A Theorem on Photomeson Production near Threshold and the Suppression of Pairs in Pseudoscalar Meson Theory*

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It is shown that the photoproduction of charged pseudoscalar mesons at threshold computed to lowest order in the meson-nucleon mass ratio but to arbitrary order in the meson-nucleon coupling constant is essentially equivalent to the weak-coupling result obtained from second-order perturbation theory. The μ/M corrections can be estimated from the ratio of the positive to the negative production cross section. It is therefore suggested that the measurement of the S-wave photoproduction of charged mesons may be one of the more reliable methods presently available for determining the meson-nucleon coupling constant. As the threshold production proceeds entirely through the mediation of negative energy states the connection of this result with the possible suppression of nucleon anti-nucleon pairs is discussed.

I. INTRODUCTION

It is well known that the matrix element for the photoproduction of charged pseudoscalar mesons as computed by second-order perturbation theory has a finite limit at threshold. The threshold production is, of course, S wave, so that in the vicinity of threshold the angular distribution of the mesons should be spherically symmetric in the center-of-mass system. The angular variation observed at about 100 Mev above threshold has led to some speculation regarding the behavior of the S-wave production which would be predicted by a more correct treatment of pseudoscalar meson theory.[1,2] Such speculations have been further stimulated by the fact that second-order perturbation theory applied to the scattering of pseudoscalar-coupled pseudoscalar mesons by nucleons predicts a much larger S-wave scattering than has been observed.[3] It has, in particular, been suggested that a large short-range meson-nucleon interaction would tend to suppress the S-wave production. It would also yield a much smaller S-wave meson-nucleon scattering than indicated by a first Born approximation.[1,4] Furthermore, it has been shown that the use of a modified propagation function for the nucleons leads to a suppression of nucleon pair (i.e., negative energy) states. Since both the S-wave photoproduction and the S-wave meson-nucleon scattering arise through the mediation of these states, the use of the modified propagation function would suppress both effects.[5]

We should like to point out first of all that the suppression of nucleon pair effects cannot be a general feature of the theory. The Thomson limit of the Compton effect for any Dirac particle arises entirely through the mediation of particle anti-particle pairs. It was, in fact, just this effect which served to demonstrate the essential role of particle pairs in the early days of the Dirac electron theory. The Thomson scattering is a classical effect and should be independent of any meson-nucleon interactions (or any other proton structure effects).

The question arises as to whether any similar statements can be made regarding photomeson production. In the next section we shall prove the following theorem: the matrix element for charged photomeson production at threshold, correct to all orders in the meson coupling constant, approaches the weak-coupling result in the limit of vanishing meson mass, provided that the meson coupling constant and nucleon mass are replaced by their renormalized values as conventionally defined.[5b]

The above stated theorem shows that the suppression of pair effects is also ineffective for photomeson production and also raises serious doubts as to whether a repulsive S-wave meson-nucleon interaction would, within the framework of a relativistic gauge invariant conventionally renormalized theory, lead to a suppression of S-wave photoproduction.

Following the proof of the theorem in the next section, we shall discuss, in Sec. III, the possibility of

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[1] S. D. Drell and E. M. Henley, Phys. Rev. **88**, 1053 (1952).

[2] J. Steinberger and A. S. Bishop, Phys. Rev. **86**, 171 (1952).

[3] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).

[4] Bethe, Dyson, Mitra, Ross, Salpeter, Schweber, Sundaresan, and Visscher, Phys. Rev. **90**, 372 (1953).

[5] Brueckner, Gell-Mann, and Goldberger, Phys. Rev. **90**, 476 (1953).

[5b] Ashkin, Simon, and Marshak, Prog. Theoret. Phys. (Japan) **5**, 634 (1950). Koba, Kotani, and Nakai, Prog. Theoret. Phys. (Japan) **6**, 849 (1951). The renormalized form of the modified propagation functions [subsequently written $S_F'(p)$, $\Delta'_F(q)$] and the modified current operators [$\Gamma_\mu(p,p')$; $Q_\mu(q,q')$] are determined by electrodynamic properties. The definition of the renormalized meson-nucleon interaction operator [$\Gamma_5(p,p')$] requires an additional specification. The condition used in this article is given explicitly in reference 10 and agrees with that of the above authors in the limit of vanishing meson mass. One may regard the theorem simply as a means of providing a physical interpretation of the conventional renormalization scheme.

determining the meson coupling constant from the photomeson production cross section.

While we have been unable to make any definite statements about the suppression of pairs in other processes, it is possible in the cases discussed above to localize the reappearance of the effects suppressed by the modified propagation function. This makes possible some discussion of the general question, which appears in Sec. IV.

II. PROOF OF THE THEOREM

In order to prove the theorem, we shall first show that the complete matrix elements for photoproduction at threshold are given, for sufficiently small rest mass of the outgoing meson, by the matrix elements of the operators:

$$U_P{}^{+}=egA_\mu{}^{+}(k)\phi'^{-}(q)\lim_{p\to p'}\bar{\psi}'(p')\times\left[\frac{\partial}{\partial p_\mu}\Gamma_5{}^{+}(p',p)S_F'(p)\right]S_F'^{-1}(p)\psi'(p')\quad(1)$$

for positive production from protons;

$$U_P{}^{0}=eg_0A_\mu{}^{+}(k)\phi_0'^{-}(q)\lim_{p\to p'}\bar{\psi}'(p')S_F'^{-1}(p')\times\left[\frac{\partial}{\partial p_\mu}S_F'(p)\Gamma_5{}^{0}(p,p)S_F'(p)\right]S_F'^{-1}(p)\psi'(p')\quad(2)$$

for neutral production from protons;

$$U_N{}^{-}=egA_\mu{}^{+}(k)\phi'^{-}(q)\lim_{p\to p'}\bar{\psi}'(p')S_F'^{-1}(p)\times\left[\frac{\partial}{\partial p_\mu}S_F'(p)\Gamma_5{}^{-}(p,p')\right]\psi'(p')\quad(3)$$

for negative production from neutrons.[6]

In the foregoing expressions, the operators $A_\mu{}^{+}$ and ψ' serve to annihilate the initial photon and nucleon, respectively, while ϕ'^{-} and $\bar{\psi}'$ create in the final meson and nucleon. The primes appearing on the various field operators imply that they are multiplied by the appropriate renormalization constants. We take $p_\mu'=(-\mathbf{k},[M^2+k^2]^{\frac{1}{2}})$ and $q_\mu=(\mathbf{0},\lambda)$. The incident and final nucleon momenta are taken equal in anticipation of the limiting process ($\lambda\to 0$) which we are going to perform. The interior nucleon momentum, p_μ, is kept different from that of a free nucleon until the end of the calculation in order to avoid the appearance of the singularities in the S_F' functions.

The function S_F' describes the propagation of a proton and is supposed to include all reactive corrections associated with the emission and absorption of mesons of mass μ. $\Gamma_5{}^{+-}(p,p')$ is the charged meson-nucleon interaction operator while $\Gamma_5{}^{0}(p,p')$ is the proton-neutral meson interaction operator. These too are to include all reactive effects associated with mass μ mesons. The operation $\partial/\partial p_\mu$ together with the associated inverse propagation functions will be shown to describe the complete interaction of the photon with the proton and with the currents associated with the meson-nucleon interaction operator. The interaction of the photon with the neutron and with the current of the outgoing meson has been omitted in (1) and (3) and will be shown separately to vanish in the limit we are considering.

As a first step in establishing (1), (2), and (3) we shall justify the omission of the outgoing meson current terms. The contribution of the outgoing meson current term to, for example, positive photomeson production is of the form

$$U_P{}^{+\prime}=egA_\mu{}^{+}(k)\phi'^{-}(q)\bar{\psi}(p+k-q)\Gamma_5{}^{+}(p+k-q,p)\times\psi(p)\Delta_F'(q-k)Q_\mu(q-k,q),\quad(4)$$

where Δ_F' is the meson propagation function and Q_μ is the meson current operator, both complete with all mesonic corrections. Simply as a consequence of covariance, $Q_\mu(q-k,q)$ must have the form

$$Q_\mu=q_\mu h_1(q^2,q\cdot k)+k_\mu h_2(q^2,q\cdot k).$$

However, $A_\mu k_\mu$ vanishes on account of the transversality of A_μ, while at threshold $A_\mu q_\mu=-A_0\lambda$ which vanishes since we take $A_0=0$. It might be noted that this result does not depend upon the assumption of a small outgoing meson mass.

Equations (1), (2), and (3) may now be seen to follow quite directly from the identities,[7]

$$-\frac{2}{(2\pi)^4}\frac{\partial S_F(p)}{\partial p_\mu}=S_F(p)\gamma_\mu S_F(p)\quad(5)$$

and

$$-\frac{2}{(2\pi)^4}\frac{\partial \Delta_F(p)}{\partial p_\mu}=\Delta_F(p)2ip_\mu\Delta_F(p),\quad(6)$$

as follows.

Recalling that $S_F'(p)=[1-S_F(p)\Sigma^*(p)]^{-1}S_F(p)$, we see that (5) and (6) imply

$$-\frac{2}{(2\pi)^4}\frac{\partial S_F'}{\partial p_\mu}=\frac{-2}{(2\pi)^4}(1-S_F\Sigma^*)^{-1}S_F\times\left[S_F{}^{-1}\frac{\partial S_F}{\partial p_\mu}S_F{}^{-1}+\frac{\partial\Sigma^*}{\partial p_\mu}\right]S_F(1-S_F\Sigma^*)^{-1}$$
$$=S_F'\left[\gamma_\mu-\frac{2}{(2\pi)^4}\frac{\partial\Sigma^*}{\partial p_\mu}\right]S_F'=S_F'\Gamma_\mu S_F',\quad(7)$$

where $\Sigma^*(p)$ is the proper nucleon self-energy operator and $\Gamma_\mu(p,p)$ is the nucleon current operator for a zero-energy photon.

[6] Irrelevant constant factors have been omitted from $U_P{}^{+}$, $U_P{}^{0}$, and $U_N{}^{-}$.

[7] J. C. Ward, Phys. Rev. **78**, 182 (1950); Abdus Salam, Phys. Rev. **79**, 910 (1950).

To see that $\gamma_\mu - 2(2\pi)^{-4}\partial\Sigma^*/\partial p_\mu$ is in fact equal to $\Gamma_\mu(p, p)$, we consider a typical constituent of $\Sigma^*(p)$ represented by the Feynman diagram in Fig. 1. Assuming $S_F'(p)$ to represent the propagation function of a proton, it must be possible to follow the proton's charge as it moves through the diagram, traveling along either meson lines or nucleon lines. One may choose the momentum variables in such a way that the external momentum p_μ appears in each segment of this charge carrying line, and nowhere else. Then the effect of the operation $-2(2\pi)^{-4}\partial/\partial p_\mu$ is, according to (5) and (6), simply to insert a photon in any one of the charge-carrying segments. The diagram may in addition have closed charge-carrying loops. It is, however, easy to see that these do not contribute. Thus, in the case of a closed charge-carrying loop, one may take as variable of integration t_μ, the momentum common to each segment of the loop. The interaction of the photon with the loop is then given by $-2(2\pi)^{-4}\partial/\partial t_\mu$, which vanishes on integration over t_μ.[8] These two observations establish Eq. (7).

We now remark that $\Gamma_\mu(p, p)$ vanishes for a neutron since $\Sigma^*(p)$ involves, for a neutron, only closed charge bearing loops. Thus the omission of the interaction between photon and neutron from (1), (2), and (3) is justified.

Confining our attention now to positive meson production we find, on inserting (7) into (1),

$$U^+ = egA_\mu{}^+(k)\phi'^-(q)$$
$$\times \lim_{p\to p'}\left[\bar\psi'(p')\Gamma_5(p', p)S_F'(p)\Gamma_\mu(p, p)\psi'(p')\frac{(2\pi)^4}{-2} + \bar\psi'(p')\frac{\partial\Gamma_5{}^+(p', p)}{\partial p_\mu}\psi'(p')\right]. \quad (8)$$

The first term corresponds to the photon-proton interaction term of second-order perturbation theory, with however, the meson-nucleon interaction, the proton propagation function, and the proton current all modified to include the reactive effects of the meson field. All of the remaining parts of the field reactions are included in the $\partial\Gamma_5/\partial p_\mu$ term which can be discussed in the same way as the $\partial\Sigma^*/\partial p_\mu$ terms in Eq. (7). In particular (see Fig. 2), $\partial/\partial p_\mu$ applied to a specific term in the perturbation theory expansion of Γ_5 yields the sum of all terms arising from the interaction of the photon with any segment of the charge-bearing line that goes through the corresponding diagram. Again the interaction with any closed charge-bearing line vanishes. The same argument can be used to establish (2) and (3).

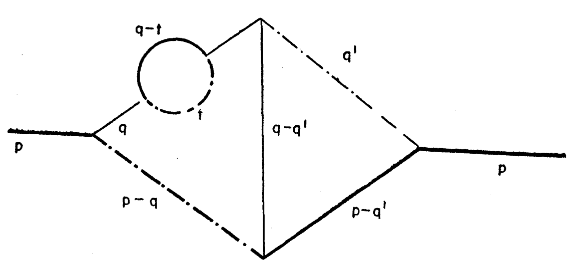

FIG. 1. A Feynman diagram representing a typical constituent of $\Sigma^*(p)$. The heavy lines are charge-bearing; the broken lines represent mesons, the solid lines, nucleons.

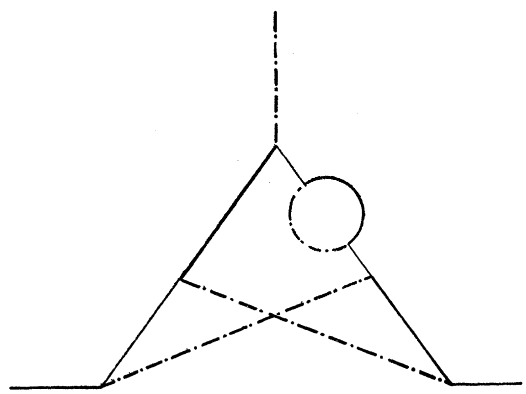

FIG. 2. A Feynman diagram representing a typical constituent of $\Gamma_5(p', p)$.

We now recall some general properties of the quantities appearing in (1).

$$\psi'(p) = Z_2^{\frac12}\psi(p);\ \bar\psi'(p) = Z_2^{\frac12}\bar\psi(p);\ \phi'^-(q) = Z_3^{\frac12}\phi^-(q);$$
$$S_F'^{-1}(p)\psi'(p') = Z_2^{-\frac12}S_F^{-1}(p)\psi(p')$$
(since we shall let $p\to p'$);
$$\bar\psi'(p')\Gamma_5{}^+(p', p)S_F'(p) = Z_1^{-1}Z_2^{\frac12}\bar\psi(p')\gamma_5 \times\{S_F(p)[1+(p^2+M^2)f_1+(p-p')^2f_2]+f_3\}. \quad (9)$$

In the above expressions Z_1, Z_2, and Z_3 are appropriate renormalization constants written in accordance with the conventions established by Dyson.[9] We have made use of the fact that $p'^2+M^2=0$ and that $\bar\psi(p')\times(i\gamma\cdot p'+M)=0$. The quantities f_1, f_2, and f_3 are functions of p^2+M^2 and $(p-p')^2$. Since these functions are finite at the zeros of their arguments when computed to any finite order in perturbation theory, we shall assume that they have this property generally.[10]

[8] A similar approach has been used by J. C. Ward in an investigation of the scattering of light by light [Phys. Rev. 77, 293 (1949)].

[9] F. J. Dyson, Phys. Rev. 75, 1736 (1949).

[10] Apart from the predictions of perturbation theory, the definition of charge and mass renormalization require that $\bar\psi(p)S_F(p)\times\Sigma_c^*(p)\psi(p)=0$ and $\lim_{p'\to p}(\bar\psi(p')\Gamma_{5c}(p', p)\psi(p)/\bar\psi(p')\gamma_5\psi(p))=1$, where $\Sigma_c^*(p)$ and $\Gamma_{5c}(p', p)$ are the renormalized functions. The restrictions which these conditions imply on the f functions are still sufficient to prove the theorem if one makes use of the fact that $\bar\psi(p)\gamma_5\psi(p)=0$. In Sec. III, however, we shall attempt to apply the theorem to the photoproduction of π mesons, and some hypothesis as to the form of the λ/M corrections will be necessary. As this hypothesis will be based upon perturbation theory, there seems little point, at present, in emphasizing the greater generality which is available.

Inserting these expressions in (1), we find

$$U_P{}^+ = egZ_1{}^{-1}Z_2Z_3{}^{\frac{1}{2}}A_\mu{}^+(k)\phi^-(q)$$
$$\times \lim_{p\to p'}\left\{\bar{\psi}(p')\gamma_5\frac{\partial S_F}{\partial p_\mu}S_F{}^{-1}(p)\psi(p')+\bar{\psi}(p')\gamma_5\frac{\partial}{\partial p_\mu}\right.$$
$$\times\{S_F(p)[(p^2+M^2)f_1+(p-p')^2f_2]+f_3\}$$
$$\left.\times S_F{}^{-1}\psi(p')\right\}. \quad (10)$$

The first term yields the renormalized second-order perturbation theory result while the second term vanishes, thus completing the proof of the theorem. The proof for $U_N{}^-$ is identical.

A similar discussion applied to neutral meson production yields the result,

$$\lim_{p\to p'}\bar{\psi}(p)S_F{}'^{-1}(p)\frac{\partial}{\partial p_\mu}[S_F{}'(p)\Gamma_5{}^0(p,p)S_F{}'(p)]$$
$$\times S_F{}'^{-1}(p)\psi'(p')=0. \quad (11)$$

From this we conclude that the ratio $\langle U_P{}^0\rangle/\langle U_P{}^+\rangle$ vanishes in our limit. It is easy to see that the ratio $\langle U_N{}^0\rangle/\langle U_P{}^+\rangle$ also vanishes, since the interaction of the photon in the production of neutral mesons from a neutron always occurs on a closed charge-bearing loop.

The proof given above has assumed specifically that the meson-nucleon coupling is pseudoscalar. The same proof can, however, be carried out formally for pseudovector coupling, and leads to the same results. The theory is, of course, not renormalizable so that the functions f_1, f_2, and f_3 cannot be computed, even in perturbation theory. On the other hand, the formal advantage of pseudoscalar coupling in this respect may, in view of the large coupling constant required, be only a superficial one. It might be further remarked that the nature of the interior mesons appears in the problem only in the properties of the f functions. Thus, if one can assume the existence of these functions, the validity of the result is unaffected by the coupling of the nucleons to, say, heavier bosons or fermions.

We might note at this point that there are several other theorems relating to other processes which can be proved in the same way. The most obvious, and also the most important of these, would assert that the Thomson limit of the Compton effect is independent of meson-nucleon couplings. In this case, one has

$$U_{(\text{Compton effect})}=e^2A_\mu{}^+(k)A_\nu{}^-(k')$$
$$\times\lim_{p\to p'}\bar{\psi}'(p')S_F{}'^{-1}(p)\frac{\partial^2 S_F{}'(p)}{\partial p_\mu\partial p_\nu}S_F{}'^{-1}(p)\psi'(p'), \quad (12)$$

from which the result follows immediately on insertion of the appropriate expressions.[10b] One can also deal with the interaction of neutral scalar mesons by differentiating with respect to the nucleon mass instead of a momentum, but in view of the lack of practical examples, we shall not discuss this application.

Throughout this section we have made a distinction between the mass of the outgoing meson and that of the interior meson. One might inquire as to whether the theorem is true if one allows the mass of the interior mesons to vanish as well. The proof is affected only through the behavior of the functions f_1, f_2, and f_3. Thus one sets the interior meson mass $\mu=0$ and inquires about the behavior of f_1, f_2, and f_3 at the zeros of their arguments. For any finite set of terms from perturbation theory one can show that f_1 and f_3 are finite, while f_2 diverges logarithmically in p^2+M^2. Assuming this result to be correct for the true functions, one can easily see that the proof still holds.

III. APPLICATION TO PHOTOMESON PRODUCTION

It is interesting to note that, if the meson mass were very much smaller than the nucleon mass, then one would have in photomeson production an unambiguous means of measuring the renormalized meson-nucleon interaction constant. The situation would, indeed, be quite analogous to the fact that the Thomson scattering can be used to determine the charge of a particle irrespective of the complexity of its structure. In the case of the π meson, the mass ratio is 0.15, which is not a very small number, particularly in view of the fact that in the radiative corrections it will appear with the meson coupling constant, which one expects to be large. In spite of this we shall attempt to construct a plausible argument to the effect that the threshold photomeson production may yield a reliable measure of the interaction constant.

If one calculates to a finite order in perturbation theory, the square of the threshold matrix element summed over nucleon spin, is an expression of the form

$$T_{P,\pi^+}(\lambda)=\frac{1}{k_0q_0}[F_1+(\lambda/M)G_1+H_1(\lambda/M)],$$
$$T_{N,\pi^-}(\lambda)=\frac{1}{k_0q_0}[F_2+(\lambda/M)G_2+H_2(\lambda/M)], \quad (13)$$

where F and G are independent of λ/M and $H(x)$ goes to zero at least as fast as $x^2\ln^m(x)$. k_0 and q_0 are the energies of the photon and meson, respectively.

The theorem proved in Sec. II shows that F_1 and F_2 are the second-order perturbation theory results for a zero-mass meson. From a direct calculation one then finds $F_1=F_2$.

If H_1 and H_2 do not dominate the matrix elements T, G_1 and G_2 can be estimated from photoproduction experiments near threshold. To show this we first prove that

$$G_1/F_1=-G_2/F_2, \quad (14)$$

[10b] Our attention has been called to the fact that this method has been applied previously to the Compton effect in quantum electrodynamics. W. Thirring, Phil. Mag. 41, 1193 (1950).

i.e., $T_{P,\pi^+}(\lambda)=T_{N,\pi^-}(-\lambda)$ to terms in λ/M. The expression for T_{P,π^+} is obtained by evaluating

$$\sum_{\text{spins}} |\langle \mathbf{p}, p_0; \mathbf{k}, k_0 | U_P^+ | \mathbf{p}+\mathbf{k}-\mathbf{q}, p_0+k_0-q_0; \mathbf{q}, q_0\rangle|^2 = \frac{1}{k_0q_0}\Phi(p\cdot k, p\cdot q, k\cdot q) \quad (15)$$

at threshold in the center-of-mass system, so that

$$T_{P,\pi^+}=\sum_{\text{spin}} |\langle -\mathbf{k}, p_0; k, \lambda + M - p_0 | U_P^+ | \mathbf{0}, M; \mathbf{0}, \lambda\rangle|^2 \quad (16)$$

with $p_0=(k^2+M^2)^{\frac{1}{2}}$. We now note that the production of negative mesons from neutrons can also be expressed in terms of U_P^+ (as well as in terms of U_N^- as was done in Sec. II). Thus

$$T_{N,\pi^-}=\sum_{\text{spin}} |\langle 0, M; \mathbf{k}, -(\lambda+M-p_0) \times | U_P^+ | -\mathbf{k}, p_0; \mathbf{0}, -\lambda\rangle|^2. \quad (17)$$

On evaluating $p\cdot k$, $p\cdot q$, and $k\cdot q$ at threshold, we find

$$T_{P,\pi^+}=\frac{1}{k_0q_0}\Phi\left(-M\lambda-\frac{\lambda^2}{2}, -M\lambda-\frac{\lambda^3}{2(M+\lambda)}, -\lambda^2+\frac{\lambda^3}{2(M+\lambda)}\right), \quad (18a)$$

$$T_{N,\pi^-}=\frac{1}{k_0q_0}\Phi\left(M\lambda-\frac{M\lambda^2}{2(M+\lambda)}, M\lambda, -\lambda^2+\frac{\lambda^3}{2(M+\lambda)}\right). \quad (18b)$$

If one now compares the arguments of Φ appearing in (18a) with those appearing in (18b) it is evident that the terms linear in λ appear with opposite signs. Therefore $G_1=G_2$ and

$$T_{P,\pi^+}+T_{N,\pi^-}=\frac{1}{k_0q_0}[2F+H_1(\lambda/M)+H_2(\lambda/M)]. \quad (19)$$

An estimate of the significance of $H_{1,2}$ comes from an analysis of experiments on the photoproduction of π^0 mesons from protons. These show almost no S-wave production even very close to threshold. At threshold the matrix element for π^0 production was shown in Sec. II to vanish (relative to that for π^+ production) if the λ mass was taken to be zero.

More specifically,

$$T_{P,\pi^0}(\lambda)=\frac{1}{k_0q_0}H_3(\lambda/M), \quad (20)$$

where H_1, H_2, and H_3 are expected to be of comparable magnitude with similar λ/M behavior near threshold. The small experimental value[11] of T_{P,π^0} next to T_{P,π^+} then implies that

$$|H_{1,2,3}|\ll F+(\mu/M)G_1. \quad (21)$$

Neglecting H, $F>|(\mu/M)G|$ since both T_{P,π^+} and T_{N,π^-} are positive. The precise magnitude of G would follow from a knowledge of $\gamma+P\rightarrow\pi^+/\gamma+N\rightarrow\pi^-$ exactly at threshold.

Measurements of the π^-/π^+ ratio from deuterium have been made at various angles with meson energies between 34 and 128 Mev.[12] The mean ratio is 1.07 ± 0.7 with no significant energy variation. If this ratio remains about one down to threshold the $(\mu/M)G$ correction is unimportant. There is evidence that the π^+ production is almost all S wave below 40 Mev; these data are also consistent with a constant matrix element. If the π^- production is similar in these respects, then $|(\mu/M)G|$ is certainly much smaller than F.

At present, the limiting factor in finding $g^2/4\pi$ from the near threshold cross section are the uncertainties in the low-energy $\gamma+P\rightarrow\pi^+$ measurements. In the region below 40 Mev, a best fit to the observed cross section obtains for $g^2/4\pi\sim25$ with pseudoscalar coupling and $g^2/4\pi\sim0.19$ with pseudovector coupling.[13] The statistics of the observations and the present uncertainty in γ-ray beam calibration would permit a change of up to 50 percent in these coupling constants.

IV. ON THE SUPPRESSION OF NUCLEON PAIR EFFECTS

We shall now examine in some detail the way in which the pair suppression argument[5] breaks down in the photomeson production. This argument is based on a discussion of the properties of the renormalized nucleon propagation function, $S_F'(p)$. In particular, one may write

$$S_{Fc}'(p)=(1-S_F(p)\Sigma_c^*(p))^{-1}S_F(p). \quad (22)$$

$\Sigma_c^*(p)$ is the proper, renormalized, self-energy operator and may be taken to have the form[14]

$$\Sigma_c^*(p)=-\frac{(2\pi)^4}{2i}(i\gamma\cdot p+M)\left[\frac{p^2+M^2}{M^2}f_1(p^2+M^2)+\frac{i\gamma\cdot p+M}{2M}f_2(p^2+M^2)\right], \quad (23)$$

[11] For 45-Mev mesons at 90° in the laboratory system, $\sigma(\gamma+p\rightarrow\pi^0)/\sigma(\gamma+p\rightarrow\pi^+)\sim0.2$. A Silverman and M. Stearns, Phys. Rev. **88**, 1225 (1952); J. Steinberger and A. S. Bishop, Phys. Rev. **86**, 171 (1952). Below 45 Mev, $\sigma(\gamma+p\rightarrow\pi^0)/\sigma(\gamma+p\rightarrow\pi^+)\sim(E_\pi/45\ \text{Mev})\times0.2$. Goldschmidt-Clermont, Osborne, and Scott, Phys. Rev. **89**, 329 (1953); Janes, Kraushaar, Osborne, and Parker (unpublished). We are grateful to Professor B. T. Feld for communicating these results to us in advance of publication. The observed π^0 angular distribution indicates almost no S-wave production which alone is a measure of $H_3(\mu/M)$. Therefore it appears that $H_3(\mu/M)$ is indeed quite negligible.

[12] White, Jacobson, and Schulz, Phys. Rev. **88**, 836 (1952); R. M. Littauer and D. Walker, Phys. Rev. **82**, 746 (1951).

[13] Janes *et al.*, reference 11. Goldschmidt-Clermont, Osborne, and Winston (unpublished). We are again grateful to Professor Feld for communicating these data.

[14] The functions f_i appearing in Eqs. (23)...(27) are different from those appearing in (9), (10), as well as from those appearing in (28), (29). They are used simply to indicate certain general features of the functional dependence of the expressions on the left.

which yields

$$S_{Fc}'=\left(1+\frac{p^2+M^2}{M^2}f_1+\frac{i\gamma\cdot p+M}{2M}f_2\right)^{-1}S_F(p). \quad (24)$$

One may now consider the properties of S_{Fc}' for momenta nearly satisfying $p^2+M^2=0$, in which case

$$S_{Fc}'\approx\left[1+\frac{i\gamma\cdot p+M}{2M}f_2(0)\right]^{-1}S_F. \quad (25)$$

Now if $f_2(0)\gg 1$ as might be the case for a theory with a large coupling constant, the insertion of this function into a perturbation theory result in place of S_F tends to suppress those parts of the interaction which proceed by intermediate pairs. The large S-wave photoproduction arises through the fact that the γ_5 associated with the meson emission produces a pair which is later annihilated by absorption of the photon through $\boldsymbol{\gamma}\cdot\mathbf{A}$. The use of S_F' reduces this process by a factor $[1+f_2(0)]^{-1}$.

On the other hand, our theorem shows that such a suppression cannot take place and, in fact, indicates that the cancellation of the effect arises from the modified interaction of the photon with the nucleon. To see this in detail, we recall that

$$\begin{aligned}\Gamma_{\mu c}(p,p)&=\gamma_\mu-\frac{2}{(2\pi)^4}\frac{\partial\Sigma_c^*}{\partial p_\mu}\\&=\gamma_\mu\left[1+\frac{p^2+M^2}{M^2}f_1+\frac{i\gamma\cdot p+M}{2M}f_2\right]\\&\quad+(i\gamma\cdot p+M)\frac{1}{i}\frac{\partial}{\partial p_\mu}\left(\frac{p^2+M^2}{M^2}f_1+\frac{i\gamma\cdot p+M}{2M}f_2\right).\end{aligned} \quad (26)$$

Again assuming that $p^2+M^2\approx 0$ and that we shall multiply $\Gamma_\mu(p,p)$ on the left by $\bar{\psi}(p)$, we have effectively

$$\Gamma_{\mu c}(p,p)=\gamma_\mu\left[1+\frac{i\gamma\cdot p+M}{2M}f_2(0)\right], \quad (27)$$

so that the cancellation of the suppression is evident.

If one chooses to regard the effect of S_F' on meson-nucleon scattering as a manifestation of a strong repulsive S-state interaction, then it would appear that there are large currents associated with the interaction which serve to restore the interaction with the photon.

One may now inquire as to the validity of the pair suppression in meson-nucleon scattering or in the nuclear force problem. While we cannot make definite statements in this case, it is clear, for example, that the general form of the meson-nucleon interaction operator is

$$\begin{aligned}\Gamma_5(p',p)=&\gamma_5[1+(p-p')^2f_1+(p^2+M^2)f_2+(p_1^2+M^2)f_3]\\&+\frac{(i\gamma\cdot p'+M)\gamma_5(i\gamma\cdot p+M)}{M^2}f_4\\&+\frac{(i\gamma\cdot p'+M)}{2M}\gamma_5f_5+\gamma_5\frac{(i\gamma\cdot p+M)}{2M}f_6,\end{aligned} \quad (28)$$

where the functions $f_1,\cdots,f_6$ are functions of P^2+M^3, P'^2+M^2, $(p-p')^2$. Considering again the case that these quantities are all small, Γ_5 becomes

$$\begin{aligned}\Gamma_5(p',p)=&\frac{(i\gamma\cdot p'+M)\gamma_5(i\gamma\cdot p+M)}{4M^2}f_4\\&+\frac{(i\gamma\cdot p'+M)}{2M}\gamma_5f_5+\gamma_5\frac{(i\gamma\cdot p+M)}{2M}f_6+\gamma_5.\end{aligned} \quad (29)$$

One might again expect $f_4, f_5, f_6\gg 1$ for a large coupling constant. It is thus apparent that the appearance of large pair-enhancing terms in Γ_5 is as likely as the appearance of pair-suppressing terms in S_F'. While this is, of course, far from a complete discussion of mesonic corrections to such processes, it does suggest that, within the structure of the theory, there is no clear-cut evidence for the suppression of intermediate nucleon pairs.

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