High-Energy Meson Production*

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(Received August 24, 1953)

The angular distributions of penetrating secondary particles from showers originating in lead and in carbon have been studied. Because the distributions for the two materials are very similar, it seemed meaningful to analyze them in terms of nucleon-nucleon collisions. From the kinetics of such collisions, it is possible to estimate the energy of the primary producing the interaction. If the angular distribution in the center-of-mass system is uniform, then

$$\frac{F(\theta)}{1-F(\theta)}=\gamma_c^2\tan^2\theta$$

where $F(\theta)$ is the fraction of the shower particles emitted within an angle θ in the laboratory system and γ_c is the γ of the center-of-mass motion. By making plots of $\log[F/(1-F)]$ *vs* $\log\tan\theta$, the γ_c of the interaction is found. Also, it is established that the angular distribution of particles produced in 10–40 Bev interactions is uniform in the center-of-mass system.

The average number of mesons produced in showers of about 30-Bev primary energy is 3 to 4. From estimates of secondary momenta it appears that, on the average, the mesons are produced with about 500-Mev energy in the center-of-mass system. Thus the collision of a 30-Bev nucleon with a target nucleon would seem to be, on the average, not 100 percent inelastic.

A rough determination has been made of the mean free path for penetrating shower production *versus* energy for the secondary π-mesons. The cross section for further meson production is found to be about 0.1 geometric in lead at 1 Bev rising to geometric for energies greater than 4 Bev. The secondaries produced in meson-nucleon collisions seem to be distributed uniformly in the center-of-mass system.

Arguments are presented to show that less than 15 percent of the secondaries of penetrating showers are heavy mesons.

I. INTRODUCTION

THE present paper is a study of the angular distribution and interactions of secondary particles produced in penetrating showers. From these data it is possible to deduce some of the details of pion production in nucleon-nucleon and pion-nucleon collisions in the energy range of 10–40 Bev. It is tacitly assumed that the majority of the particles produced in the observed showers are pions although some instances of the decay of other unstable particles have been found. For some of the analysis the particle identity is not important. The data were obtained using a multiple plate cloud chamber and associated apparatus as shown in Fig. 1. The apparatus and triggering mechanism have been previously described in Part I.[1] The paper consists of a further study of the data presented in Part I.

II. THE ANGULAR DISTRIBUTION OF SHOWER PARTICLES

A measurement of the angular distribution of penetrating particles from showers in carbon and lead has been made. Penetrating particles are defined as particles which appear to be lightly ionizing and traverse at least 17 g/cm² of lead without electronic multiplication. The angle measured was the total angle between the direction of the primary particle and the secondary particle. The method of measuring the angle from the two steroscopic views is given in Appendix II. Particles which leave the shower origin at large angles with respect to the vertical are not so easily observed as those in the nearly vertical direction. A correction has been made for this effect. The results of the measurements are given in Fig. 2, in which the two histograms are normalized to represent equal numbers of showers from carbon and lead. The two distributions are very similar. The median angle of the lead showers is slightly larger than the median angle of the carbon showers. The most noteworthy difference is in the number of backward moving shower particles. The fraction of regressive particles for lead showers was 0.06; for carbon showers 0.0025. This difference has also been noted by the Bristol group.[2] In spite of this we conclude most of the shower particles emerge from the struck nucleus without appreciable scattering. It is, therefore, thought that the angular distributions from carbon should give some information on the angular distribution of the particles produced in a nucleon-nucleon encounter, which can in turn be used to estimate the primary energies of the observed showers. The analysis given below holds strictly only for a single nucleon-nucleon collision.

III. ENERGY ESTIMATES FROM THE ANGULAR DISTRIBUTIONS

The angle of emission θ of a particle in the laboratory system is related to the angle ϕ in the center-of-mass system by the following formula:

$$\tan\theta=\frac{\sin\phi}{\gamma_c[\cos\phi+\beta_c/\beta_\pi]} \qquad (1)$$

* Supported in part by a grant from the Research Corporation.
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[1] Walker, Duller, and Sorrels, Phys. Rev. **86**, 865 (1952).

[2] Camerini, Davies, Fowler, Lock, Muirhead, Perkins, and Yekutieli, Phil. Mag. **42**, 1241 (1951).

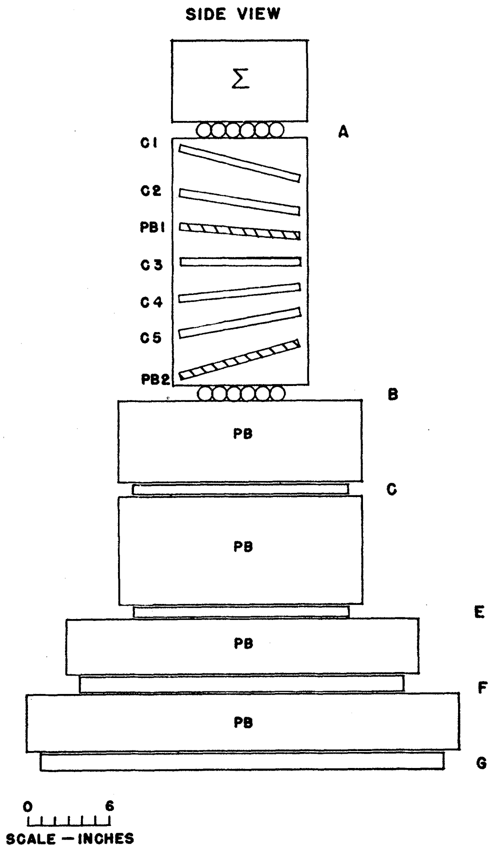

FIG. 1. Diagram of the cloud chamber and its associated apparatus.

where β_c = velocity of the center-of-mass system in units of c, β_π = velocity of the secondary particle in the center-of-mass system, and $\gamma_c = 1/(1-\beta_c^2)^{\frac{1}{2}}$. Making the approximation that $\beta_c/\beta_\pi = 1$, this expression may be written,

$$\tan\theta = \tan(\phi/2)/\gamma_c. \qquad (2)$$

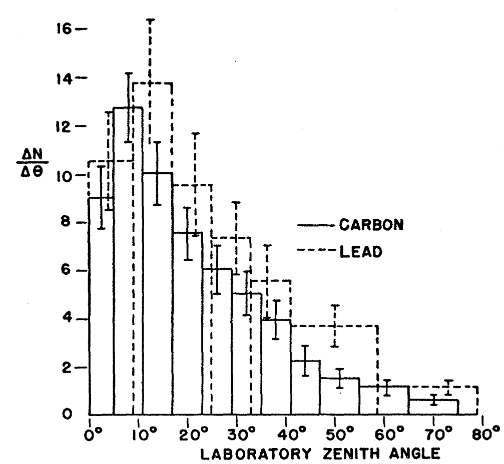

FIG. 2. A comparison of the angular distribution of shower particles arising from showers in carbon and lead.

If the secondary particles are emitted isotropically in the center-of-mass system, the fraction F of the particles that are emitted within the angle ϕ in the center-of-mass is given by $F = \sin^2(\phi/2)$. The previous expression may then be squared and written in the form:

$$\tan^2\theta = \frac{\sin^2(\phi/2)}{\gamma_c^2 \cos^2(\phi/2)} = \frac{F(\theta)}{\gamma_c^2[1-F(\theta)]}. \qquad (3)$$

To test the validity of this relationship it is necessary to have a group of showers whose energies are known by other means.[3,4] The closest approximation to such a situation is furnished by the Bristol[2] analysis made on the basis of multiple-scattering measurements of the primaries.

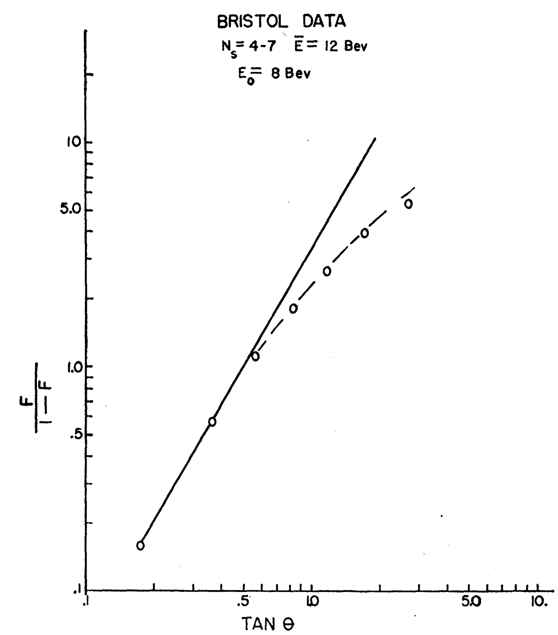

FIG. 3. F plot of the angular distribution published by Camerini *et al.* for showers of 4–7 particles.

In Fig. 3 a plot has been made of the Bristol angular distribution data.[2] Log$[F/(1-F)]$ has been plotted against log $\tan\theta$. This F plot is taken from the curves for showers of 4–7 particles. It is a straight line up to fairly large angles. The average energy of the primaries of these showers as determined from the F plot is 8 Bev. The average energy according to multiple-scattering measurements is of the order of 10–12 Bev. This discrepancy is discussed below. The Bristol group[2] has found the distribution in the center-of-mass system to be uniform for these energies; yet it is evident from

[3] Using the angular distribution of shower particles (reference 4) from interactions produced by 2.2-Bev protons from the Brookhaven Cosmotron, an estimate of the energy on the basis of an F plot gives 1.6 Bev.
[4] Private communication from Leavitt, Swartz, Shapiro, Smith, and Widgoff.

the plot that large deviations from a straight line occur at the larger angles. These deviations are probably due to two effects, both of which would produce a distribution wider than that given by the formula.

1. Scattering and interactions of the secondary particles probably take place within the struck nucleus. However, according to the data reported below and also the recent results obtained at Brookhaven,[5] the pion scattering cross section decreases above energies of a few hundred Mev.

2. The velocity of the secondary particles in the center-of-mass system is possibly greater than the velocity of the center-of-mass system. For higher-energy primaries both these velocities should converge toward the velocity of light.

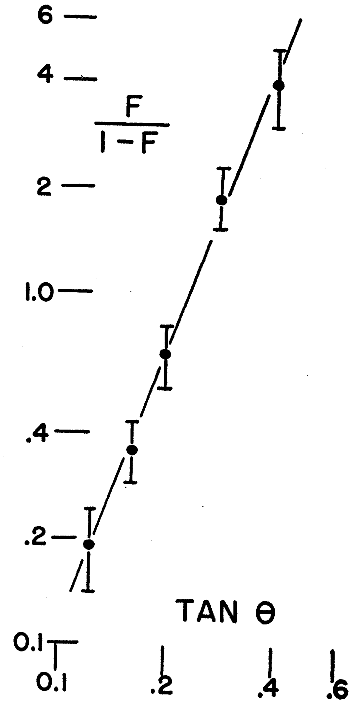

FIG. 4. F plot for showers originating in carbon whose median angles lay between 11° and 15°. $\bar{E}=34$ Bev.

Thus, in these respects, the approximations made in the analysis should be more nearly valid at higher energies, providing the secondary distribution in the center-of-mass system remains symmetric at the higher energies.

At very high energies ($\sim 10^{12}$ ev) it is possible to get an idea of the angular distribution in a single shower because of the large multiplicity. The multiplicities of the showers recorded in this experiment were on the average only 4 or 5. Thus, grouping of showers must be made in order to get meaningful statistics. In this case showers were grouped according to their apparent median angle. F plots are shown in Figs. 4, 5, 6, and 7. The slopes of the lines are usually very close to two as they should be.

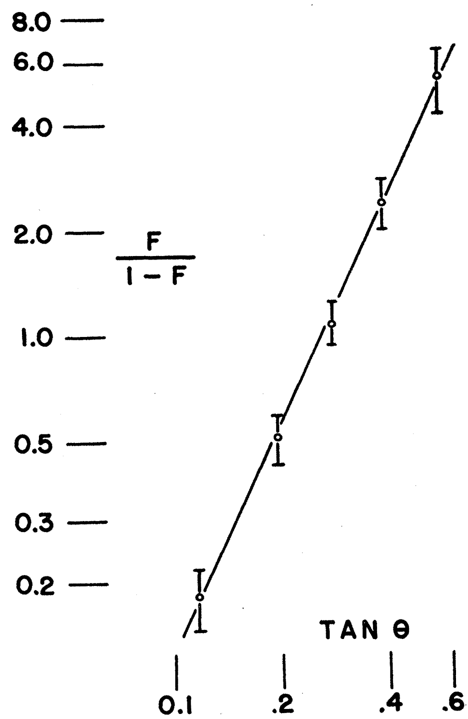

FIG. 5. F plot for showers originating in carbon whose median angles lay between 11° and 20°. $\bar{E}=26$ Bev. (Group II).

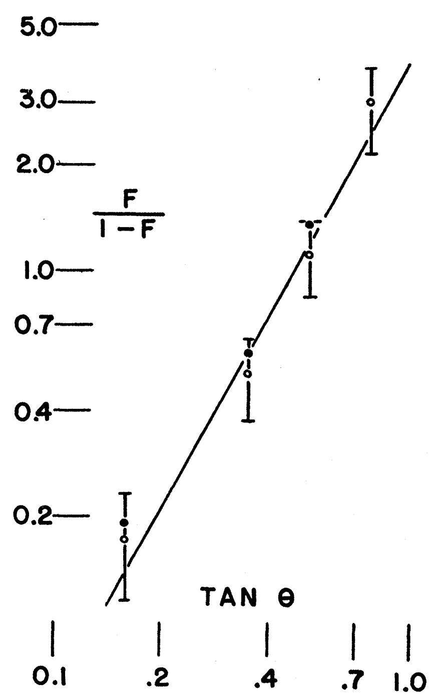

FIG. 6. F plot for showers originating in carbon whose median angles lay between 21° and 30°. $\bar{E}=7$ Bev. The solid dots are uncorrected. (Group III).

[5] S. J. Lindenbaum and L. C. L. Yuan reported these results in a post-deadline paper at the May, 1953, meeting of the American Physical Society.

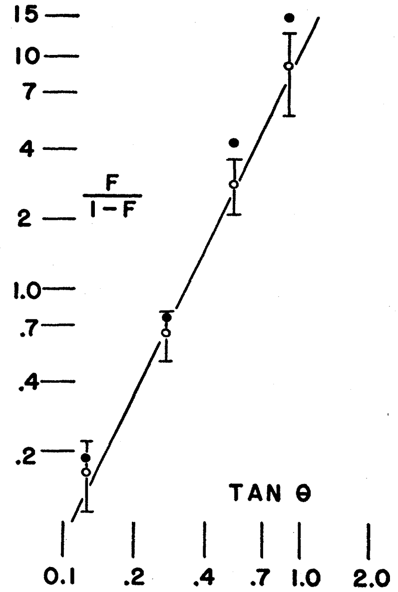

FIG. 7. F plot for showers originating in lead whose median angles were less than 25°. The solid dots are the uncorrected data points. (Group I).

The average apparent energies were determined from the $F=\frac{1}{2}$ point on the F plot for each of the groups. The average energy of the showers occurring in lead appears to be 18 Bev and in carbon 27 Bev. The normalized rate[6] of shower occurrence in lead was about 10 percent higher than the rate in carbon.[1] This means that the average energy of the showers occurring in lead and carbon should not differ by much more than 10 percent.[7] The apparent energy difference is of the order of 30 percent. There is, as one would expect, a lowering of the apparent primary energy because of scattering and multiplicative processes going on inside the struck nucleus.

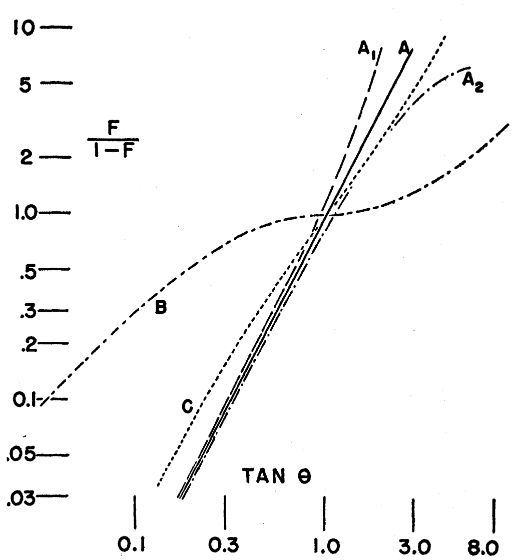

FIG. 8. F plots for various center-of-mass distributions. Curve A: $\beta_c=\beta_\pi$ and distribution isotropic. Curve B: $\beta_c=\beta_\pi$ and $N(\varphi)d\varphi=\cos^2\varphi d\varphi$. Curve C: $\beta_c=\beta_\pi$ and $N(\varphi)d\varphi=\sin^2\varphi d\varphi$. Curve A_1: $\beta_c=1.05\beta_\pi$ and distribution is isotropic. Curve A_2: $\beta_c=0.95\beta_\pi$ and distribution isotropic.

[6] Normalized rate = (observed rate of showers)$/(1-e^{-x/\lambda})$, where λ = m.f.p. for shower production and x is the thickness of generating material.

An estimate of the primary energy was made by using the proton spectrum (at the top of the atmosphere) published by Barrett *et al.*,[8] a proton mean free path in air of 120 g/cm^2, and the rate of occurrence of these showers.[1] The cut-off energy for our detector, assuming it to be sharp, is calculated to be 35 Bev. Undoubtedly, the average energy of the primaries producing the carbon showers is higher than the 27 Bev estimated from the F plots. It is interesting to note that our average energy agrees rather well with the average energy determined by Walsh and Piccioni[9] from the geomagnetic effect for showers produced in a similar shower detector.

IV. PROPERTIES OF PENETRATING SHOWERS

A. Angular Distributions of the Secondary Particles

The F plots given in Figs. 4 through 7 support the belief that the angular distribution of the secondary particles in the center-of-mass is isotropic. It is inter-

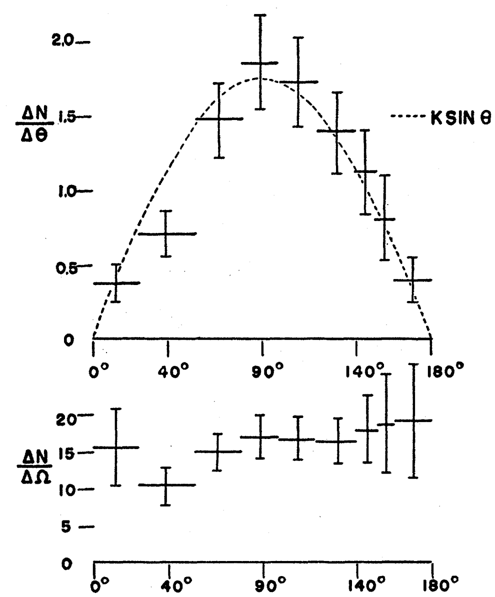

FIG. 9. Differential angular distribution of the secondary particles in the center-of-mass system for the group II carbon showers.

[7] Briefly, this argument is as follows: $\bar{E}\propto E_c$, where $\bar{E}$ is the average energy, and E_c is the lowest energy of primaries producing showers detected by the apparatus. If the differential primary spectrum is dE/E^s, with $s\cong 2$, then the shower rate is roughly proportional to $1/\bar{E}$.

[8] Barrett, Bollinger, Cocconi, Eisenberg, and Greisen, Revs. Modern Phys. **24**, 133 (1952).

[9] J. G. Walsh and O. Piccioni, Phys. Rev. **80**, 619 (1950).

esting to see how great is the deviation from formula (3) when angular distribution is not isotropic or when $\beta_c \neq \beta_\pi$. Figure 8 shows F plots of four such cases. The effect of averaging over a range of shower energies was investigated numerically, and it was found that deviations from isotropy were still detectable. Certainly a $\cos^2\varphi$ angular distribution is ruled out by the present data as is any distribution peaked more strongly in the forward or backward directions in the center-of-mass system. In Figs. 9, 10, 11, and 12 plots are given of $dN/d\theta$ and $dN/d\Omega$ for the group II, III, and IV carbon showers and group I Pb showers. The inverse Lorentz transformation was carried out using the γ_c's determined from the F plots. The distribution of the particles in the center-of-mass system is isotropic within statistics for showers up to primary energies of the order of 40–50 Bev.

2. Number of π^0 Mesons

Several investigators[10–14] have estimated the relative numbers of π^0 and $\pi^\pm$ mesons in penetrating showers. Actually in an experiment of this type the ratio of high-energy γ rays to penetrating particles is measured. Data are presented to show that the γ rays usually occur in pairs which are attributed to decays of the neutral π mesons. It is more feasible to measure the ratio of charged shower particles to π^0 mesons in showers originating in light elements than in showers in heavy elements such as lead. In the case of this experiment, γ rays originating from π^0's originating in showers in carbon travelled on the average 4 or 5 inches before materializing in lead. This allowed a greater geometrical resolution between the electron cores and the penetrating secondaries than would be possible, if the showers originated in a lead plate or traveled only one

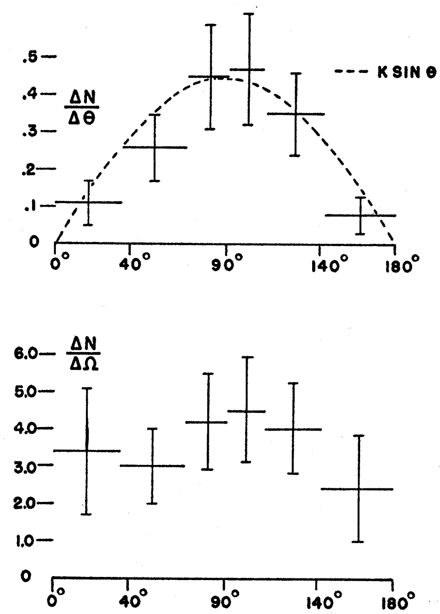

FIG. 11. Differential angular distribution for the group IV secondaries.

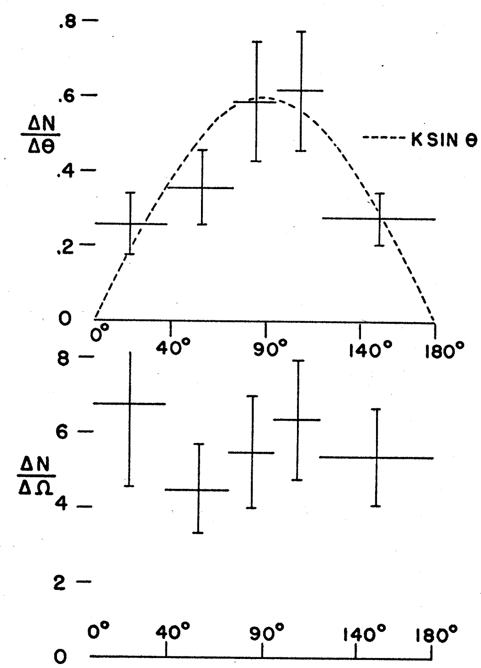

FIG. 10. Differential angular distribution for the group III secondaries.

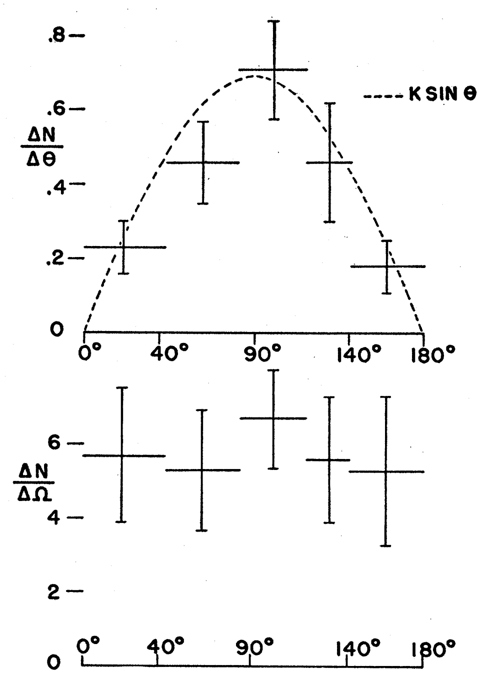

FIG. 12. Differential angular distribution in the center-of-mass system for the secondaries from the group I lead showers.

[10] Carlson, Hooper, and King, Phil. Mag. **41**, 701 (1950).
[11] B. P. Gregory and J. H. Tinlot, Phys. Rev. **81**, 675 (1951).
[12] G. Salvini and Y. B. Kim, Phys. Rev. **88**, 40 (1952).
[13] Froehlich, Harth, and Sitte, Phys. Rev. **87**, 504 (1952).
[14] Chang, Del Castillo, and Grodzins, Phys. Rev. **89**, 408 (1953).

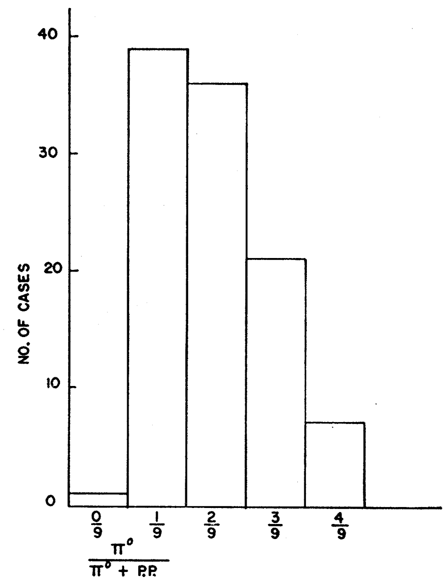

FIG. 13. Histogram of the number of cases with a given ratio of π^0's to charged particles *versus* $N_{\pi^0}/(N_{\pi^0}+\text{No. of p.p.})$. The average value of the latter fraction is 0.25. The total number of shower particles per shower is 9.

or two inches before striking a converter. It should be possible to resolve the different cores. Even for showers of the lowest energy group the average energy of the mesons is 800–900 Mev. Thus, the average γ rays would have 400-Mev energy. This means, according to Wilson's[15] calculations, that there should be 3 to 4 electrons in the core of the shower after three radiation lengths of lead. This shower should be easily detectable. As a check, all the cases in which there were one or two electron cores present were examined. These cases were presumably due to the decay of one π^0 meson. In all, there were 84 cases of one apparent core and 197 cases with two cores present. Thus, to the first approximation there were 1.7 electron cores found per π^0 decay. In Fig. 13 is plotted a histogram of the numbers of cases that were found with different proportions of π^0 and penetrating particles (p.p.). It can be seen from the diagrams that the most probable proportion is close to 0.25. This, it turns out, is also the average ratio. These diagrams were obtained by counting the number of showers with, for example, 9 penetrating particles and

TABLE I. Relative numbers of neutral mesons and charged secondaries.

Total No. particles (π^0+p.p.)	$\frac{\text{No. } \pi^0}{\text{No. } \pi^0+\text{No. p.p.}^{a}}$
5	0.204
6	0.26
7	0.232
8,9,10	0.25

[a] p.p. = penetrating particles.

$0\pi^0$, 8 penetrating particles and $1\pi^0$, 7 penetrating particles and $2\pi^0$, etc. Showers with one or two electron cores were said to have one π^0, with three or four cores, two π^0's etc. The results of this procedure are given in Table I.

Table I does not go to lower total numbers of particles because the selection system biases the results. It would be impossible for a shower composed of three π^0's to be recorded, whereas it is fairly likely that a shower of three charged mesons is recorded. Even the case with the total number of particles equal to five is probably biased slightly. In going from a shower of 5 to 10 particles the ratio No. π^0/(No. π^0+No. p.p.) does not change appreciably. The ratio of π^0's to penetrating particles is 0.33. Assuming a ratio of No. π^0/No. $\pi^{\pm}$ of $\frac{1}{2}$, the ratio of pions to protons or other particles is about 3 to 1. Thus, it seems likely that the large showers of 7 or 8 particles are the result of several interactions in the same nucleus. This agrees with the conclusion obtained from comparisons of multiplicity in light and heavy elements.[1,16]

V. MOMENTA OF THE SECONDARY PARTICLES

The momenta of the secondary particles have been estimated from their multiple scattering in the 17 g/cm² lead plates in the cloud chamber. The secondary particles were subdivided according to the median angle of the shower in which they were formed and further subdivided according to whether they occurred inside or outside the median angle for the shower. The mean square scattering angle was obtained from the distribution of scattering angles within a given group.

The scattering-angle distribution can be thought of as two distributions superposed: one the multiple-scattering distribution, which is Gaussian; the other a wide-angle tail caused by (1) single or plural Coulomb scattering and (2) nuclear diffraction and interaction. Also a few low-energy particles may be in a group and broaden the distribution. Since it is the mean square angle for the group that is needed, the procedure used was to find the rms angle which best fitted the observed data out to about three times the rms angle. The observed distribution of scattering angles for the different groups are given in Table II.

Corrections have been made for the "spurious scattering" due to distortions in the gas of the cloud chamber. This was done by measuring the scattering suffered by the particles in traversing the carbon plates in the chamber. The lead plates were about three radiation lengths thick, whereas the carbon plates were only 0.05 radiation lengths, so the scattering should be about eight times as great in lead as in carbon. The scattering in carbon was measured for the three highest energy groups of secondaries. The mean square scattering angle was calculated by the procedure given above and this was assumed to be all due to spurious scatter-

[15] R. R. Wilson, Phys. Rev. **84**, 100 (1951).

[16] McCusker, Porter, and Wilson, Phys. Rev. **91**, 384 (1953).

ing. Fortunately, the spurious scattering is much less than the observed scatterings in lead and consequently the corrections of the observed rms scatterings were small.

The average of $1/P\beta$ was calculated using the Williams formula as given by Rossi and Greisen.[17] It is unfortunately not possible to obtain the average of $P\beta$ from these measurements without knowledge of the momentum spectrum of the particles in the different groups. The results of the measurements are given in Table III. In order to calculate from the $P\beta$ measurements the energy of the pions, the following approximations were made: (1) assume that pions, protons, and other particles have the same momentum spectrum; (2) assume that each of the particle groups is homogeneous in momentum.

The seventh column of Table III gives the estimated aggregate energy per shower of the neutral and charged mesons for each of the shower groups. The last column gives the percent of the available energy used in meson production.

The average kinetic energy of the mesons in the center-of-mass system seems to be about 500 Mev. The Group IV mesons appear to have a higher energy in the center-of-mass system. Indeed, the aggregate energy of the secondaries is higher than the estimated primary energy. This is because our energy estimate is particularly low for this group of showers either because of nuclear scattering or cascading inside of the struck nucleus. Group IV is the smallest of the carbon groups. The collision elasticity in the large groups (Group II of carbon and Group I of lead) is not conclusively demonstrated by the calculations because the primary and secondary energy estimates are tenuous. However, there is other evidence for a lack of complete inelasticity. First, the F plots are very nearly straight lines for Groups II in carbon and I in lead. (These are numerically the largest groups.) This would indicate that $\beta_c \approx \beta_\pi$. Under this assumption one calculates an inelasticity of about 50 percent for the Group II carbon showers. If the collisions were completely inelastic, for the primaries of 26 Bev the fraction of secondaries detected in the backward hemisphere in the laboratory system should be of the order of 4/500. The number observed is of the order of 1/400. This also is indicative that the β of the π's in the center-of-mass system is not larger than β_c.

Some particles in the backward direction are undoubtedly due to scattering inside of the parent nucleus.

TABLE II. Multiple-scattering data of the particles in the median-angle groups. Momentum sub-groups are indicated by A and B: particles in sub-group A are at zenith angles less than the median angles of their respective showers; particles in B are at zenith angles greater than the median angles.

Group	Sub-group	$0°–\frac{1}{2}°$	$\frac{1}{2}°–1°$	$1°–1\frac{1}{2}°$	$1\frac{1}{2}°–2°$	$2°–2\frac{1}{2}°$	$2\frac{1}{2}°–3°$	$3°–3\frac{1}{2}°$
C I	A	12	2	1				
	$A+B$	19	4	3	2	1		
		No large-angle scatterings observed						
C II	A	53	9	5	2	1		
	B	22	13	8	6	4	2	
		Total of 4 large-angle scatterings in A, 6 in B.						
C III	A	15	4	6	3	1		
	B	10	5	5	6	2	2	
		Total of 6 large-angle scatterings in $A+B$						
C IV	$A+B$	9	6	5	2	2	1	1
		Total of 3 large-angle scatterings in $A+B$						
Pb I	$A+B$	20	5	3	2	3		
Pb II	$A+B$	12	6	5	3	1		
		Total of 6 large-angle scatterings for both lead groups						

VI. SECONDARY INTERACTIONS

The interactions of the secondary particles of penetrating showers have been the subject of much discus-

Table III. Results of momentum measurements.

Range of median angles	γ_c	Mean energy of primary particles in Bev	Group	Reciprocal of average $1/P_c\beta$ in Bev/c	Mean multiplicity of π mesons	Energy of π mesons in Bev	% E_{Av}
			Carbon showers				
0°–10°	9.6	170	<M.A.	4	3.3	>10	>7%
			Total	2.8			
11°–20°	3.9	26	<M.A.	4	4.2	11.7	55%
			>M.A.	1.6			
21°–30°	2.2	7	<M.A.	1.65	5.2	7.5	150%
			>M.A.	1.26			
>30°	1.3	2.3	Mean	1.44	4.7	6.8	...
			Lead showers				
0°–25°	3.3	18	Mean	2.3	4.3	10	70%
>25°	1.4	2.75	Mean	0.85	8.3	7	...

[17] B. Rossi and K. Greisen, Revs. Modern Phys. 13, 240 (1941).

TABLE IV. Mean free path measurement.

$\bar{E}_\pi$	λ_{Pb}(g/cm²)	λ_c(g/cm²)
I ⩾4 Bev	180^{+85}_{-45}	85^{+65}_{-25}
II 2 Bev	660^{+500}_{-200}	400^{+1000}_{-180}
III∼1 Bev	1700^{+1200}_{-500}	1500^{+2000}_{-500}

sion in the literature.[11,18–28] The experimenters using nuclear emulsions[23] seem to find that the secondary particles have a geometric cross section for interaction. Investigators using cloud chambers and counters usually find cross sections somewhat less than geometric. Cloud chambers are not very suitable for determining the total cross section for interaction since star prongs produced by the secondary particles are often missed because of the thickness of the plates. Most of the effort in this experiment was concentrated on measuring the mean free path of the secondary particles for producing further penetrating showers, that is, further meson production. Some data were obtained on star

TABLE V. Mean free path of secondary particles from showers of different size and origin.

Origin	Mult.	No. traversals	Material traversed	Number interactions	Mean free path (g/cm²)
Carbon and paraffin	2–4	1707	Carbon	12	490^{+200}_{-100} carbon
	2–4	779	Lead	21	660^{+180}_{-120} lead
	⩾5	1313	Carbon	3	1500^{+2000}_{-500} carbon
	⩾5	589	Lead	6	1700^{+1200}_{-500} lead
Iron and brass	2–4	1939	Carbon	12	560^{+230}_{-130} carbon
	2–4	702	Lead	15	830^{+290}_{-170} lead
	⩾5	2508	Carbon	6	1450^{+1000}_{-430} carbon
	⩾5	922	Lead	13	1260^{+480}_{-270} lead
		Carbon plate thickness =3.46 g/cm²			Lead plate thickness =17.7 g/cm²

[18] W. B. Fretter, Phys. Rev. **76**, 511 (1949).
[19] J. Harding and D. Perkins, Nature **164**, 285 (1949).
[20] Lovati, Mura, Salvini, and Tagliaferri, Phys. Rev. **77**, 284 (1950).
[21] O. Piccioni, Phys. Rev. **77**, 1 (1950).
[22] W. W. Brown and A. S. McKay, Phys. Rev. **77**, 342 (1950).
[23] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[24] Butler, Rosser, and Barker, Proc. Phys. Soc. (London) **A63**, 145 (1950); **A64**, 4 (1951).
[25] A. J. Hartzler, Phys. Rev. **82**, 359 (1951).
[26] Lovati, Mura, Succi, and Tagliaferri, Nuovo cimento **8**, 271 (1951).
[27] G. M. Branch and G. Cocconi, Phys. Rev. **84**, 146 (1951).
[28] R. Cool and O. Piccioni, Phys. Rev. **87**, 531 (1952).

production and scattering of the secondary particles. The mean free path of the charged secondary particles for the production of two or more penetrating particles has been measured. In all about 100 secondary penetrating showers were found. They were of lower average energy than the primary interactions but otherwise indistinguishable. Of the particles that produce secondary penetrating showers it was found that 87 are charged and 18 neutral. If it is assumed that the neutral ones are neutrons and that on the average there is one fast proton for each energetic neutron produced, then of the charged induced secondary interactions only about 20 percent are due to protons, assuming further, of course, that neutrons, protons, and π mesons have equal probabilities for shower production. This gives a somewhat higher multiplicity of meson production per fundamental act than deduced previously. However, it is found that most of the secondary particles that produce further mesons are produced in primary events of energies of 50 Bev or more, which perhaps accounts for the higher multiplicity.

VII. MEAN FREE PATH *vs* ENERGY OF SECONDARIES

Having subdivided the showers according to their median angles it is possible to get some information about how the π meson cross section for the production of secondary penetrating showers varies with energy. The secondary particles of the carbon showers whose angles of emission have been measured were classified as to whether they occur outside or inside of the median angle of their respective showers. From the scattering measurements described in Sec. V, estimates of the energies of the particles of the different groups have been made. The particles which are inside the median angles from the Group I and II carbon showers have an energy ⩾4 Bev. The rest of the particles in Groups I, II, III, and IV have a lower average energy of about 2 Bev and are classified together in this m.f.p. investigation. The results of the measurements are given in Table IV. The third row is explained below. There were no large-angle scatterings or stars observed among the group of particles of energies more than 4 Bev. Particles in the lower energy groups were observed to scatter and produce stars. In fact, the total mean free path for the production of stars, scatterings, and penetrating showers for the Group II particles was found to be 167_{-20}^{+40} g/cm² of lead. This agrees with the emulsion data. It would appear, then, that the cross section for charged meson production by mesons does not become large until energies greater than 4 Bev are reached.

Only a fraction of the data obtained were utilized in the tabulation, since it was possible to make angular measurements only on those showers originating inside of the chamber in which track confusion was small.

In Table V a different breakdown is given with subdivisions made as to the shower size in which the secondary originated and the type of nucleus in which the secondary originated.

The mean free paths obtained for the secondaries seem to be more dependent on shower size than on the material in which the shower was generated. The secondaries from the high-multiplicity showers have a very long mean free path. This probably indicates that the particles capable of making further mesons have been filtered out in the course of producing a high multiplicity. The Group II showers from lead have a multiplicity comparable with that of the group of large showers ($\geqslant 5$ group of Table V). The secondaries of the former have an average energy a little below 1 Bev. It is thought that the energy of the latter group must be about the same. The third row of Table IV expresses quantitatively this correlation of low energy and long mean free path of the high-multiplicity shower particles from light elements. Thus, it may be seen that the cross section for further meson production by mesons is about 0.1 the geometric cross section of lead at 1 Bev and rises to nearly geometric cross section at meson energies of 4 Bev and above.

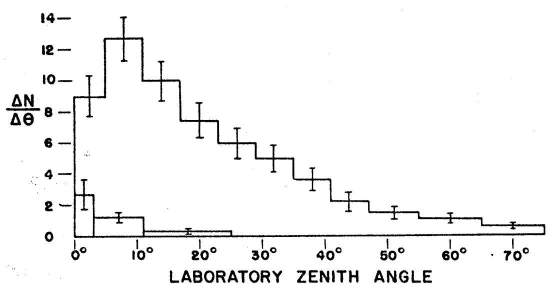

FIG. 14. A comparison of the angular distribution of secondary particles that produce further mesons with the angular distribution of all the penetrating secondary particles.

VIII. ANGULAR DISTRIBUTION OF SECONDARY PARTICLES PRODUCING SHOWERS

Figure 14 compares the angular distribution of the particles which produce further penetrating showers with the distribution of all the shower particles. It can be seen that the angular distribution of the shower-producing secondaries is much narrower than that of the ordinary secondaries. In Fig. 15 a plot is shown of the angular distribution of the shower-producing secondaries measured in units of the median angle of the shower in which the secondary particle was produced. Almost all of the particles producing secondary showers come from inside the median angles of their respective showers. From these data it is calculated that the average energy of the shower that produces a secondary capable of producing further mesons is close to 50 Bev. Using this fact and the meson energies in the center-of-mass determined in Sec. V, a mean energy of 4.5 Bev is calculated for the energy of the secondaries producing further interactions.

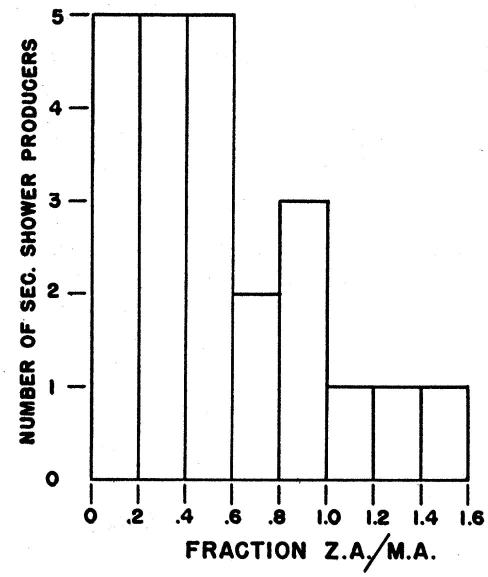

FIG. 15. A histogram of the number of secondary shower producers *vs* the ratio of their zenith angle of the median angle of the shower in which they occur.

IX. ANGULAR DISTRIBUTION OF THE PARTICLES PRODUCED IN SECONDARY INTERACTIONS

In Fig. 16 an F plot is made for 13 especially clear secondary penetrating showers. The average energy of the particles producing these showers is determined to be about 8.5 Bev which is in fair agreement with our other estimates. Perhaps the most interesting feature of these secondary showers is that their progeny seem

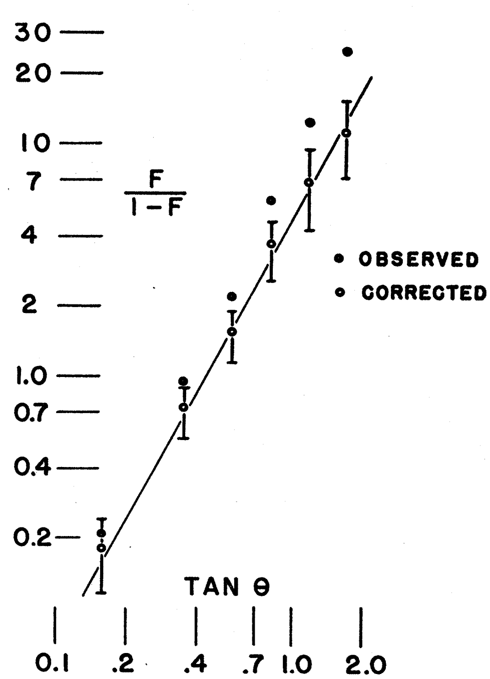

FIG. 16. An F plot for the secondary penetrating showers.

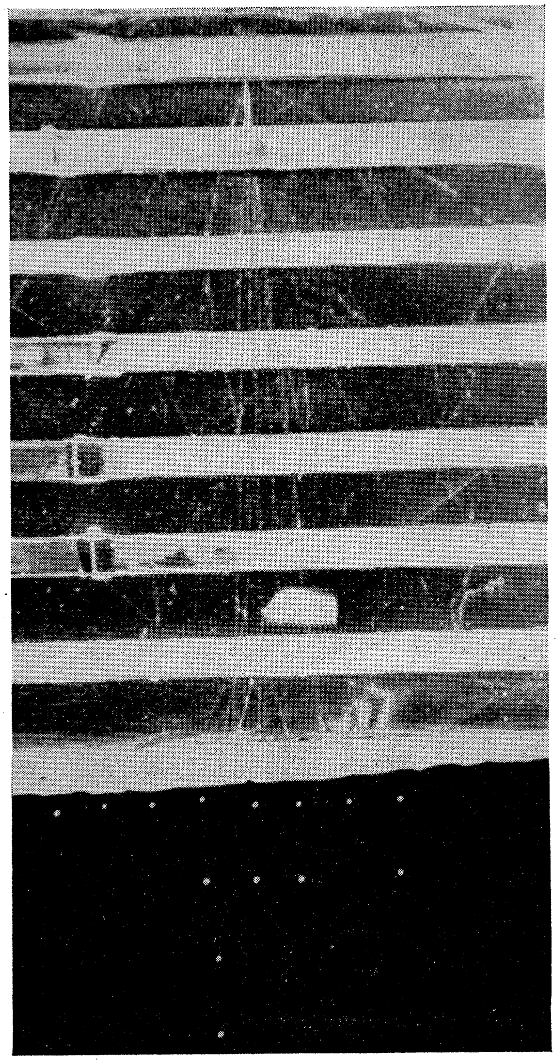

FIG. 17. A very energetic shower originating in carbon. There appears to be a secondary interaction produced in the last lead plate in the chamber. The neon bulbs beneath the chamber indicate that at least some of the particles traverse a foot or more of the lead.

also be be distributed uniformly in the center-of-mass system. Since we believe that at least 60 percent of the particles producing these secondary interactions are π mesons, there seems to be evidence that mesons produced in meson-nucleon collisions are distributed uniformly in the center-of-mass. Figure 17 gives an example of a shower recorded in this experiment.

X. HEAVY MESONS

In this experiment two unstable particles, one $V^{\pm}$ and one S particle[29] were observed to decay in the chamber. Figure 18 shows one view of the $V^{\pm}$ observed. Of showers that originated in carbon inside the chamber, and whose primary energy was 20 Bev or greater, an aggregate secondary track length of about 40 meters was scanned. The total track length was the result of 160 secondaries traversing an average distance of 25 cm in the gas between the plates. Only one decay in flight was observed. The net laboratory time in which decays could have occurred was 13×10^{-8} sec. Since the average dilation for the mesons is about 15, about one $\pi-\mu$ decay would be expected. None was observed.

The relative number of K particles among the secondaries of penetrating showers has been the subject of some speculation.[30] We can set an upper limit on the fraction of K's using the facts given above in conjunction with some information obtained by Bridge *et al.*[29] First we assume that the K's have about the same momentum in the laboratory system as the π mesons. This would give them a dilation of about four. The argument is as follows: if the lifetime of the K is 10^{-9} sec or less, then from the number observed to decay in flight in this experiment, the fraction of the secondaries which they constitute is calculated to be not greater than 0.1.

If, however, the lifetime is 10^{-8} sec or more, we would expect a considerable fraction of the K's to stop and decay in dense material. Bridge *et al.* saw 6 S's and 260 π's, all secondaries from penetrating showers, stop in their chamber. Since their cloud chamber was triggered by a counter arrangement similar to this one, the shower energies should be comparable. From these facts we set 0.05 as the upper limit of the fraction of K's among the secondaries.

If the lifetime is between 10^{-8} and 10^{-9} sec, then comparable fractions of the particles should stop and decay in flight. In this case, the numbers from this experiment and the Bridge experiment should be added, giving a maximum fraction of 0.15.

Thus, we infer that, regardless of the lifetime, the fraction of K's among the secondaries cannot be more than 15 percent for showers of about 27 Bev.

As was pointed out previously,[31] the number of V^0's per interaction found in this investigation was much smaller than the number found by Fretter *et al.*[32] The rate of production of V^0's in carbon showers seems to be a factor of four or five less than the rate observed by Fretter, even when corrected for the latter's higher search efficiency.

XI. DISCUSSION

One of the most interesting results of the experiment is that the observed angular distribution of the secondary particles is consistent with a uniform distribution in the center-of-mass system. The relative momenta of the colliding nucleons for the average shower is of the order of 7 Bev/c. This means that quite often nucleon-

[29] Bridge, Peyrou, Rossi, and Safford, Phys. Rev. **90**, 921 (1953).

[30] Report of D. H. Perkins in *Proceedings of Third Annual Rochester Conference* (Interscience Publishers, Inc., New York, 1953), pp. 43–46.

[31] W. D. Walker and N. M. Duller, Phys. Rev. **90**, 320 (1953).

[32] Fretter, May, and Nakada, Phys. Rev. **89**, 168 (1953).

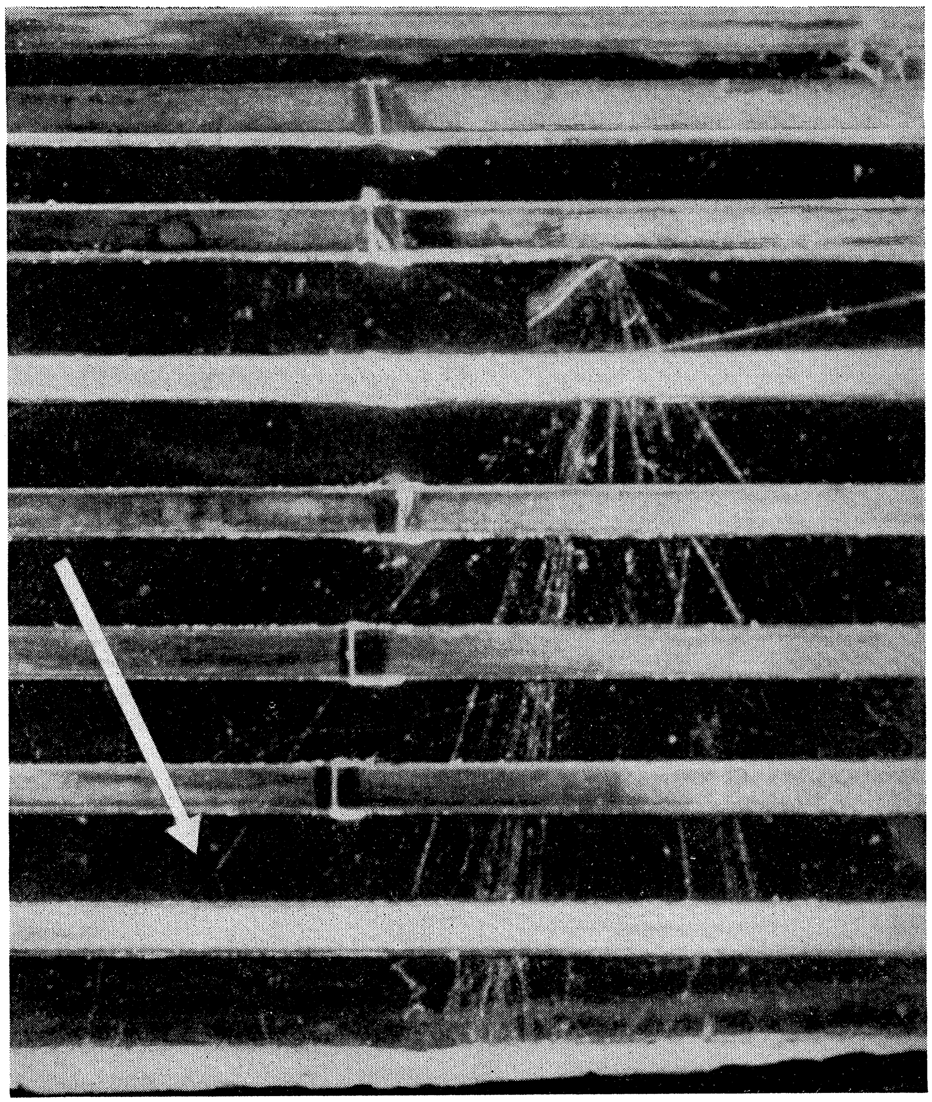

FIG. 18. The decay of a $V^{\pm}$ particle. The V appears in a rather energetic shower from carbon. The decay product scatters through about 15° in traversing the lower lead plate.

nucleon interactions will occur with $20\hbar$ or more angular momentum, which might lead one to expect center-of-mass distributions somewhat peaked in the forward and backward directions. It is possible that the momenta of the secondary particles depend strongly on the angle of emission in the center-of-mass system, even though the data of the present experiment give no indication of such dependence.

The result concerning the apparent lack of complete inelasticity is tenuous because of the method of momentum measurement. However, there are indications from other experiments that such an effect exists: for example, the absorption mean free path in air,[33] which is about twice as long as the interaction mean free path, and the neutral to charged ratio for the primaries,[1,34–36] which produce penetrating showers at mountain altitudes. Both of these phenomena can be explained by assuming a non catastrophic-type collision.

The interactions of the secondaries give a clue as to the processes which go on in the nuclear cascade. The meson-nucleon interactions certainly are very important in producing high-multiplicity showers. This is indicated by the fact that there are relatively more interacting particles emerging from the small than from the large showers. Probably the interacting particles have largely been absorbed in interactions which produced the high multiplicity.

In view of the abundant evidence for cascade processes,[1] it is at first sight rather difficult to understand how the analysis in terms of nucleon-nucleon collisions is at all meaningful. The angular distribution of the particles which produce secondary-penetrating showers show that most of these particles travel in almost the

[33] J. Tinlot, Phys. Rev. **73**, 1476 (1948).
[34] K. Greisen and W. D. Walker, Phys. Rev. **90**, 915 (1953).
[35] W. D. Walker, Phys. Rev. **77**, 686 (1950).
[36] Walker, Walker, and Greisen, Phys. Rev. **80**, 546 (1950).

direction of the primary particle. If the nucleon-nucleon collisions are not completely inelastic, probably the degraded primary would be expected to continue on a path close to its original direction. In a shower of primary energy of the order of 50 Bev, probably after the initial interaction, a sizeable fraction of the original energy comes out in the degraded primary and perhaps one other secondary in a direction close to the primary's original path. Perhaps these particles produce coherent collisions[37–40] in which both of the particles interact with a nucleon farther on in the nucleus. The resultant angular distribution is thus a superposition of more than one distribution. As stated before this means, of course, that our kinematic estimates of the primary energy are usually low.

The results concerning the angular distributions and elasticity are difficult to understand in terms of Fermi's model of meson production. A slight modification of the model could, perhaps, bring better agreement between theory and experiment. Fermi's model supposes that all the energy of the two colliding nucleons is concentrated in a volume containing a hot mesic gas. The mesons emerge from this volume. However, if one supposes that meson production is a more localized phenomenon and occurs essentially only in the region of overlap of the fields of the two nucleons, the isotropic distribution follows more naturally. The lack of complete inelasticity fits better into this picture since not all of the nucleon takes part in the interaction. Granting a uniform distribution of mesons, it is difficult to see how angular momentum is conserved. The colliding nucleons, according to such a model, would carry off most of their initial angular momentum. Perhaps it is necessary to include in the model particles of high intrinsic angular momentum.

ACKNOWLEDGMENTS

The authors wish to thank Mr. John D. Sorrels for his invaluable aid at the beginning of this work. The authors also wish to thank Mr. J. F. Van der Henst and his staff for constructing the cloud chamber and some of the associated apparatus. The continued support and interest of Professor T. W. Bonner was very important to the success of the investigation. The authors wish to acknowledge discussions with Professor K. Sitte and Professor J. R. Risser and Dr. D. M. Ritson and Dr. C. M. Class. Finally the authors wish to express their gratitude to the Research Corporation for grants which made this work possible.

APPENDIX I. ANGULAR DISTRIBUTION CORRECTIONS DUE TO CHAMBER GEOMETRY

Penetrating secondary particles at large zenith angles were recorded with less efficiency than those near the shower axis, since the former were less likely to traverse a lead plate. Approximate estimates of the effect of chamber geometry on the observation of particles at large zenith angles were made by means of a scale drawing of the cloud chamber and target diagrams representing the observed average angular distributions. The target diagrams were drawn by use of the relation $r=h\tan\theta$ in which r is the radius of the circle corresponding to zenith angle θ and h is the average distance of shower production from the absorbing lead plates. If the concentric rings of the target diagram are divided into a convenient number of sections by radial lines, one can estimate the fraction of particles missed in various zenith-angle intervals by moving the center of the diagram about the axis of the illuminated chamber volume and noting what fraction of each interval falls outside the chamber. The correction for chamber geometry is noticeably greater for lead showers than for those from carbon because the only producing layer of lead is about 10 inches from the absorbing layer, whereas the average distance of producing layers of carbon from lead is less than 5 inches.

APPENDIX II. METHOD OF MEASURING THE TOTAL ZENITH ANGLE FOR EACH SHOWER SECONDARY

The total zenith angles of all penetrating secondary particles were measured by means of a target diagram which consists of a large number of concentric circles of radii given by $R=H\tan\theta_v$. In this relation, H is the arbitrarily selected height of all shower origins above the center-of-the-target diagram and θ_v is the angle with respect to the vertical which corresponds to radius R. An oblique set of coordinates (two families of parallel lines) is drawn on the diagram in the two directions corresponding approximately to the lines of sight of the two lenses into the cloud chamber. Each line of a given family then corresponds to a value of the projected angle with respect to the vertical as viewed from the position of the lens for which that family of lines was drawn. In this experiment the angle between the two sets of lines was about 11°.

The procedure used in the actual zenith-angle measurements is as follows. First, the projected angles with respect to the vertical are measured from the two stereoscopic views for the primary particle and each of the secondaries of a shower. This measurement determines the two oblique coordinates of the point of intersection of each particle track with the plane of the target diagram. Two fine black threads, tied at the top of a rigid brass rod which extends vertically to the height H above the center of the diagram, are then stretched taut to the points of intersection of the primary particle and, in turn, each of the secondary particles. In this manner the entire shower is reconstructed, and the measurement of the angles between the threads by means of a large protractor gives the total zenith angles by a purely mechanical technique. The error inherent in the method is about 2°, though poor experimental conditions can cause larger errors to accumulate.

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38 M. F. Kaplon and D. M. Ritson, Phys. Rev. **88**, 386 (1952).
39 Kaplon, Ritson, and Walker, Phys. Rev. **90**, 716 (1953).
40 F. C. Roesler and C. B. A. McCusker, Nuovo cimento **10**, 127 (1953).

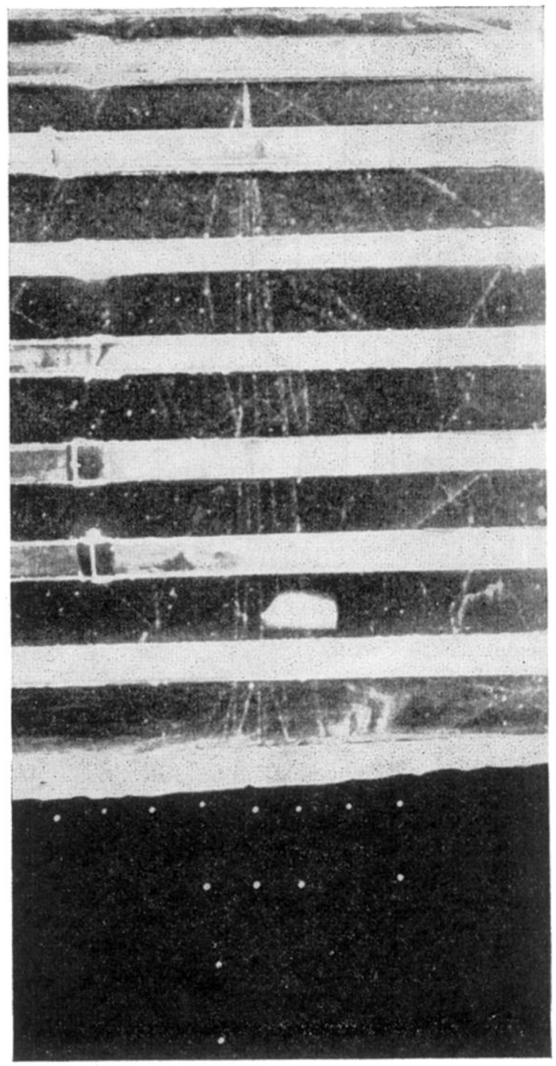

FIG. 17. A very energetic shower originating in carbon. There appears to be a secondary interaction produced in the last lead plate in the chamber. The neon bulbs beneath the chamber indicate that at least some of the particles traverse a foot or more of the lead.

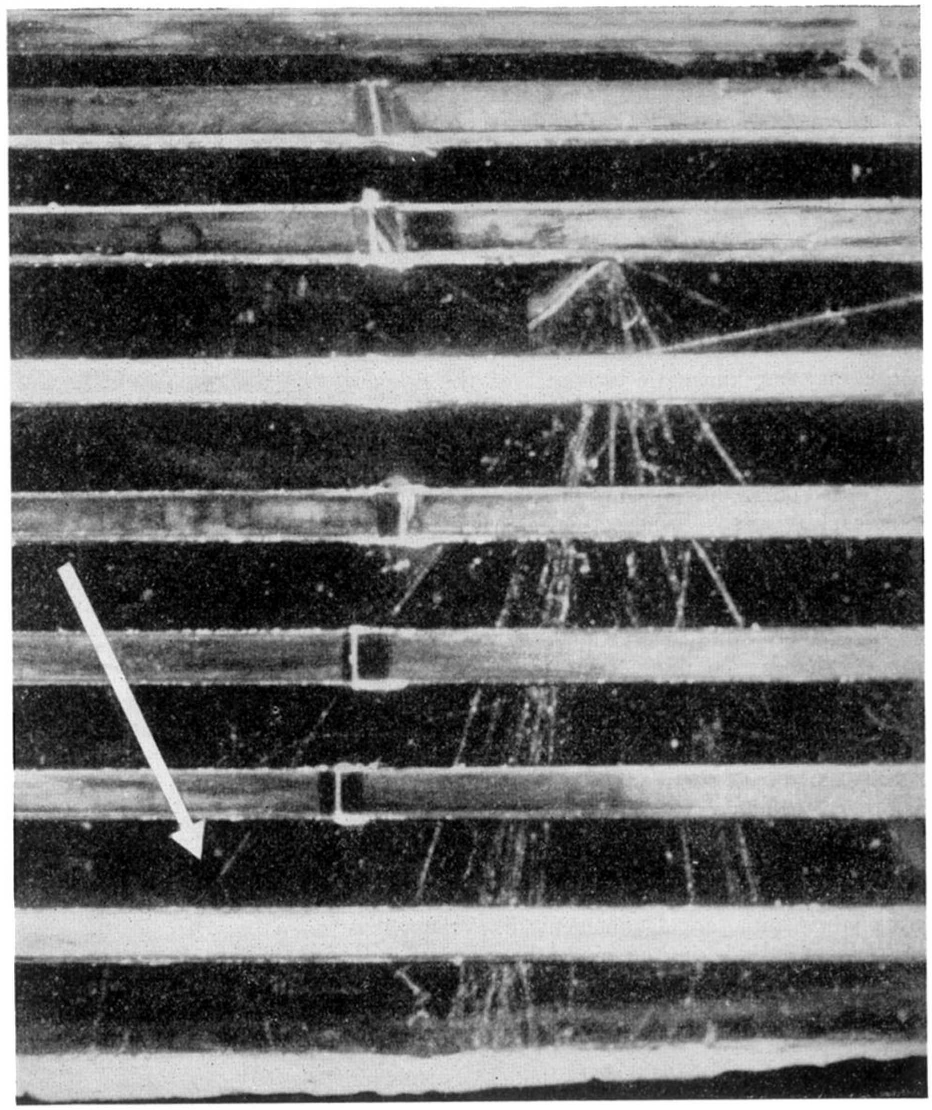

FIG. 18. The decay of a $V^{\pm}$ particle. The V appears in a rather energetic shower from carbon. The decay product scatters through about 15° in traversing the lower lead plate.