measuring the transmission of a spherical shell with a threshold detector.[27,28]

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Inelastic Scattering of Neutrons near Threshold*

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Using the compound nucleus model, and assuming a statistical distribution of levels of the compound state, the inelastic neutron scattering cross sections for excitation of metastable states of Cd^{111} and In^{115} are calculated. The energy range just above threshold is considered. The sensitivity of the cross sections to the spin and parity of the metastable state is demonstrated in the case of Cd^{111}. Comparison is made with experiment, and the calculations are seen to be useful in learning about spins and parities of excited states of nuclei.

INTRODUCTION

HAUSER and Feshbach[1] have pointed out that just above threshold energy the cross section for production of an excited state of a target nucleus by inelastic neutron scattering is particularly sensitive to the spins of the ground and excited states and to a lesser extent to the parities of these states. It is clear then that measurements of cross sections for the inelastic scattering of neutrons are of interest in nuclear spectroscopy. In particular, if the target nucleus has an excited state with a fairly long lifetime, it is possible to measure the cross section for excitation of this state by measuring its radioactivity. This is precisely the case when the cross sections are most sensitive to spin and parity difference. The larger the spin difference between the ground and excited states the more sensitive is the cross section to these spins and parities. Experiments of this type have been carried out by Francis, Goodman, and McCue[2] and by Martin, Diven, and Taschek.[3]

In this paper cross sections will be computed for excitation of metastable levels of Cd^{111} and In^{115} having energies 0.396 Mev, and 0.335 Mev, respectively.

THEORY

Using the compound-nucleus model and assuming a statistical distribution of levels for the compound states, Hauser and Feshbach[1] derive the following expression for direct excitation of an excited state of spin i' and energy above ground E' by inelastic scattering of neutrons of energy E and wavelength $2\pi\lambda$ from a target of spin i:

$$\sigma(i|i')=\frac{\pi\lambda^2}{2(2i+1)}\sum_{l=0}^{\infty}T_l(E)\sum_{J=0}^{\infty}\epsilon_{jl}{}^J(2J+1)\frac{\sum_{l'}\epsilon_{j'l'}{}^J T_{l'}(E-E')}{\sum_{E'',l''}\epsilon_{j''l''}{}^J T_{l''}(E-E'')}. \quad (1)$$

The expression (1) takes into account competition due to elastic scattering and inelastic scattering of the neutrons by other levels. The competition from the (n,γ) process is omitted since, in the energy range under consideration, the radiation width is at most only a few percent of the total neutron width. This competition can be included easily when necessary.[4]

In formula (1) the energies and spins of the levels of the target nucleus that can be excited by a neutron of energy E are denoted by E'' and i'', respectively. J represents the possible spins of the compound nucleus; l, l', and l'' are the orbital angular momentum quantum numbers of an incoming neutron, of an outgoing neutron leaving the residual nucleus in the metastable state E', and of any outgoing neutron, respectively. The sum over the l'' must be taken so as to include only those terms that conserve the parity as well as the angular momentum of the system. $j_{1,2}''=i''\pm\frac{1}{2}$ and

$$\epsilon_{jl}{}^J=\begin{cases}2 \text{ if both } j_1 \text{ and } j_2\\ 1 \text{ if } j_1 \text{ or } j_2, \text{ not both}\\ 0 \text{ if neither } j_1 \text{ nor } j_2\end{cases}\quad \text{satisfy } |J-l|\leq j_i\leq J+l. \quad (2)$$

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TABLE I. Energies, spins, and parities of low-lying levels of Cd^{111} and In^{115}.

Target	Cadmium 111						Indium 115					
n	0	1	2	3	4	5	0	1	2	3	4	5
E_n	0	0.247	0.340	0.396	0.419	0.72?	0	0.335	0.500	0.860	0.960	1.30
i_n	1/2	5/2	3/2	11/2	7/2	···	9/2	1/2	5/2	3/2	7/2	9/2
π_n	e	e	e	0	e	···	e	0	e	e	e	e

The $T_l(E)$ are wave-mechanical transmission coefficients for penetration of the nuclear surface by neutrons of energy E and orbital angular momentum quantum number l. They also depend on the nuclear radius R. Feshbach and Weisskopf[5] show that

$$T_l(E)=4xXv_l/[X^2+(2xX+x^2v_l')v_l], \tag{3}$$

where $x=R/\lambda\!\!\!^{-}$, $X^2=X_0^2+x^2$ with $X_0\simeq 10^{13}\times(R \text{ in cm})$, $v_l=|xh_l^{(1)}(x)|^{-2}$, and $v_l'=|d/dx[xh_l^{(1)}(x)]|^2$, $h_l^{(1)}(x)$ being the spherical Hankel function of order l of the first kind. The notation in (3) is that of Blatt and Weisskopf,[6] not that of the original authors.

Formula (1) depends on the assumption that the neutron widths can be written in the form[7]

$$\Gamma(E)^{(J,l)}=T_l(E)D_c^J/2\pi, \tag{4}$$

where D_c^J is the spacing of energy levels of spin J of the compound nucleus at the proper excitation energy. Fairly large fluctuations from (4) must be expected for individual levels. Since the cross sections are the results of competitions among several emissions by the compound nucleus, it is expected these fluctuations are canceled out to a large extent.

CALCULATIONS

The nuclear radius R is determined by comparing measured values of total neutron cross sections with the theoretical results of Feshbach and Weisskopf.[5] Measurements for Cd^{111} and In^{115} have been made by Fields *et al.*[8] and Barschall *et al.*,[9] respectively. In both cases, one gets $R\simeq 7.8\times 10^{-13}$ cm in the energy range under consideration. These are the effective values of the nuclear radius that must be used to be consistent with the total neutron cross-section theory to which the inelastic-scattering theory herein corresponds.

The decay schemes and spins, parities and energies of the low-lying states of Cd^{111} and In^{115} are given in Figs. 1 and 2, and Table I. They have been obtained from McGinnis,[10,11] and Goldhaber and Hill,[12] respectively. The spins, parities, and energies of these states are denoted by i_n, π_n and E_n, respectively; $n=0$ represents the ground state, $n=1$, the first excited state and so on.

CADMIUM 111

The decay scheme of Fig. 1 shows that the E_4 level does not decay to the $E_3=0.396$-Mev metastable level in its cascade decay to the ground state. Hence for energies of the bombarding neutrons up to E_5 at least the cross section for excitation of the E_3 level is given by a formula like (1) with $E'=E_3$ and $i'=i_3$. The computed values of this cross section, σ, are tabulated in Table II and plotted in Fig. 3 for a range of energies using (i) the values of spin and parity for the E_3 level determined by McGinnis[10,11] by studying the decay scheme of the 111 isobars and verified by Sunyar,[13] namely 11/2, odd; (ii) the values 13/2, even, which gives properties for the decay scheme closest to those for the choice 11/2, odd.

As an example, the term of (1) for $l=0$ in case (i) is, putting $E-E_n=\epsilon_n$,

$$\sigma^{(0)}=\pi\lambda\!\!\!^{-2}(a_0+3a_1)/4,$$

where

$$a_0=T_5(\epsilon_3)/[T_0(\epsilon_0)+T_2(\epsilon_2)+T_5(\epsilon_3)+T_4(\epsilon_4)],$$

$$a_1=[2T_5(\epsilon_3)+T_7(\epsilon_3)]/[T_0(\epsilon_0)+T_2(\epsilon_0)+2T_2(\epsilon_1)+T_4(\epsilon_1)+T_0(\epsilon_2)+2T_2(\epsilon_2)+2T_5(\epsilon_3)+T_7(\epsilon_3)+T_2(\epsilon_4)+2T_4(\epsilon_4)].$$

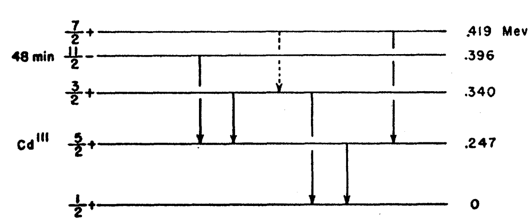

FIG. 1. Decay scheme for low-lying energy levels of Cd^{111}.

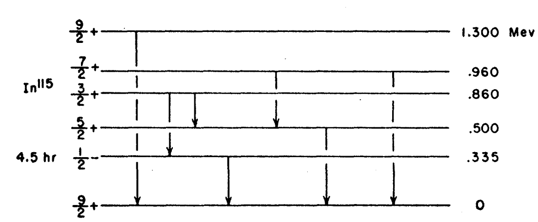

FIG. 2. Decay scheme for low-lying energy levels of In^{115}.

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TABLE II. Theoretical cross section for excitation of 0.396-Mev metastable level of Cd^{111} by inelastic neutron scattering taking the spin and parity of this level to be (i) 11/2, odd and (ii) 13/2 even.

E(Mev)	i_3	π_3	0.396	0.4	0.419	0.45	0.50	0.55	0.60	0.65	0.70	0.75
σ(millibarns)	11/2	0	0	0.0843	0.445	0.618	1.27	2.21	3.52	5.23	7.36	9.93
	13/2	e	0	0.00808	0.0207	0.0543	0.162	0.309	0.505	0.763	1.10	1.53

For three-figure accuracy, the calculations required terms up to $l=5$ in case (i) and $l=6$ in case (ii). The large number of l values contributing to the cross section so near threshold is a result of the large spin difference between the ground state and the metastable state of the target nucleus. In the case $i_3=11/2$, σ is seen to be 8.1 times as large as in the case $i_3=13/2$ at $E=0.5$ Mev and 6.6 times as large at $E=0.75$ Mev. In fact, it is seen from Table II that the ratio of the cross sections for the two choices of spin and parity gets closer and closer to 1 as one goes to higher neutron energies E. Measurements of σ made to determine the spins and parities of metastable nuclear states should be made as close as possible to the threshold energy of the scattering process then.

Francis, McCue, and Goodman[2] find experimentally $\sigma \simeq 10$ millibarns at $E=0.72$ Mev. This value agrees with the assignment 11/2, odd for the spin and parity of the E_3 level as can be seen from Table II or Fig. 3. The experimental values for σ much below this energy are not too certain so no comparison is made with theory.

INDIUM 115

In this case because of a fuller knowledge of the energy levels involved than for Cd^{111}, σ is calculated for incident neutron energies up to 1.3 Mev and the results are compared with the measurements of Martin, Diven, and Taschek.[3] These measurements are subject to experimental uncertainties of the order of 15 percent. One should expect then a test of the shape and magnitude of the calculated cross sections within the limits of the experimental accuracy.

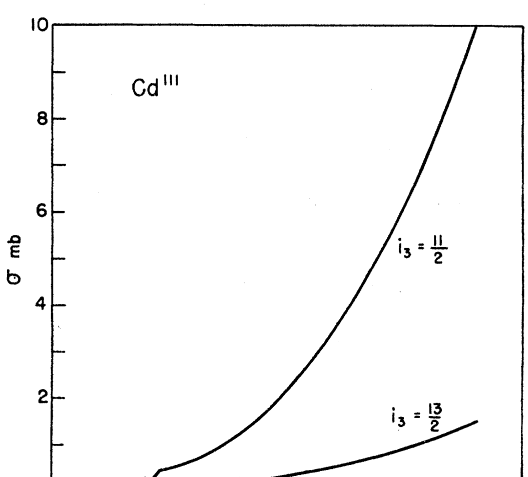

FIG. 3. Theoretical cross section for excitation of 0.396-Mev metastable level of Cd^{111} by inelastic neutron scattering taking the spin and parity of this level to be (i) 11/2 odd and (ii) 13/2 even.

The decay scheme of Fig. 2 shows that only the E_3 level decays to the ground state by cascading through the $E_1=0.335$ Mev metastable level. The total cross section, σ, for excitation of the E_1 level by inelastic neutron scattering at least up to energies of the sixth excited state of In^{115} is computed as follows then. One evaluates (1) in the cases $E'=E_1$, and $E'=E_3$. One then adds to the former cross section f times the latter where f is the fraction representing the relative number of times the E_3 level decays to the ground state through the E_1 level. (For lack of better knowledge, it was assumed that $f \simeq 1$. The value of f has no effect on σ for energies $E<E_3$ and only a small effect for energies above E_3 in the region that has been considered. A smaller value of f lowers σ slightly.) The experimental values of σ of Martin, Diven, and Taschek[3] are plotted in Fig. 4 together with the theoretical values, which are also given in Table III.

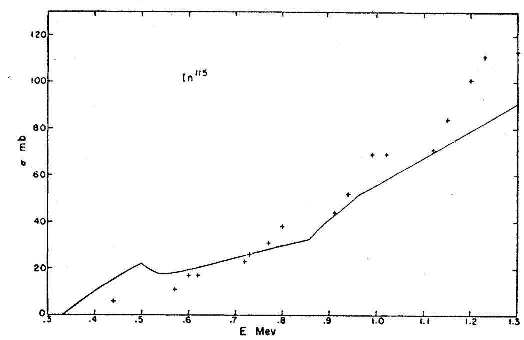

FIG. 4. Theoretical and experimental cross-section values for excitation of 0.335-Mev metastable level of In^{115} by inelastic neutron scattering.

One sees in Fig. 4 that $d\sigma/dE$ is discontinuous at neutron energies $E=E_n$, the energy of an excited state of the target nucleus. If the excited state E_n decays to the ground state by passing through the metastable state, the slope increases, otherwise the slope decreases at these points of discontinuous slope. The most notable apparent deviation of theory from experiment is in the energy region above $E=1.15$ Mev. This may indicate an unreported level of In^{115} near this energy. This level

TABLE III. Theoretical cross section for excitation of 0.335-Mev metastable level of In^{115} by inelastic neutron scattering.

E(Mev)	0.335	0.4	0.45	0.5	0.54	0.6	0.75	0.860	0.875	0.9	0.960	1.0	1.1	1.3
σ(millibarns)	0	10.3	16.5	22.1	17.7	19.7	27.1	32.7	36.4	40.7	50.7	55.6	67.5	90.2

would cascade to the ground state through the metastable state because of the suddenly increasing slope, $d\sigma/dE$.

CONCLUSIONS

The agreement of experiment with theory is good. This is notable since (i) there are no parameters in the theory aside from those obtained in fitting the total cross-section theoretical expressions to experiment, *viz.* X_0 and R, and (ii) immediate compound nucleus formation is assumed in all the neutron-scattering processes. Recent calculations by Feshbach, Porter, and Weisskopf[14] have shown that for total neutron cross sections much better agreement with experiment is obtained, if one does not assume immediate compound nucleus formation. For best agreement with experiment they find that, in an energy range including the one considered above, the neutron travels a distance of about 2×10^{-12} cm in nuclear matter before being incorporated in a collective compound motion.

It can be seen from the particular cases considered above that it is possible to investigate energy levels and their properties by analyzing the inelastic-scattering experiments with this theory.

ACKNOWLEDGMENTS

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[14] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953).