partition function can be written in the form,

$$Q=(\lambda^{3N}/N!)\int_z \exp(-\beta V)\Big[\sum_P \exp\Big[-(\pi/\lambda^2)\times\sum_{i=1}^{N}|\mathbf{z}_i-P\mathbf{z}_i|^2\Big]+O(h^2)+O(h^4)\cdots\Big]d\mathbf{z}, \quad (2)$$

where V is the interaction potential of the atoms. If we ignore the terms of order $h^2,h^4\cdots$, etc., in the above expansion we get

$$Q''=(\lambda^{3N}/N!)\int_z \exp(-\beta V)\times\sum_P \exp\Big[-(\pi/\lambda^2)\sum_{i=1}^{N}|\mathbf{z}_i-P\mathbf{z}_i|^2\Big]d\mathbf{z}. \quad (3)$$

It is seen that Q' only differs from Q'' in that m is replaced by m' and $\exp(-\beta V)$ by $K_\beta \exp[-\beta V'(\beta)]$. In I it is stated that m' will probably not differ very much from the true mass m, near the λ temperature, and that K_β can be ignored as far as the nature of the transition is concerned. We have already noted that $V'(\beta)$ is fairly insensitive to changes in β and is qualitatively similar to the actual potential V. In other words the modifications to the zero-order term in Eq. (2) proposed by Feynman are quite small; we therefore have to consider whether the effects of all the other terms, that have been omitted, can be supposed to be small. This question is very difficult to decide, although it is quite easy to show that the coefficients of the leading terms of the series are large. However, even if we cannot evaluate any of the higher-order terms rigorously we can still argue that they are unlikely to remove any discontinuities that may be present in the derivatives of the first term of the series. This follows from the fact that the above series is an expansion in powers of $\Lambda^{*2}(=h^2/\sigma^2 m\epsilon)$, and therefore if the higher-order terms were to remove any discontinuities predicted from the first term this would imply a critical dependence on Λ^*. This would seem to be unreasonable on physical grounds. We therefore conclude that although the first term may be a poor approximation quantitatively it may still yield useful qualitative information.

Feynman next considers a typical term in the summation over all the permutations P in Eq. (1). Let us suppose that in this particular term there are m_l cycles of l "atoms" so that $\Sigma_l l m_l = N$. Then a typical term in the sum in Eq. (2) can be written,

$$\int_z \rho_N(\mathbf{z}_1\cdots\mathbf{z}_N)\prod_l\prod_{k=j}^{j+m_l l}\prod_{p=1}^{l}\exp[-(\pi/\lambda^2)|\mathbf{z}_{k+p}-\mathbf{z}_{k+p+1}|^2]d\mathbf{z}, \quad (4)$$

where the product Π_l is over a set of values of l such that $\Sigma_l l m_l = N$ and $\mathbf{z}_{k+l+1}\equiv\mathbf{z}_{k+1}$, Feynman now approximates to ρ_N by the product

$$\prod_l\prod_{k=j}^{j+m_l l}\prod_{p=1}^{l}\rho_2(\mathbf{z}_{k+p},\mathbf{z}_{k+p+1}),$$

where ρ_2 is proportional to the pair or "radial" distribution function. Clearly with this approximate form for ρ_N each of the terms in the sum of permutations can be evaluated explicitly, provided the form of ρ_2 is assumed. We shall now show that this approximation is closely related to the well-known superposition approximation of Kirkwood.[5] The superposition approximation can be formulated by saying that we approximate to ρ_N by the product

$$\prod_{i\le j=1}^{N}\prod\rho_2(\mathbf{z}_i,\mathbf{z}_j).$$

Basically, therefore, Feynman's approximation rests on this approximation. However, Feyman has also replaced all factors of the form $\rho_2(\mathbf{z}_i,\mathbf{z}_{i+p}),p>1$, in the superposition approximation, by unity. As he has pointed out, this means that we are completely neglecting (a) correlations between atoms that are not 'nearest neighbors' in a cycle and (b) correlations between all the atoms in any one cycle and those in any other cycle. It is well known that it is very difficult to estimate the validity of the superposition approximation. However it is very reasonable to assume that the more "gas-like" the structure of the liquid the more accurate the approximation will be. We may therefore be reasonably sure that for helium, which is a "gas-like" liquid with an abnormally low density, the approximation will be quite accurate. This conclusion lends support to Feynman's basic approximation and we would agree with him that the neglect of correlations between atoms in different cycles is probably much more important; it may be vital in deciding the order of the lambda transition. It is interesting to note that for a "classical liquid" the superposition approximation is, by itself, sufficient to allow us to calculate many of its properties. This is unfortunately not so for a "quantum liquid."

Finally we would like to mention that Matsubara[2] has developed a similar theory of liquid helium using essentially the same mathematical approximations as Feynman. Equations (4.13) and (4.14) of Matsubara's paper are equivalent to Eqs. (25) and (26) of Feynman's. Matsubara concludes that the lambda transition predicted by his theory is of the second order: this conclusion appears to us to be due to an error in the later stages of his analysis.

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Speculations on the Behavior of Positrons in Superconductors

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THE measurements of Graham and Bell[1] and those of Ferguson and Lewis[2] indicate that the mean lifetime of positrons in metals is constant and roughly independent of the metal. In other materials one may describe the observed facts by using two separate lifetimes for the positrons, indicating two distinct modes of annihilation. Inasmuch as there appears to be some empirical connection between the characteristics of the material and the type of positron annihilation process, one may inquire whether the annihilation characteristics of positrons are affected when a metal goes over from the normal to the superconductive state. A precise calculation of these characteristics should be based on a detailed description of the thermalization and annihilation processes in superconductive solids. Since even for a normal solid the mechanisms involved in these processes are only partially known[3,4] and since there does not appear to be a completely satisfactory wave function of a superconductor, it seems best to investigate the qualitative features of these processes. Several of the results are amenable to experimental verification, so one may decide on the basis of the experimental result whether a more detailed treatment is worth while.

1. It would appear that in so far as the lifetime of positrons depends on the average electron density only, the lifetime should not change in a superconductive transition, since the electron density is the same in the two phases (unless one wanted to exclude the "super electrons" *a priori* from taking part in the annihilation process, which seems unreasonable). It should be stressed however that even for normal metals the relationship between lifetime and electron density is not completely clear. In fact the experimentally obtained lifetimes are constant, whereas the electron densities may vary by a factor 20.[1] It may be shown[3] that the annihilation mechanism of a thermalized positron in a solid is determined by the matrix element $M_{ij}(p)=\int u_i^* v_j \times\exp(ipx/\hbar)dx$. Here p is the total momentum of the annihilating pair, u_i and v_j are the wave functions of the electron and positron in the lattice (specified by wave numbers i and j). To obtain a mean lifetime from this matrix element, one has to perform certain averaging processes over the electron and positron distributions. Now the wave functions u_i and v_j certainly change when the solid

goes from a normal to a superconductive state, and the electron distribution will be affected to some extent. Since the factors on which the lifetime depends change in the superconductive transition, it seems quite possible that the lifetime itself changes. Crude estimates indicate the lifetime might be lengthened.

2. It seems quite certain now[1,4] that positronium formation plays an important role in the annihilation process. In fact the experimental data can be explained best by assuming that the positrons slow down rapidly, then form positronium, and finally annihilation takes place from the ground state of the positronium. The long-lived triplet state of positronium will be converted rapidly to the singlet state, by the interaction of the positronium with the lattice vibrations, or by collisions with other electrons. One of the typical features of the superconductive state is the inability of the electron system to exchange energy with the lattice system or with impurities. It has been recognized[5] that in the superconductive state one has to deal with a wave function which is "adjusted" to the electron-lattice interaction, the electron-electron interaction and also to the interaction between the electron system and impurities. ("Adjusted" is used here in the sense that these interactions cannot cause transitions.) It seems quite possible therefore that the positronium exists in the system as some kind of impurity. In that case the interactions responsible for the conversion from the triplet to the singlet state would not operate, or would operate much less efficiently, in the superconductive state. Consequently the triplet state would persist longer in a superconductor than in a normal conductor. This would give rise to a complex lifetime decay of positrons in a superconductor. It is true that a second electron could in principle annihilate with the positron in the positronium; one finds, however, that when positronium formation itself is likely, this pick-off annihilation probability becomes quite small. Further, the average electron density around positronium is smaller than that around a positron, and this too reduces the efficiency of the "pick-off annihilations." (This type process was in fact invoked by Garwin[4] to explain the long-lived component of the positron lifetime in some materials.)

3. Experiments done by Stump and Talley[6] and independently by Millett[7] indicate that the decay of positrons in lead at liquid nitrogen temperatures differs from that at liquid helium temperatures. The experiments at liquid nitrogen tempreature yield the usual one-lifetime annihilation process (with the usual numerical value for metals), whereas those at liquid helium temperatures indicate a complex lifetime scheme with a longer-lived component.

4. On the basis of the model developed, one would expect more three-quantum annihilation processes in a superconductor since the triplet state has less chance to be converted. Experiments to check this feature are now in progress. If these same lifetime measurements were made at liquid helium temperatures in lead and if a magnetic field strong enough to destroy the superconductive state were applied, one should on the basis of previous considerations reobtain one-lifetime type annihilation. This last experiment would also decide whether the effects already observed are indeed characteristic of the superconductive state, or whether they represent some general property of metals at low temperatures.

Further details will be published in a more extensive communication.

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[6] The author is indebted to these authors for communicating their results before publication. These experiments will be reported at the Austin Meeting of the American Physical Society.
[7] The author wants to express his gratitude to Dr. Millett for informing him about these results in advance of publication. These results, too, will be reported at the Austin Meeting.

Creation of Displacements in Radiation Damage*

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A THEORETICAL investigation has been started into the formation and character of the displacements created in face-centered cubic metals by radiation damage. (Displacements, or Frenkel defects, are caused when energetic particles drive atoms of the lattice to interstitial sites leaving vacancies in their original positions.) Attention has been concentrated here on low-energy collisions because most of the displacements are generally created by collisions with energies only a few times the threshold—in the range of one or two hundred electron volts. The procedure has been to consider that an atom at a lattice site, or in an interstitial position, receives a considerable amount of energy at a given instant and to attempt to determine the end state of the lattice as a function of the magnitude and direction of the original impulse. In this way the threshold energy for displacement creation can be calculated and presumably, for a given distribution of primary particle energies, the distribution in length of the resulting displacements, their spatial distribution, and their total number could be obtained.

It has been assumed that for close-packed metals at low energies the principal interaction between colliding atoms is the repulsion of closed shells, which may be approximated by a semiempirical formula of the Born-Mayer type, $V(r)=A\exp\{-(r-r_0)\rho/r_0\}$. The constants in the closed-shell repulsive potential depend to some extent on the assumptions used in determining them. It has been shown[1] that for copper the value for ρ can be bracketed between 13 and 17. Since this force law is very short-range, the procedure has been to predict the result of a particular collision on the basis of a billiard ball model where the size of the balls is determined by the interatom distance at the instant of contact, i.e., when the relative kinetic energy vanishes. At these low energies many-body collisions are important, and reasonable techniques have been evolved to treat such cases in a manner consistent with the rigid sphere model. So far only collisions have been considered where the high-energy particle moves in a crystallographic direction. The high degree of symmetry in these cases facilitates their treatment. From their study, one finds two close competitors for the mechanism of displacement creation with minimum energy.

For ρ equals 13 (for which the corresponding value of A is 0.053 ev), it is found that an atom at a lattice site needs about 18.5 ev to move in the (111) direction through the triangle formed by three of its nearest neighbors to the interstitial position. Comparable energy (17.5 ev) is required by a lattice atom moving in the (100) direction for displacement creation in a "knock-on" mechanism where the original fast atom moves to another lattice site displacing its neighbor to an interstitial position. The history of all these collisions is rather complex and we have had to resort to reasonable, though somewhat arbitrary, criteria for determining the thresholds. A further difficulty is introduced by uncertainty as to the equilibrium configuration of the interstitial atom.[1]

Rough estimates have been made on the magnitude of these threshold energies on the basis of an alternate force law with $\rho=17$ and $A=0.038$ ev. They give 43 ev for the (111) mechanism and 34 ev for the knock-on collision. Experiments with electron radiation by Eggen and Laubenstein[2] report a threshold value of 25 ± 1 ev. Apparently this value lies well inside the rather wide limits set by the foregoing alternate calculations.

It was mentioned above that consideration was also being given to the history of energetic interstitial atoms. Such interstitials can be studied as typical atoms undergoing displacements longer then the minimum stable length. It appears that displacement length increases somewhat more rapidly than the logarithm of the energy.

Among the subjects planned for future study are (1) the replacement of the rigid body model by an improved treatment, and (2) the investigation of collisions undergone by atoms moving along directions of low symmetry. It appears that the rigid body