Letters to the Editor

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Elastic Scattering of 1-Mev Electrons from Aluminum and Gold

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KINZINGER and Bothe[1] have reported significant deviations from Mott theory in the large-angle scattering of 245-kev electrons from heavy nuclei. For gold at 150° they found that the scattering is more than 35 percent low. Subsequent measurements by Kinzinger[2] at 120° show a consistent deviation which is 25 percent low and is independent of energy between 150 and 400 kev. Paul and Reich[3] report similar behavior for 2.2-Mev electrons, the deviation being about 15 percent for platinum at 90° and 120°.

A preliminary measurement of the scattering of 1-Mev electrons by aluminum and gold has been made. Electrons from a Van de Graaff generator are focused on Parlodion (0.006 mg/cm²) backed foils of aluminum (0.22 mg/cm²) and gold (0.02 mg/cm²), and the relative number of electrons scattered at 30°, 60°, 90°, 120°, and 150° is measured. The two detectors are photomultipliers with anthracene crystals mounted at the end of collimators. An integral pulse-height discriminator is biased to minimize the x-ray and low-energy electron background so that the electrons elastically scattered from the foil nuclei are counted. The ratios of scattering at 30°, 60°, 120°, and 150° to that at 90° are measured and compared with the theoretically predicted values. Account has been taken of background counting rate, counting losses due to finite resolving time of the scaling circuits, multiple scattering, plural scattering (transmission-reflection asymmetry), and scattering in the Parlodion foil support. The maximum contribution for each of these was of the order of one percent and was in most cases less than this.

The results for aluminum and gold are given in Table I. The uncertainties shown are the standard deviations associated with counting. The results for aluminum are compared with the α^4 approximation to the Mott series given by McKinley and Feshbach[4] rather than the α^2 approximation formula, since the latter does not fit the experimental values as well as the more accurate α^4 approximation. For gold, the α^4 approximation is not sufficiently accurate, since $\alpha=Z/137=0.58$ and the series does not converge rapidly enough. The theoretical values given in Table I for gold were obtained by interpolating the calculations of Bartlett and Watson[5] for mercury and multiplying the result by the ratio of gold to mercury scattering given by the α^4 approximation. This procedure should give results accurate to about 1 percent.[4] As may be seen in Table I, the experimental results do not differ from these theoretical values by more than 1.5 percent and the differences exhibit no systematic trend. Thus, there is no evidence for deviation from the Mott theory of electron scattering at 1 Mev. Work is continuing on the measurement of relative scattering at other energies as well as of the absolute scattering.

TABLE I. Relative scattering with respect to 90°.

Angle	Aluminum Experimental	Theoretical	Deviation (%)
30°	89.06 ±0.27	89.08	−0.0
60°	5.504 ±0.011	5.496	+0.2
120°	0.2654 ±0.0007	0.2665	−0.4
150°	0.08572 ±0.00029	0.08544	+0.3
Angle	**Gold Experimental**	**Theoretical**	**Deviation (%)**
30°	39.50 ±0.12	39.87	−0.9
60°	3.898 ±0.009	3.840	+1.5
120°	0.3204 ±0.0008	0.3178	+0.8
150°	0.09610 ±0.00039	0.09693	−0.9

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Feynman's Theory of Liquid Helium

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RECENTLY, Feynman[1] has developed a new theory of liquid helium. In the present note we wish to show how the approximations that Feynman has to make to evaluate the partition function may be related to well-known approximations in statistical mechanics. We should also like to point out that Matsubara[2] has developed a theory of liquid helium using almost the same mathematical approximations as Feynman. It appears to us that the relationships that exist between Feynman's approximations and the standard approximations of statistical mechanics throw new light on the nature of Feynman's work.

We shall confine ourselves in this note to the first of Feynman's papers—hereafter referred to as I— in which he discusses the origin of the lambda transition of liquid helium. In I, three approximations are made in order to evaluate the partition function for the system. Feynman first argues that it is very reasonable to assume that the helium atoms move about as if they were practically free, and therefore that to a high degree of accuracy the distribution function in configuration space will be essentially the same as that for an ideal Bose-Einstein gas. This distribution function is proportional to

$$\Sigma_P \exp\Big[-(\pi/\lambda^2)\sum_{i=1}^{N}|\mathbf{z}_i-P\mathbf{z}_i|^2\Big],$$

where $\mathbf{z}_i$ is the coordinate of the ith atom, P is any permutation of the N indices, $\lambda^2=h^2/2\pi mkT$, and Σ_P stands for a summation over all permutations P. In order to take into account the effects of the interactions between the atoms Feynman proposes to modify this distribution function by replacing the true mass m by an effective mass m' and by multiplying it by an additional distribution function $K_\beta\cdot\rho_N(\mathbf{z}_1\cdots\mathbf{z}_N)$, where K_β is a normalization constant. The function ρ_N is stated to be "$\cdots$ qualitatively very similar to the distribution function for a classical gas." We can therefore write it in the form $\exp[-\beta V'(\beta)]$, where $V'(\beta)$ is a quasi-interaction potential, with properties similar to that for a classical system except that it depends slightly on the temperature. With these approximations the partition function reduces to

$$Q'=\lambda'^{3N}\frac{K_\beta}{N!}\int_{\mathbf{z}}\exp[-\beta V'(\beta)]\times\sum_P\exp\Big[-(\pi/\lambda'^2)\sum_{i=1}^{N}|\mathbf{z}_i-P\mathbf{z}_i|^2\Big]d\mathbf{z}, \quad (1)$$

where $\lambda'^2=h^2/2\pi m'kT$ and $\mathbf{z}$ stands for the set of all coordinates $(\mathbf{z}_i)$. We now wish to compare this approximate partition function with another that can be obtained from the semiclassical expansion due to Wigner[3] and Kirkwood.[4] These authors showed that the

partition function can be written in the form,

$$Q=(\lambda^{3N}/N!)\int_z \exp(-\beta V)\Big[\sum_P \exp\Big[-(\pi/\lambda^2)\times\sum_{i=1}^{N}|\mathbf{z}_i-P\mathbf{z}_i|^2\Big]+O(h^2)+O(h^4)\cdots\Big]d\mathbf{z}, \quad (2)$$

where V is the interaction potential of the atoms. If we ignore the terms of order $h^2,h^4\cdots$, etc., in the above expansion we get

$$Q''=(\lambda^{3N}/N!)\int_z \exp(-\beta V)\times\sum_P \exp\Big[-(\pi/\lambda^2)\sum_{i=1}^{N}|\mathbf{z}_i-P\mathbf{z}_i|^2\Big]d\mathbf{z}. \quad (3)$$

It is seen that Q' only differs from Q'' in that m is replaced by m' and $\exp(-\beta V)$ by $K_\beta \exp[-\beta V'(\beta)]$. In I it is stated that m' will probably not differ very much from the true mass m, near the λ temperature, and that K_β can be ignored as far as the nature of the transition is concerned. We have already noted that $V'(\beta)$ is fairly insensitive to changes in β and is qualitatively similar to the actual potential V. In other words the modifications to the zero-order term in Eq. (2) proposed by Feynman are quite small; we therefore have to consider whether the effects of all the other terms, that have been omitted, can be supposed to be small. This question is very difficult to decide, although it is quite easy to show that the coefficients of the leading terms of the series are large. However, even if we cannot evaluate any of the higher-order terms rigorously we can still argue that they are unlikely to remove any discontinuities that may be present in the derivatives of the first term of the series. This follows from the fact that the above series is an expansion in powers of $\Lambda^{*2}(=h^2/\sigma^2 m\epsilon)$, and therefore if the higher-order terms were to remove any discontinuities predicted from the first term this would imply a critical dependence on Λ^*. This would seem to be unreasonable on physical grounds. We therefore conclude that although the first term may be a poor approximation quantitatively it may still yield useful qualitative information.

Feynman next considers a typical term in the summation over all the permutations P in Eq. (1). Let us suppose that in this particular term there are m_l cycles of l "atoms" so that $\Sigma_l l m_l = N$. Then a typical term in the sum in Eq. (2) can be written,

$$\int_z \rho_N(\mathbf{z}_1\cdots\mathbf{z}_N)\prod_l\prod_{k=j}^{j+m_l l}\prod_{p=1}^{l}\exp[-(\pi/\lambda^2)|\mathbf{z}_{k+p}-\mathbf{z}_{k+p+1}|^2]d\mathbf{z}, \quad (4)$$

where the product Π_l is over a set of values of l such that $\Sigma_l l m_l = N$ and $\mathbf{z}_{k+l+1}\equiv\mathbf{z}_{k+1}$, Feynman now approximates to ρ_N by the product

$$\prod_l\prod_{k=j}^{j+m_l l}\prod_{p=1}^{l}\rho_2(\mathbf{z}_{k+p},\mathbf{z}_{k+p+1}),$$

where ρ_2 is proportional to the pair or "radial" distribution function. Clearly with this approximate form for ρ_N each of the terms in the sum of permutations can be evaluated explicitly, provided the form of ρ_2 is assumed. We shall now show that this approximation is closely related to the well-known superposition approximation of Kirkwood.[5] The superposition approximation can be formulated by saying that we approximate to ρ_N by the product

$$\prod_{i\le j=1}^{N}\prod\rho_2(\mathbf{z}_i,\mathbf{z}_j).$$

Basically, therefore, Feynman's approximation rests on this approximation. However, Feyman has also replaced all factors of the form $\rho_2(\mathbf{z}_i,\mathbf{z}_{i+p}),p>1$, in the superposition approximation, by unity. As he has pointed out, this means that we are completely neglecting (a) correlations between atoms that are not 'nearest neighbors' in a cycle and (b) correlations between all the atoms in any one cycle and those in any other cycle. It is well known that it is very difficult to estimate the validity of the superposition approximation. However it is very reasonable to assume that the more "gas-like" the structure of the liquid the more accurate the approximation will be. We may therefore be reasonably sure that for helium, which is a "gas-like" liquid with an abnormally low density, the approximation will be quite accurate. This conclusion lends support to Feynman's basic approximation and we would agree with him that the neglect of correlations between atoms in different cycles is probably much more important; it may be vital in deciding the order of the lambda transition. It is interesting to note that for a "classical liquid" the superposition approximation is, by itself, sufficient to allow us to calculate many of its properties. This is unfortunately not so for a "quantum liquid."

Finally we would like to mention that Matsubara[2] has developed a similar theory of liquid helium using essentially the same mathematical approximations as Feynman. Equations (4.13) and (4.14) of Matsubara's paper are equivalent to Eqs. (25) and (26) of Feynman's. Matsubara concludes that the lambda transition predicted by his theory is of the second order: this conclusion appears to us to be due to an error in the later stages of his analysis.

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Speculations on the Behavior of Positrons in Superconductors

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THE measurements of Graham and Bell[1] and those of Ferguson and Lewis[2] indicate that the mean lifetime of positrons in metals is constant and roughly independent of the metal. In other materials one may describe the observed facts by using two separate lifetimes for the positrons, indicating two distinct modes of annihilation. Inasmuch as there appears to be some empirical connection between the characteristics of the material and the type of positron annihilation process, one may inquire whether the annihilation characteristics of positrons are affected when a metal goes over from the normal to the superconductive state. A precise calculation of these characteristics should be based on a detailed description of the thermalization and annihilation processes in superconductive solids. Since even for a normal solid the mechanisms involved in these processes are only partially known[3,4] and since there does not appear to be a completely satisfactory wave function of a superconductor, it seems best to investigate the qualitative features of these processes. Several of the results are amenable to experimental verification, so one may decide on the basis of the experimental result whether a more detailed treatment is worth while.

1. It would appear that in so far as the lifetime of positrons depends on the average electron density only, the lifetime should not change in a superconductive transition, since the electron density is the same in the two phases (unless one wanted to exclude the "super electrons" *a priori* from taking part in the annihilation process, which seems unreasonable). It should be stressed however that even for normal metals the relationship between lifetime and electron density is not completely clear. In fact the experimentally obtained lifetimes are constant, whereas the electron densities may vary by a factor 20.[1] It may be shown[3] that the annihilation mechanism of a thermalized positron in a solid is determined by the matrix element $M_{ij}(p)=\int u_i^* v_j \times\exp(ipx/\hbar)dx$. Here p is the total momentum of the annihilating pair, u_i and v_j are the wave functions of the electron and positron in the lattice (specified by wave numbers i and j). To obtain a mean lifetime from this matrix element, one has to perform certain averaging processes over the electron and positron distributions. Now the wave functions u_i and v_j certainly change when the solid