

Second Sound in He³-He⁴ Mixtures below 1°K*†

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An investigation has been made of second sound in mixtures of He³-He⁴ in the temperature region obtained by demagnetization of a paramagnetic salt. Several concentrations of He³ were used ranging between 0.0017 percent and 4.3 percent. The velocity of second sound for the mixtures was found to reach a maximum value of about 35 meters/sec just below 1°K and to approach 20 meters/sec near 0.2°K. In addition, essentially no dispersion of second-sound pulses in mixture below 1°K was observed. This is in marked contrast to heat pulses in pure He⁴ in this temperature region. Our results show general agreement with the theoretical predictions of Pomeranchuk, provided the effective mass μ of the He³ atom is allowed to vary from 2.0 m_3 at 0.2°K to 3.6 m_3 at 1.8°K.

INTRODUCTION

SECOND sound is the name given to a type of wave motion, as yet found only in liquid helium II, in which a temperature variation, propagated through the liquid, obeys a wave equation. Experimental studies of the velocity, absorption, dispersion, and shock effects of second sound in pure He⁴ made by a number of investigators,¹⁻⁶ have been quite valuable in checking and guiding the theories of liquid helium. Likewise of interest has been the effect of impurities, such as He³, added to the He⁴ on the second-sound properties. Pomeranchuk,⁷ in 1949, treated this problem theoretically and showed that the velocity of second sound, u_2 , should be drastically changed by quite small additions of He³ in He⁴. In the higher-temperature region above 1°K, the principal effect of the He³ is to increase the entropy and, thus, the velocity. In 1950 Lynton and Fairbank⁸ reported second-sound velocity measurements in dilute mixtures of He³ in He⁴ from 1.2°K to the lambda point which showed this increase in velocity and substantially supported the predictions of Pomeranchuk. Recently, Weinstock and Pellam,⁹ using a thermal Rayleigh disk, have measured the velocity down to 0.9°K in 4 percent He³ with similar results.

In the low-temperature region below 1°K, small amounts of impurities should affect u_2 even more drastically on Pomeranchuk's theory. He shows that as the temperature is reduced, u_2 for dilute solutions should pass through a maximum value and decrease

toward zero. This is in striking contrast to the behavior in pure He⁴, where the velocity increases several-fold below 0.8°K to $u_1/\sqrt{3}$,¹⁰ where u_1 is the velocity of first sound in liquid helium II at 0°K. On the other hand, Dingle¹¹ has suggested that the energy of interaction between He³ atoms at very low temperatures may be sufficiently large that Pomeranchuk's treatment would no longer be valid and u_2 should approach the same limiting velocity $u_1/\sqrt{3}$ as for He⁴ at 0°K.

We have extended the second-sound measurements into the ultra-low temperature region down to 0.2°K and obtained results in substantial agreement with the predictions of Pomeranchuk, leaving little doubt as to the validity of his basic approach. No evidence of the increase in u_2 suggested by Dingle appeared, although this does not rule out such behavior at still lower temperatures. A preliminary report of our measurements of u_2 below 1°K has already appeared.¹² This paper elaborates on the experimental techniques, gives more complete results, and discusses in more detail the theoretical implications.

II. EXPERIMENTAL METHOD

The low temperatures were obtained by the adiabatic demagnetization of a pressed paramagnetic salt pill, thermally connected to the second-sound cavity as shown in Fig. 1. The salt-pill and second-sound cavity assembly was contained within an outer cylindrical cavity, closed at the top with a helium-tight "O"-ring seal made of gold.¹³ Around this cavity were wound the primary and secondary coils used to measure the magnetic susceptibility and hence the temperature of the

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¹ V. Peshkov, J. Phys. (U.S.S.R.) 8, 381 (1944); 10, 389 (1946).

² Lane, Fairbank, and Fairbank, Phys. Rev. 71, 600 (1947).

³ D. V. Osborne, Proc. Phys. Soc. (London) A64, 114 (1951); Nature 162, 213 (1948).

⁴ J. R. Pellam, Phys. Rev. 75, 1183 (1949).

⁵ K. R. Atkins and D. V. Osborne, Phil. Mag. 41, 1078 (1950).

⁶ R. D. Maurer and M. A. Herlin, Phys. Rev. 76, 948 (1949); 81, 444 (1951).

⁷ I. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 19, 42 (1949).

⁸ E. A. Lynton and H. A. Fairbank, Phys. Rev. 80, 1043 (1950); 79, 735 (1950).

⁹ B. Weinstock and J. R. Pellam, Phys. Rev. 89, 521 (1953).

¹⁰ This is the theoretical value of u_2 at absolute zero for pure He⁴, assuming the phonon mean free path to be small. Recent measurements of deKlerk, Hudson, and Pellam [Phys. Rev. 89, 326 (1953)], Kramers, van den Burg, and Gorter [Phys. Rev. 90, 1117 (1953)], as well as some of our own [Fairbank, King, and Wheeler (to be published)], give velocities that approach the velocity of first sound near 0°K with a large spreading of the received pulses, indicating long phonon mean free paths. This is no longer second-sound transmission in the ordinary sense.

¹¹ R. B. Dingle, Phil. Mag. 42, 1080 (1951).

¹² J. C. King and H. A. Fairbank, Phys. Rev. 90, 989 (1953).

¹³ Wexler, Corak, and Cunningham, Rev. Sci. Instr. 21, 259 (1950).

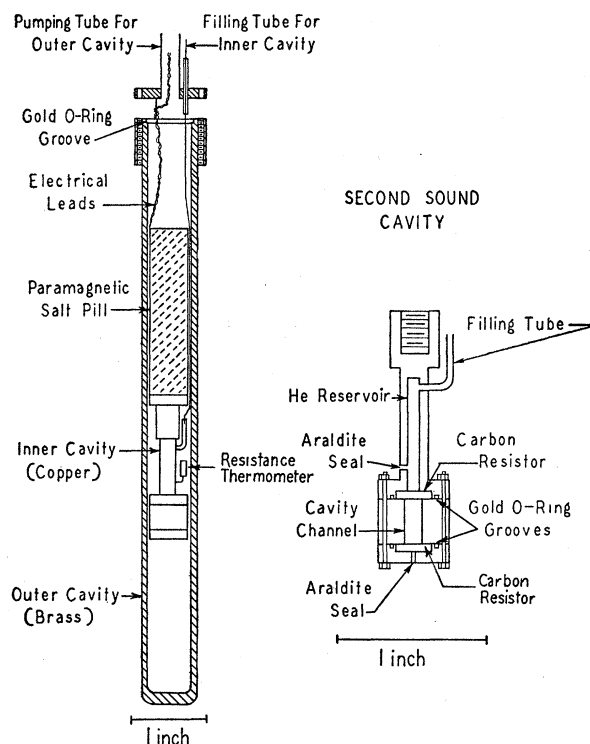


FIG. 1. The ultra-low temperatures assembly. An enlarged drawing of the second-sound cavity is shown on the right.

salt.¹⁴ The pumping tube, soldered to the outer top, was connected outside the helium Dewar flask to a small He⁴ reservoir and a 50-l/sec oil-diffusion pump. A pressure of 10^{-3} mm Hg of helium was allowed in the outer cavity when the magnetic field was turned on, and then, just before demagnetization, the cavity was pumped to a pressure between 10^{-5} and 10^{-6} mm Hg. In this manner, a heat switch between the outside helium bath and the paramagnetic salt was effected. The helium bath, surrounding the outer cavity, was held at a temperature near 1°K by pumping on it with a high-capacity Kinney forepump.

A 10-kilogauss magnetic field was provided by a Weiss-type electromagnet with 6-in. pieces and a gap of 2 in. The magnet was mounted on steel runners and could readily be moved several feet from the paramagnetic salt to effect the demagnetization.

The salt pill, illustrated in Fig. 2, was made of 11 grams of chromium potassium sulfate. Crystals about 1 mm in diameter were pressed into a cylindrical form, about a grid of copper, with a pressure of 2.5×10^3 atmos.¹⁵ The filling factor f for the specimen, given by the density of the pill over the density of the pure crystal, was 0.94. The effective area of the embedded copper was calculated to be 40 cm² and was so dis-

tributed in the pill that no part of the salt was greater than 3 mm from a portion of the copper. This greatly improved the thermal contact between the inner cavity and the salt.

The inner cavity, or the second-sound cavity, shown in detail at the right in Fig. 1, was made of machined copper. The top section, which was threaded to screw onto an extension of the copper embedded in the salt, functioned as a small helium reservoir as well as a mount for a carbon disk-type resistor.¹⁶ A second such resistor was mounted at the bottom of the cavity. One resistor served as a second-sound transmitter and the other as the receiver. The body of the inner cavity, containing the second-sound channel, was sealed between the resistor retainers by gold "O"-rings. Electrical leads were brought to the carbon disks through Araldite¹⁷ seals. The channel was 8.54 mm in length and 2.8 mm in diameter. However, the design of the cavity makes any alteration in the geometry of the channel relatively easy. This type of cavity construction proved to be quite simple, reliable, and flexible. Our earlier designs, in which the electrical leads were brought in through Kovar-glass seals and the cavity was sealed with solder rather than with "O"-rings, were much more troublesome to assemble and considerably more susceptible to misfortunes such as leaks.

The inner cavity was filled via a small-bore, stainless-steel capillary having an inside diameter of 0.15 mm.

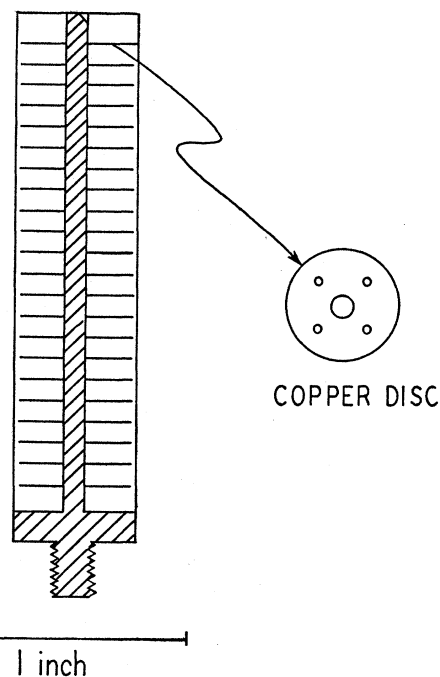


FIG. 2. The pressed paramagnetic salt pill showing the imbedded copper network.

¹⁴ N. Kurti and F. Simon, Proc. Roy. Soc. (London) **149**, 152 (1935).

¹⁵ E. Mendoza, Ceremonies Langevin-Perrin, Paris, 1948 (unpublished), p. 53.

¹⁶ H. A. Fairbank and C. T. Lane, Rev. Sci. Instr. **18**, 525 (1947).

¹⁷ Obtained from CIBA Company, Inc., New York.

The small-diameter tubing was chosen to reduce the heat leak to the cavity brought about by the Rollin¹⁸ film. The calculated heat leak after demagnetization, caused by the creeping film's evaporating at a higher temperature up the filling tube and recondensing in the inner cavity, is of the order of 10^4 erg/min. In view of the extremely poor thermal conductivity of the salt in the ultra-low temperature region,¹⁹ a heat influx to the copper cavity of this amount will naturally lead to an appreciable temperature difference between the copper and the salt.

With these considerations in mind, a secondary thermometer in the form of a $\frac{1}{2}$ -watt 270-ohm I.R.C. resistor was locked thermally to the inner cavity and calibrated *versus* the magnetic or Curie temperature T^* . The warm-up time for the calibration demagnetization, with the cavity void of helium, was over 80 min. This allowed for temperature equilibrium between the salt and the resistor. The resistance calibration was constant during a single run with a number of successive demagnetizations. However, the calibration shifted slightly between runs.

All the electrical leads were thermally locked to the helium bath at the outer cavity top and brought outside the helium flask through the main pumping tube.

The gas handling system, consisting of two Toepler pumps and associated manometry, provided the means to dilute accurately small amounts of the original He³-He⁴ mixture (supplied by the Stable Isotopes Division of the U. S. Atomic Energy Commission) prior to condensing it in the inner cavity.

The original gas sample, as analyzed with a mass spectrograph by the U. S. Atomic Energy Commission, was $4.30 \text{ percent} \pm 0.07 \text{ percent}$ He³. As an added check on the value of the concentration, we measured the vapor pressure of this mixture at several temperatures below the lambda point in a specially designed apparatus and found good agreement with the vapor pressure values of Sommers²⁰ for the concentration given above. The lower concentration samples, prepared by diluting the original mixture by measured amounts of pure He⁴, are estimated to be in error in concentration no greater than ± 5 percent of the concentration.

The vapor volume above the inner cavity was kept sufficiently small to insure that the He³ concentration in the liquid was never depleted by more than 1 percent. This is a rather important consideration, since the vapor He³ concentration is much higher than that of the liquid.^{20,21}

The velocity of second sound was measured using the pulse method^{3,4,8} whereby the time of flight of a single heat pulse between the transmitter and receiver, a known distance apart, is measured from the photograph

of an oscilloscope trace, using standard radar techniques. The error in the determination of u_2 is no greater than 1 percent.

The electronic components employed to generate and detect the heat pulse were essentially the same as those previously described.⁸ The square voltage pulse sent to the transmitting resistor was usually between 20 and 60 microseconds wide. The power dissipation at the resistor during the pulse was 0.2 watt/cm^2 . The receiver current, on for less than a second per measurement, dissipated about 60 ergs/sec. Care was taken to keep the power fed to the transmitter and the receiver resistance small in each pulse for the following reasons:

(1) The isotope equilibrium can be easily upset by the "heat flush" effect,^{22,23} the He³ atoms being swept away from the heat source. For very low He³ concentrations this effect was observed and will be discussed later. However, there was ample evidence that no appreciable isotope separation was occurring for He³ concentrations of 0.3 percent and above for the power inputs used.

(2) Shock-wave effects with larger apparent velocities will show up for large transmitter pulse amplitudes as observed earlier by Osborne³ in pure He⁴. However, the pulse amplitude has to be increased considerably above that normally used before the shock-wave effects become noticeable. In this temperature range, shock waves can be detected by the sharpening of the leading edge of the heat pulse and by the increase in velocity with increasing pulse amplitude.

(3) The specific heat of the liquid becomes very low in this low-temperature region (although considerably higher in the mixtures than in pure He⁴). It is therefore necessary to reduce the transmitter pulse power accordingly as the temperature is reduced, if one is to keep the temperature amplitude of the pulse to a reasonable size.

The procedure for making the measurements was as follows. After demagnetization from a field of 10 kilogauss, heat pulses were triggered and photographed on the average of every 30 seconds, during the warm-up period. For the mixtures, this warmup time varied between 22 and 52 minutes. During this period, the resistance of the secondary thermometer was recorded as a function of time and compared with the calibration curve of R *versus* T^* to determine the Curie temperature at the time the pulse was triggered.

III. TEMPERATURE MEASUREMENT

In determining the thermodynamic temperature of the liquid, using the susceptibility of the salt pill as a primary thermometer and the carbon resistor attached to the copper second-sound cavity as a secondary thermometer, three corrections must be considered:

¹⁸ B. V. Rollin, Proc. Intern. Congr. Refrig., 7th Congr. 1, 187 (1936).

¹⁹ Kurti, Rollin, and Simon, Physica 3, 266 (1936).

²⁰ H. S. Sommers, Phys. Rev. 88, 113 (1952).

²¹ Taconis, Beenaker, Nier, and Aldrich, Physica 15, 733 (1949).

²² Lane, Fairbank, Aldrich, and Nier, Phys. Rev. 73, 256 (1948).

²³ Reynolds, Fairbank, Lane, McInteer, and Nier, Phys. Rev. 76, 64 (1949).

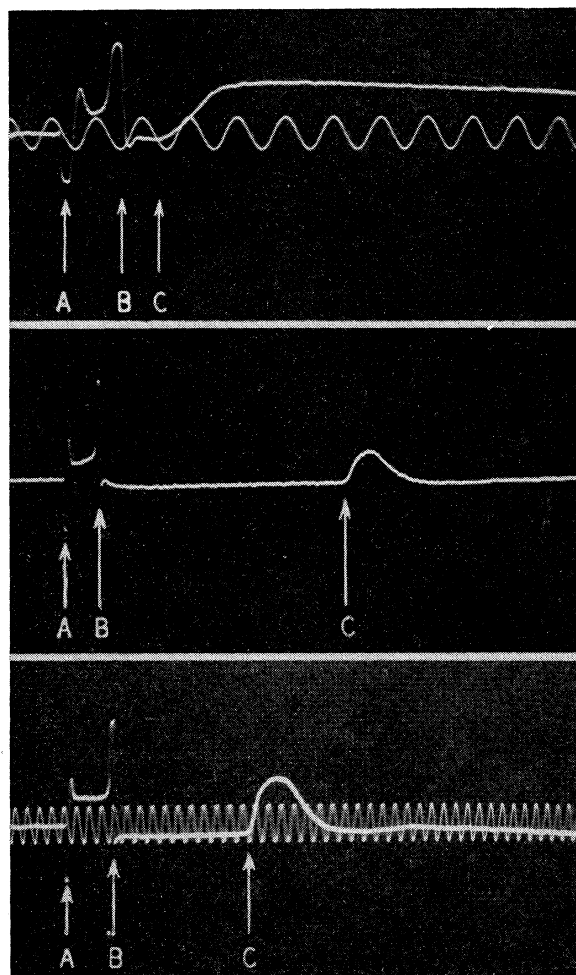


FIG. 3. Cathode-ray oscilloscope trace of second-sound pulses. The transmitted pulse, beginning at *A* and ending at *B*, is picked up in the receiver by stray electromagnetic coupling. The beginning of the received pulse occurs at *C*. Top trace: 20- μ sec pulse at 0.45°K in pure He⁴ (250- μ sec sweep). Middle trace: 30- μ sec pulse at 1.6°K in pure He⁴ (1000- μ sec sweep). Bottom trace: 35- μ sec pulse at 0.45°K in 0.32 percent He³ mixture (1000- μ sec sweep). A 50-kc/sec sine wave is superimposed on two of the traces for calibration.

(a) The Curie temperature T^* of the salt may deviate slightly at the lowest temperatures from the thermodynamic temperature.

(b) Since the salt pill is not spherical, a correction for the actual shape of the pill must be made in calculating T^* .

(c) There may be a difference between the temperature of the salt and the resistance thermometer during the calibration warmup period.

The correction under (a) can be shown to be small for potassium chrome alum in the temperature range of these measurements. At the lowest temperature, 0.2°K, $T^* - T$ is no more than 0.01°K on the theory of Hebb and Purcell.²⁴ This correction has not been made, and

²⁴ M. H. Hebb and E. M. Purcell, *J. Chem. Phys.* **5**, 338 (1937).

the temperatures reported are Curie temperatures. The shape-factor correction (b) was made by calculating T^* from the modified form of Curie's law

$$I/H_0 = a/(T - \Delta),$$

in which I is the intensity of magnetization of the salt, H_0 is the external applied field, a is the Curie constant of the salt, and $\Delta = f(4\pi/3 - D)$ is the factor introduced to account for the shape of the pill (Δ is zero for a sphere). Here f is the filling factor of the compressed salt pill and D is the demagnetization factor.²⁵ Δ for our pill was 0.020°K \pm 0.002, the uncertainty arising mostly from the difficulty of calculating the demagnetizing factor of the pill with the copper network present. Our actual procedure was to measure the mutual inductance M of the coils surrounding the salt pill (by the ballistic galvanometer method)¹⁴ at several temperatures between 1 and 4°K. The temperatures were obtained from the vapor pressure of the surrounding bath, using the 1949 scale of temperature. A plot of M (or galvanometer deflection) vs $1/(T - \Delta)$ gave a straight line which then could be extrapolated to give T^* for temperatures below 1°K.

There remains the possibility, (c), that there was a temperature difference between the carbon-resistance thermometer and the salt pill during the calibration run. We estimate this to be small, less than 0.01°K at the lowest temperature. Resistance and salt-temperature measurements were taken simultaneously on several runs where the heat flow from cavity to salt was several times greater than during the calibration run. By comparing the results from different runs a good check of the above estimate was obtained.

IV. RESULTS

Second-sound pulses in dilute mixtures at ultra-low temperatures exhibit two distinct features which are in contrast to heat pulses in pure He⁴ at this temperature. First, u_2 , after reaching a maximum value of about 35 m/sec just below 1°K, decreases with decreasing temperature. Secondly, there is almost a complete absence of the spreading of the heat pulses which occurs for heat pulses propagated in pure He⁴ below 0.6°K. These features can be observed in Fig. 3. The top picture is that of a scope trace showing the time of flight of a 20-microsecond heat pulse in pure He⁴ at 0.45°K. It can readily be seen that the pulse suffers considerable dispersion and, too, that its velocity is of the order of 200 m/sec. (The calibration signal is a 50-kc/sec sine wave.) The scope picture at the bottom of Fig. 3 is of a pulse in a 0.32-percent He³ mixture at the same temperature. The calibration signal is unchanged. In addition to the almost tenfold decrease in u_2 , the pulse is very little distorted.

In Table I are listed the values of u_2 together with their corresponding T^* values for 4.30-, 0.62-, and

²⁵ E. C. Stoner, *Magnetism and Matter* (Methuen and Company, London, 1934), p. 38.

0.32-percent He³ concentration, respectively. These u_2 and T^* values are used in plotting the curves shown in Fig. 4. The curve, shown without points in this figure, is for pure He⁴ and represents the composite results of several investigators.^{1,2,6,8,10}

V. DISCUSSION

General theoretical treatments of the effects of addition of He³ to He⁴ upon the thermodynamics of He II and upon the propagation of second sound have been put forth by Pomeranchuk,⁷ Koide and Usui,²⁶ and recently by Price.²⁷ Pomeranchuk starts with the two-fluid hypothesis and further assumes that the He³ contributes only to the normal component density ρ_n . His theory applies to dilute solutions ($\epsilon < 0.01$, where ϵ is the mole fraction of He³) at temperatures above 0.2°K where classical statistics can be applied to the He³ atoms. He also treats the behavior very close to 0°K, where the He³ can be considered a completely degenerate Fermi-Dirac gas. Price's approach is somewhat more general in that he does not restrict the He³ to the normal component only. However, if this assumption, which now has considerable experimental support,^{21-23,28} is imposed on his theory, his expression for the velocity of second sound reduces to that of Pomeranchuk. We will attempt no comparison of our data with the theory of Koide and Usui, since it is restricted by the authors to the region between the lambda point and 1.5°K.

TABLE I. Velocity of second sound *versus* temperature for various He³ concentrations.

0.32% He ³		0.62% He ³		4.30% He ³	
$T^*(^{\circ}\text{K})$	$u_2(\text{m/sec})$	$T^*(^{\circ}\text{K})$	$u_2(\text{m/sec})$	$T^*(^{\circ}\text{K})$	$u_2(\text{m/sec})$
0.260	24.95	0.206	21.35	0.293	21.35
0.267	25.10	0.215	21.68	0.309	22.25
0.287	25.40	0.226	21.90	0.324	22.82
0.312	25.72	0.244	22.83	0.350	23.70
0.337	26.36	0.264	23.52	0.381	24.80
0.359	27.02	0.300	24.53	0.415	25.70
0.377	27.55	0.324	25.11	0.433	26.52
0.397	28.10	0.350	25.87	0.453	26.50
0.418	28.65	0.368	26.52	0.473	27.55
0.453	29.65	0.397	27.36	0.517	28.85
0.477	30.50	0.433	28.45	0.567	30.07
0.503	30.95	0.473	29.65	0.624	31.40
0.531	32.10	0.517	30.95	0.687	32.60
0.561	32.83	0.567	32.10	0.722	33.36
0.629	34.43	0.624	33.50	0.761	34.16
0.667	34.70	0.687	34.42	0.806	34.70
0.730	35.15	0.761	35.15	0.856	35.55
0.779	35.00	0.856	34.72	0.914	35.85
0.837	34.16	0.974	32.60		
0.903	32.58	1.017	31.40		
0.973	30.50				
1.010	29.05				
1.035	28.45				
1.060	27.55				
1.072	26.84				

²⁶ S. Koide and T. Usui, Progr. of Theoret. Phys. (Japan) **6**, 506 (1951).

²⁷ P. J. Price, Phys. Rev. **89**, 1209 (1953).

²⁸ Daunt, Probst, Johnston, Aldrich, and Nier, Phys. Rev. **72**, 502 (1947).

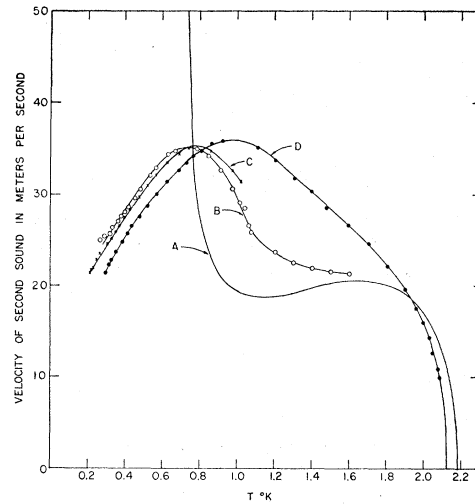


FIG. 4. Second-sound velocity *versus* temperature for several He³ concentrations. Curve A, pure He⁴. Curve B, 0.32-percent He³. Curve C, 0.62-percent He³. Curve D, 4.30-percent He³.

For the velocity of second sound in dilute solutions above 0.2°K, Pomeranchuk obtains the expression

$$u_2^2 = \frac{\rho_s}{\rho_n} \frac{T}{C} \left[\left(S_0 + \frac{k\epsilon}{m_4} \right)^2 + \frac{k\epsilon}{m_4} C \right], \quad (1)$$

where S_0 and ρ_s at temperature $T^{\circ}\text{K}$ are the entropy of pure He⁴ and the superfluid component density, respectively. In this expression ϵ , the mole fraction of He³, enters into the expression for C , the specific heat of the mixture, as well as for ρ_n , the normal component density of the mixture. Pomeranchuk suggests that the elementary excitations connected with foreign atoms, namely He³, dissolved in He⁴, will give rise to one of two possible energy spectra, either

$$E = E_0 + p^2/2\mu \quad (2)$$

or

$$E = E_0 + (p - p_0)^2/2\mu, \quad (3)$$

where, by Eq. (2), the minimum energy corresponds to $p_0 = 0$ and by Eq. (3), to $p_0 \neq 0$. The most important effect of the energy spectrum upon the velocity of second sound enters into the expression for ρ_n , which can be expressed as

$$\rho_n = \rho_{n0} + \rho_{ni}, \quad (4)$$

where ρ_{n0} is the contribution to the normal component density due to pure He⁴ and ρ_{ni} is the contribution due to the He³ impurity atoms. For the two postulated spectra Pomeranchuk, following Landau,²⁹ shows that

$$\rho_{ni} = \frac{\rho}{m_4} \mu \epsilon \quad [\text{for } p_0 = 0, \text{ assumption (2)}], \quad (5)$$

$$\rho_{ni} = \frac{\rho}{m_4} \frac{p_0^2}{3kT} \epsilon \quad [\text{for } p_0 \neq 0, \text{ assumption (3)}], \quad (6)$$

²⁹ L. Landau, J. Phys. (U.S.S.R.) **5**, 71 (1941); **11**, 91 (1947).

where ρ is the density of the mixture, and μ is the effective mass of the He^3 atom in solution. Since ρ_{ni} is independent of T in one case and inversely proportional to T in the other, an experimental determination of ρ_{ni} as a function of T can determine which energy spectrum is obeyed.

Qualitatively, the effect of impurity atoms on the second-sound velocity as given by Eq. (1) is the same on either assumption for the energy spectrum. In the higher-temperature region above 1°K the velocity is increased by the presence of the He^3 through the terms involving ϵ . At lower temperatures the effect of ϵ on ρ_{ni} becomes the predominating effect because $\rho_{ni} \gg \rho_{n0}$, and u_2 falls to low values. Below about 0.6°K , when $k\epsilon/m \gg S_0$, Eq. (1) reduces to

$$u_2^2 = 5kT/3\mu \quad (\text{for } p_0=0), \quad (7)$$

$$u_2 = 3kT/p_0 \quad (\text{for } p_0 \neq 0). \quad (8)$$

The experimental values of u_2 from 0.2°K to 1.9°K for dilute solutions less than 1 percent He^3 are in excellent qualitative agreement with these predictions of Pomeranchuk and in good quantitative agreement if the first assumed energy spectrum, with $p_0=0$, is used [Eq. (2)]. However, the adjustable parameter μ turns out to be larger than the actual mass of the He^3 atom and slightly temperature-dependent, as will be seen.

The confirmation of Pomeranchuk's theory in the higher-temperature region above 1.25°K was evident from the earlier u_2 measurements of Lynton and Fairbank⁸ taken in 0.09, 0.21, 0.41, and 0.80 percent He^3 in He^4 . Using these experimental values of u_2 in Eqs. (1) and (4), Khalatnikov³⁰ and Lynton and Fairbank³¹ independently calculated ρ_{ni} for several temperatures and concentrations and found it to be proportional to ϵ and nearly independent of temperature, in agreement with Eq. (5). This then confirmed the energy spectrum with $p_0=0$ for the impurity atoms.³² The effective mass was found to be about three times the mass of a He^3 atom.³³

In the lower-temperature region the data of this paper

³⁰ I. M. Khalatnikov, Akad. Nauk S.S.S.R. (Doklady Akad. Sci.) **79**, 57 (1951).

³¹ E. A. Lynton and H. A. Fairbank, Proc. Internat. Conf. on Low Temp. Physics, Oxford University, Oxford, p. 88 (1951).

³² The suggestion made in the earlier paper of Lynton and Fairbank (see reference 8) that their data were in better agreement with the assumption $p_0 \neq 0$, was based on the assumption that μ was the mass of a He^3 atom. This assumption is not required in Pomeranchuk's theory, and by letting μ take on larger values than m_3 , Lynton and Fairbank (see reference 31) came to the conclusion outlined above (i.e., $p_0=0$). There is little doubt that this conclusion is correct, especially in view of the results of this paper below 1°K .

³³ Khalatnikov found $\mu=2.85m_3$ to within ± 5 percent between 1.2 and 1.7°K . Lynton and Fairbank found $\mu=3.3m_3$ at 1.25°K , rising to about 3.8 between 1.4 and 1.9°K . This discrepancy is unexplained, although some difference could easily arise from a different choice of values of the other parameters in Eq. (1). The apparent increase in μ with increasing temperature, seen in Lynton and Fairbank's calculations, remained when we recalculated the values using Kramer's S_0 and C_0 values. This is consistent with the temperature dependence of μ at lower temperatures (see Fig. 5).

likewise are in good agreement with Pomeranchuk's predictions with the same energy spectrum assumption ($p_0=0$). However, to obtain good agreement between experiment and theory, μ must be allowed to vary from about $2.0 m_3$ at 0.2°K to $3.6 m_3$ at 1.8°K , as shown in Fig. 5. Our procedure in arriving at these results was to calculate μ from Eqs. (1), (4), and (5), using the experimental values and u_2 and the S_0 and C_0 values of Kramers, Wasscher, and Gorter.³⁴ ρ_{n0} was calculated from the equation $u_2^2 = (T/C_0)(\rho_s/\rho_{n0})S_0^2$, where the second-sound velocities were the combined results of several investigators. In the temperature region above 1.25°K , the values of μ in Fig. 5 were recalculated from the u_2 values of Lynton and Fairbank in solutions up to 0.8 percent He^3 , using the latest specific-heat and entropy data for pure He^4 . The points at 1.0 and 1.1°K were taken from our values of u_2 in a 0.95 percent He^3 mixture, and the lower points calculated from the 0.6 percent He^3 mixture results (Table I). The estimated error in the temperature determinations below 1°K is indicated in this figure by a line through each point.

Below 0.6°K it should be noted that our results show u_2 to be nearly independent of He^3 concentration, as required by Eq. (7). Likewise, the experimental values of u_2 below 0.6°K are nearly proportional to $T^{1/2}$, as predicted by Eq. (7). The deviation from this prediction is a reflection of the temperature-dependence of μ .

In the temperature region below 0.6°K , the normal component density of the He^3 becomes exceedingly small; hence, the density attributed to the He^3 atoms given by Eq. (4) becomes increasingly dominant.³⁵ (This holds as well for the entropy and specific heat of the mixture.) In other words, the primary excitation energy in a dilute mixture at ultra-low temperatures can be attributed to the He^3 atoms which apparently have a roton type of spectra.

In pure He^4 , on the other hand, the excitations are

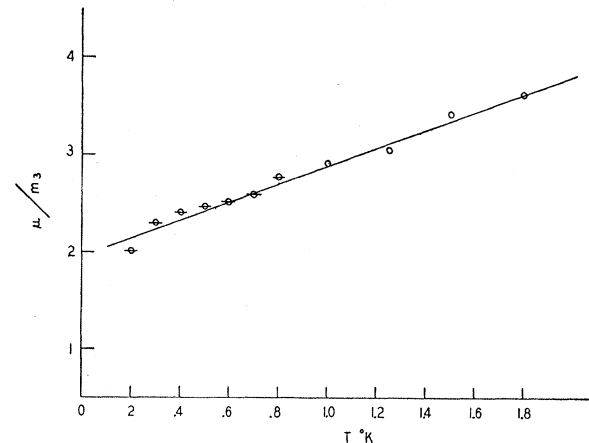


FIG. 5. The effective mass of He^3 as a function of temperature.

³⁴ Kramers, Wasscher, and Gorter, Physica **18**, 329 (1952).

³⁵ Indeed, a concentration of 0.6 percent He^3 is sufficiently large to give to the mixture a normal density equal to the normal density of pure He^4 at 1.0°K .

almost entirely phonons below 0.6°K, with the rotons predominating at higher temperatures. Thus, the large rise in the velocity and the spreading of the received pulse which occurs in pure He⁴ below 0.6°K (e.g., see Fig. 2) would not be expected in the mixtures, since this behavior is a consequence of the phonon-type excitation spectrum. Instead, one might expect the propagation of a heat pulse in the mixtures at low temperatures to be similar to the propagation in pure He⁴ in the higher-temperature region where the excitation spectrum is similar. Qualitatively, the shape of the heat pulse in a dilute mixture (see Fig. 3) does indeed resemble a heat pulse in pure He⁴ at higher temperatures and shows none of the spreading characteristic of pulses in pure He⁴ at low temperatures. Putting this another way, the phonon mean free path is so long at low temperatures that heat propagation in pure He⁴ becomes a diffusion process. However, in the mixtures the He³ excitation mean free path is still short enough to allow genuine second-sound propagation.

It is of interest to speculate upon how little He³ is necessary to change effectively the velocity of second sound in He⁴ in the ultra-low temperature region. At low enough temperatures any impurities present would govern the behavior on Pomeranchuk's theory. An experimental attempt to this end was made using a He³ concentration of 0.017 percent. The results are shown in Fig. 6, together with the theoretical curve calculated from Eq. (1) using values of μ taken from Fig. 5. For purposes of comparison, u_2 for pure He⁴ is included, taken with the same apparatus under identical conditions.³⁶ Apparently the large deviations between the theoretical curves are brought about by heat-flush effects^{22,23} in the cavity produced by the heat influx from the creeping film. This effect essentially sweeps out most of the He³ atoms from the second-sound channel below about 0.4°K, since u_2 for this mixture approaches the value for pure He⁴ below this temperature. Reliable measurements of u_2 for $\epsilon \leq 10^{-4}$ thus appear impossible unless the heat leak to the cavity is greatly reduced or the mixture stirred. For the above reasons it is not likely that the small concentration of He³ in ordinary He⁴ is likely to affect appreciably the second-sound properties even at the lowest temperatures. Commercial helium has a He³ concentration ratio of 1.3×10^{-6} in atmospheric helium and 1.6×10^{-7} in well helium.³⁷

It should be observed, however, that since u_2 below about 0.6°K is virtually independent of He³ concen-

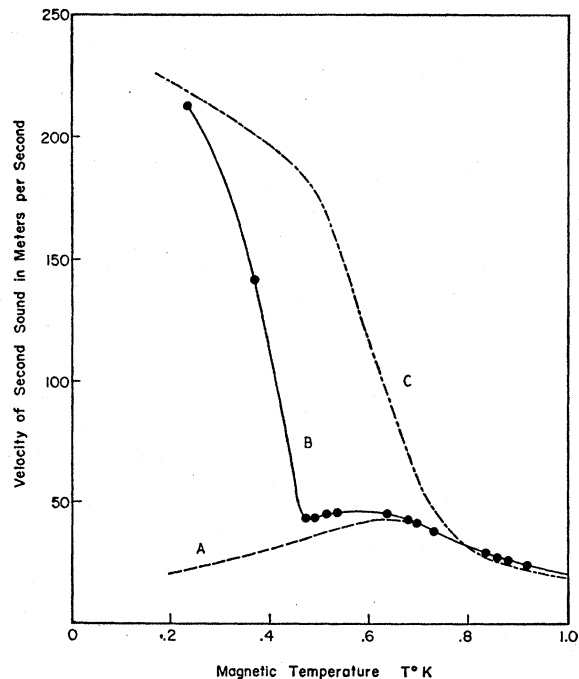


Fig. 6. Second-sound velocity as a function of temperature in 0.017 percent He³ mixture, illustrating the effect of heat-flush separation of the isotopes at low temperatures and concentrations. Curve A represents the predictions of Pomeranchuk's theory. Curve B gives the experimental results. Curve C is a comparison curve for pure He⁴ under the same conditions.

tration for dilute solutions, gradients in the isotope concentration caused by heat flush would not be expected to effect materially the measured velocities unless nearly complete heat flush occurred.

Pomeranchuk's theory outlined above is a valid approximation only in the region where classical statistics can be applied to the He³ atoms. In the very low temperature region, assuming complete Fermi-Dirac degeneracy for the He³ atoms, Pomeranchuk derives another expression for u_2 in which the limiting value of the velocity of second sound at 0°K is small but finite and proportional to the cube root of the He³ concentration (for $p_0=0$). This is in contrast to Dingle's suggestion that u_2 should approach $u_1/\sqrt{3}$ near 0°K in He³-He⁴ mixtures. Measurements at lower temperatures are clearly required.

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³⁶ The u_2 values for pure He⁴ in Fig. 6 were taken in the same cavity with the same pulse amplitude as for the He³-He⁴ mixture curves for direct comparison. These values are slightly high in the low-temperature region because of a shock effect which shows up in pure He⁴ for these pulse amplitudes. They are also higher than values of u_2 obtained with longer cavities, probably as a result of the long mean free path of the phonons below about 0.6°K. We are publishing the details of our helium 4 measurements in a separate paper (see reference 10).

³⁷ L. T. Aldrich and A. O. Nier, Phys. Rev. **70**, 983 (1946); **71**, 911 (1947).

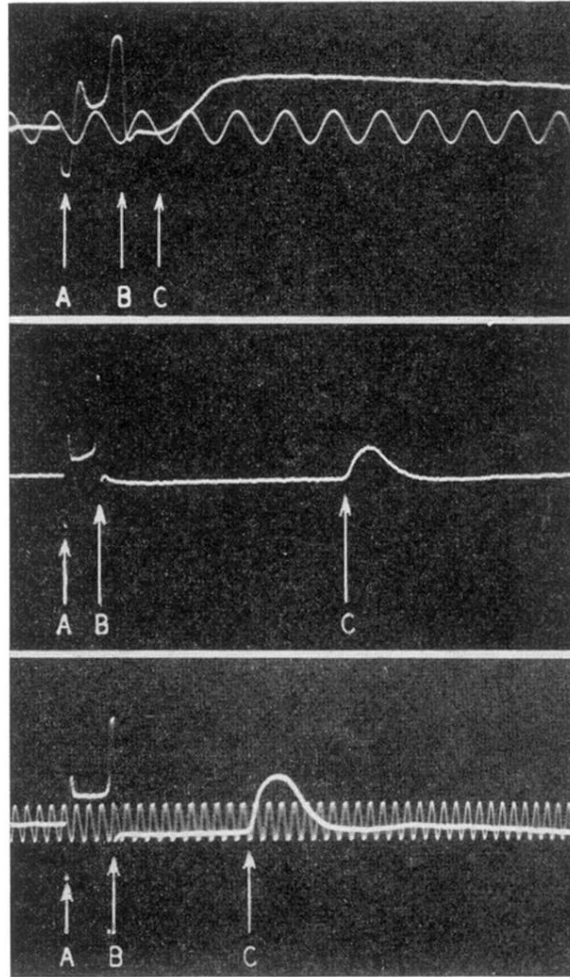


FIG. 3. Cathode-ray oscilloscope trace of second-sound pulses. The transmitted pulse, beginning at *A* and ending at *B*, is picked up in the receiver by stray electromagnetic coupling. The beginning of the received pulse occurs at *C*. Top trace: 20- μ sec pulse at 0.45°K in pure He⁴ (250- μ sec sweep). Middle trace: 30- μ sec pulse at 1.6°K in pure He⁴ (1000- μ sec sweep). Bottom trace: 35- μ sec pulse at 0.45°K in 0.32 percent He³ mixture (1000- μ sec sweep). A 50-kc/sec sine wave is superimposed on two of the traces for calibration.