\rho_n)_{\text{roton}}. \tag{2}$$

We will discuss the behavior of $(\rho_n)_{\text{rot}}/\rho$ in connection with Landau's calculations which also include C_{roton} and S_{roton}.

A tabulation of (ρ_n/ρ), $(\rho_n)_{\text{ph}}/\rho$, and $(\rho_n)_{\text{rot}}/\rho$ determined from these measurements appears in Table II.

Landau's Roton Evaluation

Using only Peshkov's second sound velocity measurements down to 1.35°K,[3] and the specific heat measurements available to him at the time (1947), Landau deduced expressions for the entropy S_{rot}, specific heat C_{rot}, and normal fluid concentration $(\rho_n)_{\text{rot}}/\rho$ for the roton gas. He expressed these quantities, which he used for predicting second sound behavior below 1°K, in terms of three arbitrary constants adjusted to fit the existing data. Thus

TABLE II. Tabulation of normal fluid concentrations [total ρ_n/ρ, phonon contribution $(\rho_n)_{\text{phonon}}/\rho$, and roton contribution $(\rho_n)_{\text{roton}}/\rho$].

T in degree K	S^2T/C in joule/g	u_2 in cm/sec	ρ_n/ρ	$(\rho_n)_{\text{phonon}}/\rho$	$(\rho_n)_{\text{roton}}/\rho$
0.02	4.18×10^{-3}	2.15×10^{4}	9.03×10^{-12}		
0.04	6.68×10^{-2}	2.01_5	1.65×10^{-10}		
0.06	0.338	1.94_5	8.94		
0.08	1.07	1.91	2.93×10^{-9}		
0.10	2.61	1.88_5	7.35		
0.15	13.2	1.83	3.95×10^{-8}		
0.20	41.8	1.76_5	1.33×10^{-7}		
0.30	2.11×10^{2}	1.61_5	8.06		
0.40	6.68	1.49	$3.01_0\times10^{-6}$	$2.34_4\times10^{-6}$	0.67×10^{-6}
0.50	1.63×10^{3}	1.39_5	8.38_3	5.95_7	2.43
0.60	3.36	1.10	$2.77_7\times10^{-5}$	$1.27_4\times10^{-5}$	1.50×10^{-5}
0.70	5.44	0.521	$2.00_4\times10^{-4}$	2.42_7	1.76×10^{-4}
0.80	8.13	0.294	9.60_5	4.23_6	9.18
0.90	1.38×10^{4}	0.218	$2.90_0\times10^{-3}$	6.99_8	2.83×10^{-3}
1.00	2.71	0.194	7.14_5	$1.07_6\times10^{-4}$	7.04
1.10	5.32	0.188	$1.48_4\times10^{-2}$	$1.59_6\times10^{-4}$	1.47×10^{-2}

$$S_{\text{rot}} = \frac{2(k\mu)^{\frac{1}{2}}p_0^2\Delta}{(2\pi)^{\frac{3}{2}}\rho T^{\frac{1}{2}}\hbar^3}\left[1+\frac{3kT}{2\Delta}\right]e^{-\Delta/kT}, \tag{3}$$

$$C_{\text{rot}} = \frac{2\mu^{\frac{1}{2}}p_0^2\Delta^2}{(2\pi)^{\frac{3}{2}}\rho k^{\frac{1}{2}}T^{\frac{3}{2}}\hbar^3}\left[1+\frac{kT}{\Delta}+\frac{3}{4}\left(\frac{kT}{\Delta}\right)^2\right]e^{-\Delta/kT}, \tag{4}$$

$$(\rho_n)_{\text{roton}} = \frac{2\mu^{\frac{1}{2}}p_0^4}{3(2\pi)^{\frac{3}{2}}\rho(kT)^{\frac{1}{2}}\hbar^3}e^{-\Delta/kT}, \tag{5}$$

where the constants were specified as

$$\Delta/k = 9.6°, \quad p_0/\hbar = 1.95\times10^8\ \text{cm}^{-1}, \quad \mu = 0.77m_{\text{He}}. \tag{6}$$

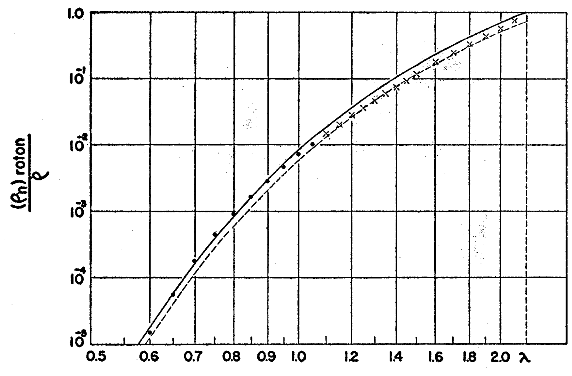

FIG. 4. Roton contribution to normal fluid concentration $(\rho_n)_{\text{roton}}/\rho$ *versus* temperature. ● Values derived from present second sound velocity measurements and the calorimetric data of Kramers *et al.* (see reference 9) × Values derived from the data of Andronikashvilli and of Maurer and Herlin (see Fig. 3). - - - Landau's theoretical curve (see text). —— Modified Landau curve for $(\rho_n)_{\text{roton}}/\rho$ using modified constants, Eq. (7).

The subsequent measurements of specific heat (and entropy) by Kramers *et al.*[9] and our more recent second sound results provide the necessary data for a direct numerical check of Landau's predictions regarding roton behavior. Actually the Leiden workers did check their specific heat results for an $e^{-\Delta/kT}$ dependence, obtaining a value of $\Delta/k = 8°$. The variation between this and Landau's 9.6° value is not surprising since their exponential factor had to be less sensitive to temperature to atone for neglecting the $T^{\frac{3}{2}}$ factor in the denominator [see Eq. (4)]. We have compared both the results of Kramers *et al.* and our own with the complete Landau expressions for C_{rot} and $(\rho_n)_{\text{rot}}/\rho$. We find *full* agreement between the Leiden specific heat measurements (modified to give the roton contribution alone) and Landau's expression for C_{rot} [Eq. (4)] using his constants (6).

The comparison between Landau's roton expression for $(\rho_n)_{\text{rot}}/\rho$ and our determined values [obtained by

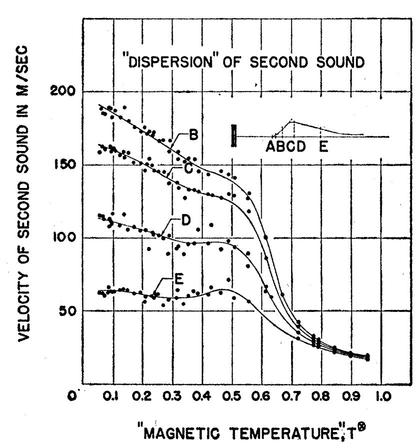

FIG. 5. "Dispersion" of second-sound pulses. Pseudovelocity *versus* temperature.

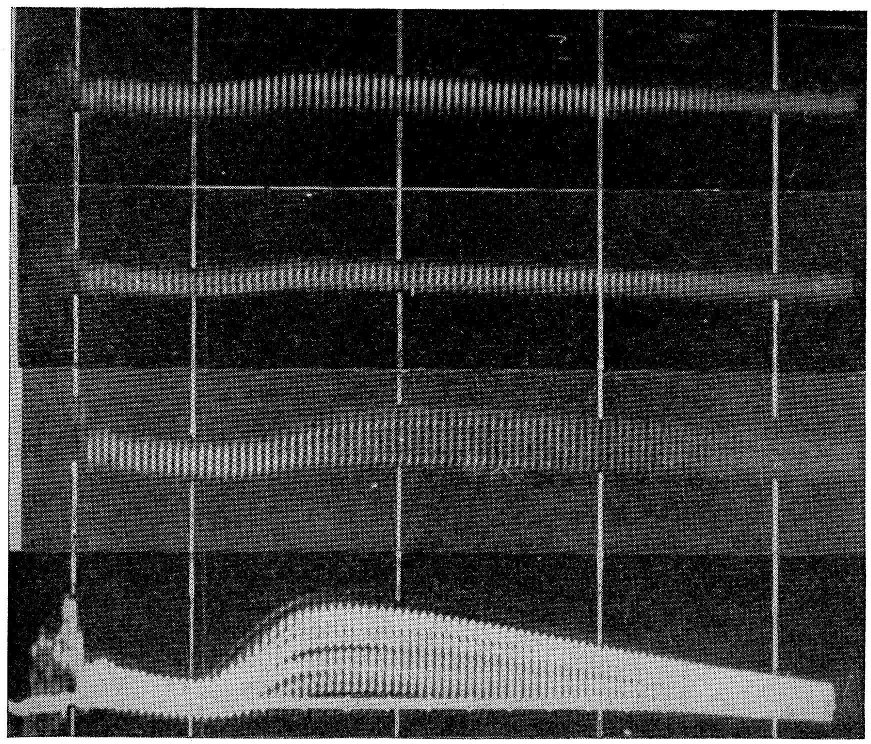

FIG. 6. Effects of power level on pulse characteristics. Sequence of oscillograms illustrating the progressive changes (a–d) in the appearance of the received signals as $\dot{H}$ is increased. Note that while the time at which first observable portion of the signal arrives seems to be independent of $\dot{H}$, the received pulse width increases systematically with input power level.

subtracting the $(\rho_n)_{\text{ph}}/\rho$ from $(\rho_n)_{\text{total}}/\rho$ as explained] is illustrated in Fig. 4. Here the "measured" values are plotted *versus* temperature on a log-log scale, with the dotted curve representing Landau's predictions using (5) with the same constants (6). The Landau curve agrees adequately with the observed results above about 1.2°K (except near the λ point) as should be expected, since Landau evaluated his constants with data available to him in this temperature range. One does observe, however, a moderate deviation below 1°K between Landau's predictions for $(\rho_n)_{\text{rot}}/\rho$ and our observed values. This would apparently indicate the need for a slight adjustment of the constants p_0 and μ. That is, the nearly exact correlation with specific heat data appears to verify the Landau values for both Δ/k and the product $(\mu^{\frac{1}{2}}p_0^2)$, leaving only the possibility of modifying μ and p_0 so as to leave this product unchanged. An attempt to do so is illustrated by the solid line of Fig. 4, which was drawn both to fit our data for $(\rho_n)_{\text{rot}}/\rho$ below 1°K and to intercept unity (nearly) at the λ point. Such an adjustment gives the following evaluation of the constants:

$$\Delta/k=9.6 \text{ deg},\ p_0/\hbar=2.30\times10^8 \text{ cm}^{-1},\ \mu=0.40m_{\text{He}}. \quad (7)$$

C. Pulse Behavior

The spreading of the pulses observed below about 0.75°K, which has been reported by several authors,[5–7] has not been satisfactorily explained, although tentative explanations have been given—ranging from true dispersion to extraneous causes such as thermal barrier effects in the elements of the apparatus.

That the phenomenon is truly a property of the liquid helium and not the result of certain features of the apparatus appears to have been confirmed by the fact that no spreading occurs if a He^3-He^4 mixture[19] is used instead of pure He^4. For pure He^4, the observed spreading disappears at the temperature where the roton density begins to mount rapidly (about 0.5°K), and it would therefore appear that, crudely speaking, rotons in the one case and He^3 atoms in the other play a similar role in preserving the compactness of the pulse.

Taking the shape of a pulse to be as given in the inset of Fig. 5, and approximately defined by the points A, B, C, D, E, its variation with temperature may be illustrated by taking the time delay for each of these points and deriving a pseudo-velocity. This has been

[19] J. C. King and H. A. Fairbank, Phys. Rev. 90, 989 (1953).

done in Fig. 5; the first-series points in Fig. 2(b) correspond to point A and are practically coincident with the points of curve B of Fig. 5.

The most striking feature of the latter figure is that the pulse spreading changes but little as the temperature rises from very low values to about 0.5°K. Then, coincident with the rapid fall in true wave velocity and rapid overpowering of the phonon concentration by that of the rotons, the broadening diminishes until by 0.8° the pulse shape is well preserved during transmission.

The data in Fig. 5 were taken from the first series of measurements (see Sec. IV) wherein relatively high-energy pulses were used. The second series, made at pulse-energy levels varying from the lowest practical value ($\dot{H}=4800$ erg sec^{-1} cm^{-2}) up to the order of intensity used in the first series ($\dot{H}=2\cdot 10^5$), was carried out in the hope of obtaining further insight into both the pulse-spreading phenomenon and the anomalous velocity behavior. In Fig. 6 are shown 4 oscillograms for pulses of different intensities and the same ambient temperature, 0.05°K. It is to be noted that the lowest-energy pulse is well bunched, and one is tempted to draw the conclusion that no spreading at all would occur if sufficiently small pulses could be employed (and detected!). But if the pulse energy is a dominant factor in causing spreading, the ratio of pulse "temperature" $\tau(=\Delta T)$ to ambient temperature T calculated from the formula

$$\dot{H}=\rho u_2\int_T^{T+\tau} C dT \qquad (8)$$

should be highly significant. (It should be noted that these values of τ/T are computed on the basis of undistorted pulse propagation and so constitute upper limits only.)

For pulses of such magnitude that $\tau/T\geq 1$, the basic differential relationships from which the wave equation is derived will be radically altered. For example, the normal condition of heat-flow continuity must be modified to take into account the rate of conversion of thermal energy to mechanical energy (or *vice versa*) $(\dot{H}/T)\cdot\nabla T$, so that the heat-flow equation becomes

$$\rho C(T)\frac{dT}{dt}+\nabla\cdot\dot{H}+\frac{\dot{H}}{T}\cdot\nabla T=0. \qquad (9)$$

Whereas for small signals the effects of conversion between thermal and mechanical energy are small compared with the actual heat flows involved, these may become very important in the nonlinear case of large pulses. It is not surprising, then, that such an added "sink" for heat should result in a diminished velocity for large portions of the heat carried by the pulse.

In Fig. 6, approximate values of τ/T have been calculated, and there is certainly some internal consistency in that, at this particular temperature, the spreading is slight for the case $\tau<T$ and very marked for $\tau\geq T$. Further evidence of the significance of τ/T (at very low temperatures) may be seen in Fig. 7, which shows a series of oscillograms for a constant pulse-energy setting [the same as for (a) of Fig. 6] and varying temperature. As noted above, the pulse at

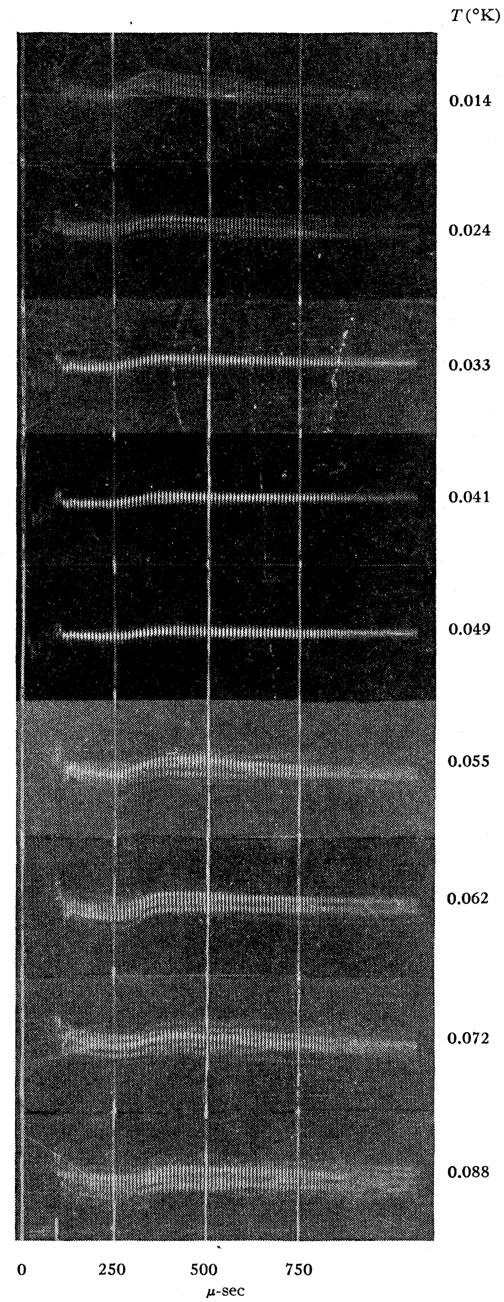

FIG. 7. Second pulses at extremely low temperatures. Sequence of received pulses at lowest practicable power input (constant at 4800 ergs sec^{-1} cm^{-2}) during warm-up from 0.015°K. Increase of specific heat of the helium with temperature causes progressive diminution of signal strength, resulting in barely visible signals even with the increased amplification used for the lower four photographs.

0.05°K, for which $\tau \approx \frac{1}{2}T$, is very little spread out. In contrast, the pulse at 0.015°K, for which $\tau \approx 4T$, closely resembles (d) of Fig. 6 in shape and for which $\tau \approx 3T$. [The decrease of τ with rising temperature, due to the increase in the specific heat of liquid helium, is illustrated in Fig. 7 in that the received signal has all but faded into the background noise in the last 3 pictures, despite the fact that for these three the gain of the amplifier was increased and that the sensitivity of the receiver element (i.e., dR/dT) has increased.]

The phenomenon, however, cannot be explained solely in terms of the magnitude of τ/T since, for fixed input power, the general shape of the pulse such as in (d) of Fig. 6 is maintained throughout the temperature range up to about 0.5°K (compare the separation between curves C and E, Fig. 5) at which point τ/T has fallen to approximately 1/1500 of its value at 0.05°K, i.e., from 3 to 0.002.

Another explanation of the pulse-spreading *at very low temperatures* has been put forward by Atkins,[20] and by Mayper and Herlin.[21] These authors consider the effects of the increased mean free path for phonons at such temperatures, the flow of phonons becoming analogous to Knudsen flow in a gas. Mayper and Herlin envision the result as a building-up of the received pulse to appear "as if it were caused by diffusive heat flow," whereas on Atkins' picture the "decay" of the received pulse beyond its maximum, at least, is accounted for by a time lag in the escape of the injected heat to the main part of the low-temperature enclosure. [This latter effect would, of course, vary according to the precise structural details.]

Finally, Gorter[22] has examined further the suggestion[5] that at the lowest temperatures one may actually have diffusive heat flow, rather than second sound proper.

D. Anomalous Velocity Behavior

The continued increase in wave velocity past the $u_1/\sqrt{3}$ value observed for second sound below 0.5°K was suspected at first of being spurious. It appeared at that time that shock-wave effects might be enhancing the unperturbed wave velocity by an amount corresponding to the normal fluid particle velocity v_n. Effects of this nature might be expected to increase the observed velocities by varying amounts depending upon pulse size. As pointed out earlier,[7] at low enough temperatures a situation might ultimately be reached where even the smallest observable heat pulse would have to proceed at about the first sound velocity u_1.

The normal fluid particle velocity v_n may be obtained by combining relationship (8) with the entropy flow condition,

$$\dot{H}/(T+\tau)=\rho S v_n, \tag{10}$$

and taking into account the T^3 nature of phonon specific heat and entropy, to give

$$v_n/u_2 \simeq \frac{3}{4}\left[1-\left(\frac{T}{T+\tau}\right)^4\right]. \tag{11}$$

We thus see that for extremely low temperatures, for which observable signals would require $\tau \geq T$, the observed pulse velocity would be, very roughly,

$$\begin{aligned} u &\simeq u_2+v_n \\ &\simeq (1+3/4)u_2 \simeq u_1, \end{aligned} \tag{12}$$

as given in reference 7.

Although this type of argument might conceivably explain our observed velocity decrease at the very low temperatures between 0.015°K and 0.05°K [see Figs. 2(b) and 7] it could not account for the anomalous velocity at a few tenths of a degree Kelvin where $\tau/T \ll 1$.

In fact, contrary to expectations, our attempts to observe such energy dependent velocity effects even at temperatures in the 0.05°K region produced singularly negative results. Thus the constant temperature series of Fig. 6 shows the initial precursor arrival time to be effectively independent of the heat pulse magnitude. This behavior of course lends credence to the velocity measurements obtained by observing point A of the pulse front. Despite distortion, the leading edge of the pulse proceeds as a precursor at a characteristic fixed velocity.

V. CONSIDERATIONS OF TEMPERATURE EQUILIBRIUM

In the region below 0.1°K where thermal conductivities become very small, any heat influx, whether the natural "drift" or that due to pulsing, will cause temperature inhomogeneity and errors in temperature measurement.

The most obvious effect is that the liquid helium should warm up ahead of the bulk of the salt. In our experimental arrangement this will be the case for pulse heat input, but, as far as "drift" is concerned, the divergence in temperature will tend to be lessened by the fact that the incoming heat reaches both the bulk of the liquid and of the salt via high-resistance paths.

It was observed that the major portion of the heat generated during a pulsing was absorbed by the salt in less time than the interval between the end of the pulsing and the first ensuing susceptibility determination (a few seconds). A small thermal "after-effect" was evident, however, (except in the case of the very low energy pulses) which persisted for a period of up to one minute, depending on the size of the pulse.

Multi-trace pictures were only observed in the region of 0.6°K, where the velocity is extremely sensitive to temperature variation, and only for the larger pulses.

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[20] K. R. Atkins, Phys. Rev. **89**, 526 (1953).
[21] V. Mayper and M. A. Herlin, Phys. Rev. **89**, 523 (1953).
[22] C. J. Gorter, Phys. Rev. **88**, 681 (1952).

We invariably obtained too low values of u_2 whenever the warm-up time was short—i.e., a few minutes—and also when employing pulses large enough to produce a significant jump in the temperature of the system. Under our experimental conditions (best 1800 ergs/min, normal 4000 ergs/min) it is most likely that the indicated temperatures are rather lower than the true values below 0.1°K, although the errors in the velocity curve [Fig. 2(b)] are most probably small above 0.05°K where u_2 is varying rather slowly with temperature.

VI. SUMMARY

In conclusion, our investigation of second sound própagation below 1°K has shown:

A. In the region immediately below about 0.8°K, the second sound velocity rises quite abruptly, reaching the Landau $u_1/\sqrt{3}$ velocity by 0.5°K. This is nearly thrice the value reported by Atkins and Osborne at 0.5°K.

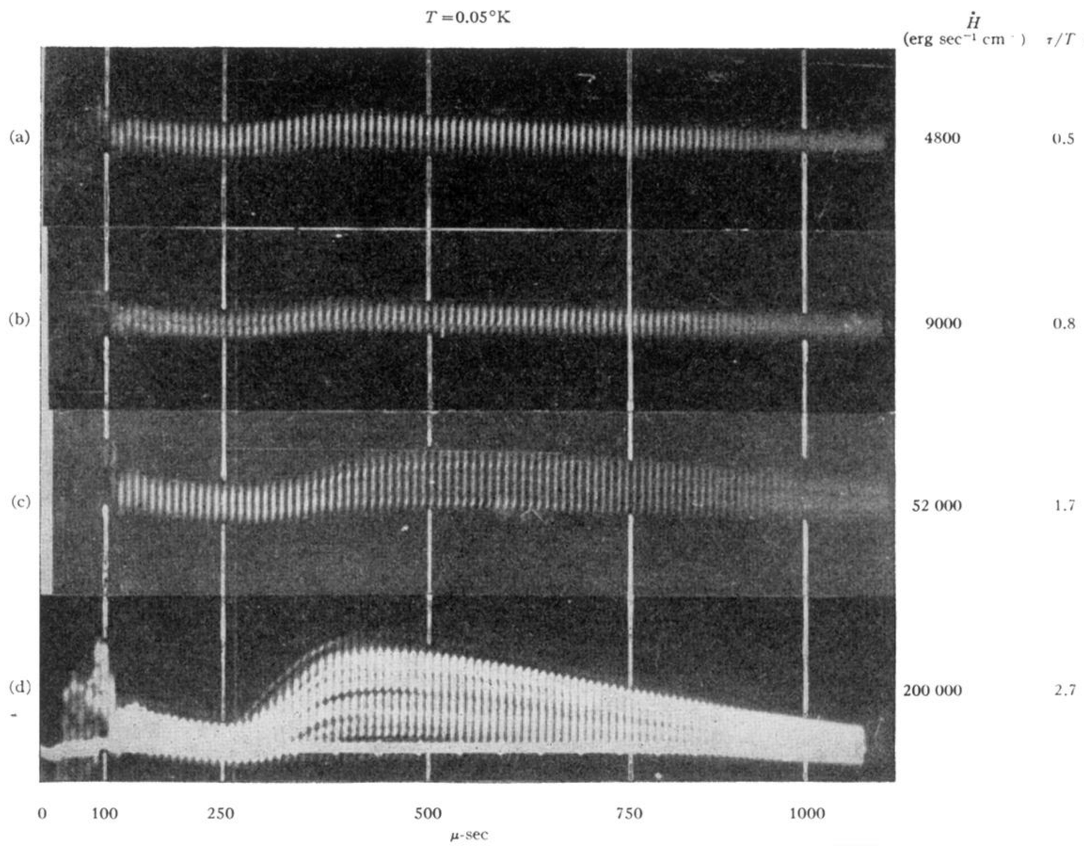
B. Although the velocity curve tends to level off somewhat in the 0.4°K–0.5°K region, the values continue to rise above the Landau value at the still lower temperatures, appearing to approach first sound velocity u_1 at absolute zero.

C. With regard to the thermodynamic quantity ρ_n/ρ, the phonon region below 0.5°K shows very nearly the fourth power (actually 4.18) dependence on temperature predicted by Landau. The gross increase of ρ_n/ρ past the fourth-power behavior above 0.5°K represents the excitation of rotons, and our values for the roton contribution to the normal fluid concentration $(\rho_n)_{\text{rot}}/\rho$ check Landau's predictions very closely. Our results allow a small modification of the values of the Landau constants.

D. Heat pulse distortion at a very low temperature such as 0.05°K appears to be small when the pulse temperature is smaller than the ambient value. At these temperatures and higher, even in the presence of pulse-spreading, the first observable signal precursor appears to be a significant and reproducible quantity.

VII. ACKNOWLEDGMENT

We are indebted to Dr. Keith McLane, on leave from the University of Wisconsin, for his valuable cooperation during the second series of runs. It is also a pleasure to acknowledge the efforts of Mr. A. K. Stober and Mr. J. O'Leary in liquefying the helium necessary for these experiments.



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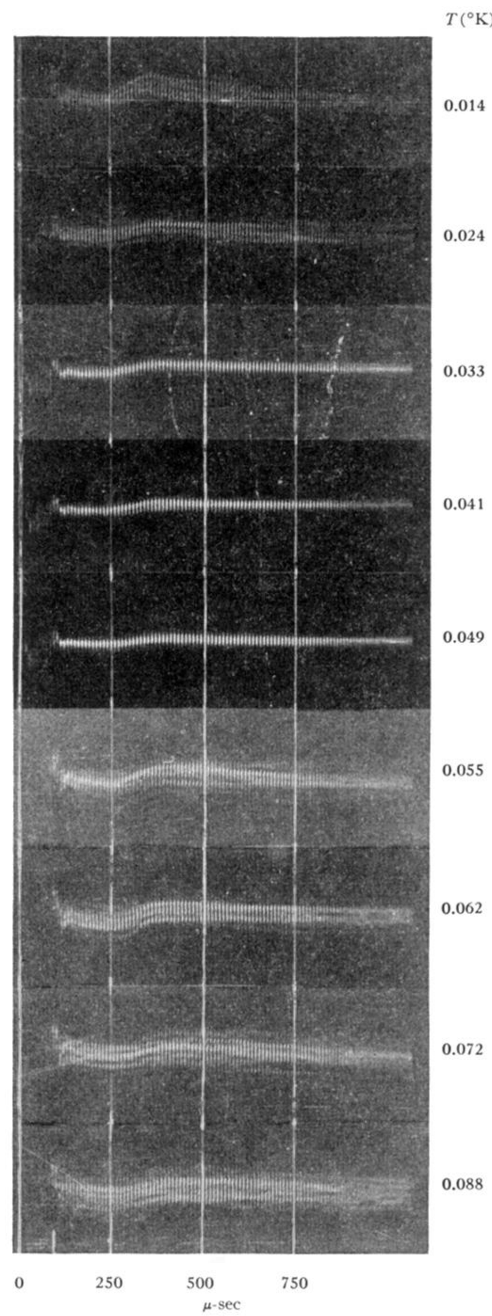

FIG. 7. Second pulses at extremely low temperatures. Sequence of received pulses at lowest practicable power input (constant at 4800 ergs sec^{-1} cm^{-2}) during warm-up from 0.015°K. Increase of specific heat of the helium with temperature causes progressive diminution of signal strength, resulting in barely visible signals even with the increased amplification used for the lower four photographs.