The Altitude and Angular Dependence of Cosmic-Ray Air Showers*

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The angular distribution of the axes of large air showers incident upon a typical coincidence counter detector has been computed in detail from the altitude dependence.

The distribution may be approximated by the function $\cos^{5.5}\theta$ at 3500 meters elevation and by $\cos^{7.5}\theta$ at sea level. These distributions have been tested experimentally by comparing the rates of horizontally and vertically oriented counters. The vertical-horizontal counting ratio (R) was measured to be 2.12 ± 0.07 at 3500 meters, and 2.68 ± 0.07 at sea level. Values of R calculated from the above angular distributions are somewhat higher. The difference is attributed principally to scattering of shower particles about the primary axis.

THE angular distribution of extensive air showers is related to the processes involved in their formation. If they are formed solely by interactions with the matter of the atmosphere, the angular distribution must be related to their altitude dependence by a transformation similar to that first given by Gross. If, on the other hand, decay processes are important in transferring energy into or out of the showers, these decay processes will be more prevalent in the rare atmosphere traversed at large zenith angles, than in the same mass of air traversed vertically, and the angular dependence of the showers may be different from that deducible from the altitude variation by a Gross transformation.

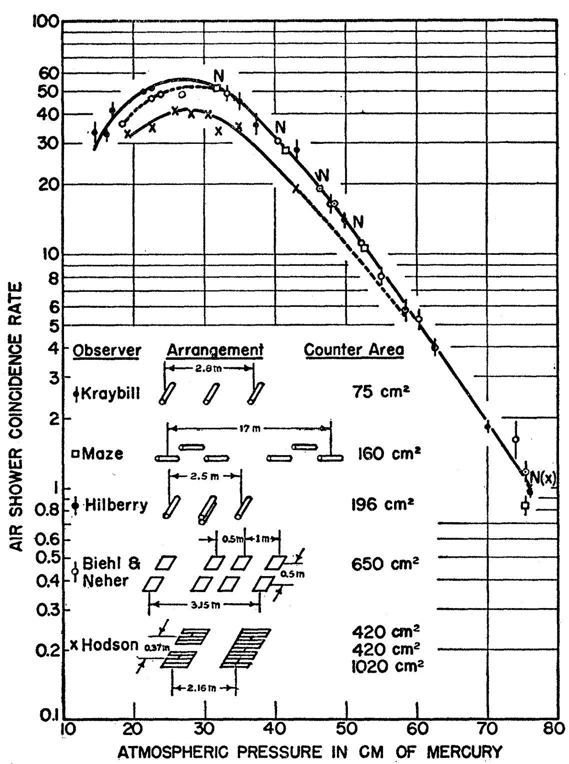

FIG. 1. Altitude dependence of extensive air showers. Each observer's data are normalized to the solid curve at the point marked N. The solid curve is normalized to unity at 76 cm of mercury.

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Previous measurements[1–5] of the angular distribution of air showers, performed mostly with cloud chambers, have yielded varying results. The results of Deutschmann, Daudin, and McKay and Brown are in approximate agreement with a Gross transformation from the known altitude dependence. Other observers,[3,5] however, have found distributions considerably steeper. I have, therefore, undertaken to examine closely the relation between angular distribution and altitude dependence of air showers, and to investigate the angular distribution by a different experimental method from those used previously.

This method consists of comparing the shower counting rate of long cylindrical Geiger counters with their axes oriented horizontally, to the rate of the same counters when their axes are vertical. This horizontal-vertical ratio is quite sensitive to the steepness of the zenith angle distribution of the incident showers. It was measured at mountain altitudes and at sea level, and it has been compared with values calculated theoretically from the angular distributions deduced from the altitude curve of large air showers. Results obtained at mountain altitudes have been reported previously.[6] Since then, measurements based upon the same method have been reported by Bassi *et al.*[7] The present work includes additional measurements at sea level, and explains the method of calculating the angular dependence from the altitude dependence.

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[2] M. Deutschmann, Z. Naturforsch. **2**, 61 (1947).
[3] R. W. Williams, Phys. Rev. **74**, 1689 (1948).
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ALTITUDE DEPENDENCE OF AIR SHOWERS

In Fig. 1, measurements of air shower altitude dependence by different observers,[8–12] using coincidence counters, are plotted on the same graph. Each observer's data are normalized to fit the solid curve at a single point. The solid curve has been drawn to fit all of the data as well as possible, and has been normalized to unity at 76 cm of mercury. In spite of differences in experimental arrangements, it is possible to draw a curve which is reasonably consistent with most of the data below 22 000-ft altitude. An altitude curve is thus determined which is insensitive to detector geometry within rather wide limits in the lower atmosphere.

This insensitivity of the altitude variation to counter geometry is related to the empirical observation that the variation of shower rate with counter area and with counter separation is similar at different altitudes, especially between mountain altitude and sea level.[13,14] This is particularly true for counter areas between 10 and 1000 cm^2 and for separations between 2 and 10 meters. All of the experimental arrangements used for the data of Fig. 1 fall within these ranges. On the other hand, the angular distribution changes with altitude, as shown in this paper. This factor should cause single horizontal cylinders to have a slightly steeper altitude variation in counting rate than a closely packed tray of cylinders, but the difference is only a few percent in the ratio of rates between sea level and mountain altitudes. At higher elevations, the zenith angle distribution should change rapidly near the maximum of the altitude curve. This will cause counter trays to show a maximum rate at lower altitudes than single cylinders, and a more rapid decrease in rate at altitudes above the maximum. The existence of this effect at altitudes above 22 000 ft is apparent in Fig. 1.

CALCULATION OF ANGULAR DISTRIBUTION

Shower detectors usually have a sensitivity which varies with incident direction of the shower axis. Therefore the angular distribution of showers counted by a detector will depend on its anisotropy. To obtain an angular distribution which is independent of detector shape, we will make the following definition. The angular distribution of showers *incident upon* a given detector is defined as: the directional variation of showers recorded by an isotropic detector having the same sensitivity in every direction which the actual detector has towards the zenith direction. This distribution will be denoted by the function $M(x,t)$, where

TABLE I. Comparison of normalized values of the directional dependence of Auger showers, deduced for three different counter shapes, for atmospheric pressure of 50-cm Hg. $M(x,t)$ is the directional dependence of showers incident upon the detector. $x=\cos\theta$, and t= atmospheric depth.

Zenth angle	$M(x,t)$, assuming isotropic detector	$M(x,t)$, assuming cylindrical counters	$M(x,t)$, assuming trays of counters
0°	1.000	1.000	1.000
10°	0.940	0.937	0.935
20°	0.750	0.746	0.742
30°	0.497	0.493	0.485
40°	0.238	0.235	0.229
50°	0.070	0.067	0.064

t is the depth below the top of the atmosphere, and $x=\cos\theta$ (θ= zenith angle).

The angular distribution defined in the above manner should be nearly independent of counter area and spacing for most commonly used areas (between 10 cm^2 and 1000 cm^2) and separations (between 2 m and 10 m) in the lower atmosphere. If this were not so, the change in counting ratio of different detectors with zenith direction would imply a corresponding change in their counting ratio with altitude. However, it has been noted above that this ratio is nearly constant with altitude.

The altitude curve of Fig. 1 was used to compute the angular dependence of air showers *incident upon* the detector at 50 cm and at 75 cm of Hg. Details of the computation are given in Appendix I. The results at 50 cm are given in Table I. This table shows that the deduced angular variation depends very weakly on the shape of detector.

The computation was based upon the following premises: (1) the shower primaries are isotropic at the top of the atmosphere, (2) the multiplication and disappearance of the particles depends only on the mass traversed, not upon the air density or time of traversal, and (3) the shower particles have the same direction as the incident primary. Further assumptions, which were used in refining the calculations, are (4) the shape of the lateral structure of the showers depends only upon the mass of air traversed, and the extent of the lateral structure is inversely proportional to the air density, (5) the variation of counting rate (C) with counter area (A), for showers incident from the zenith only, is given by $C\propto A^{1.5}$ at all atmospheric depths, and (6) the variation of counting rate with counter separation (S) for showers incident from the vertical direction is given by $C\propto S^{-0.2}$ at all atmospheric depths.

Assumption (5) is supported by the work of Cocconi *et al.*[13] Although they actually measured γ for showers from all directions rather than those incident only vertically, showers in the lower atmosphere come mostly from small zenith angles. Also, the near constancy of γ with atmospheric depth implies that it is also constant with direction at a given altitude. Assumption (6) is obtained from the work of Wei.[14] Deviations from

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[9] H. L. Kraybill, Phys. Rev. **73**, 632 (1948); Phys. Rev. **76**, 1093 (1949).
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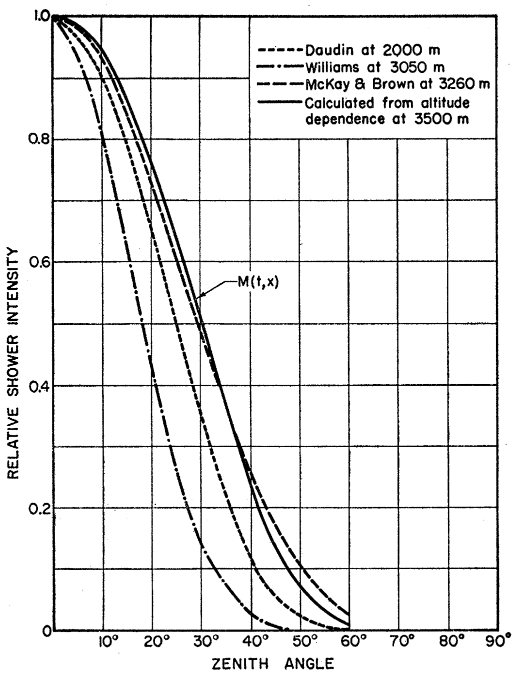

FIG. 2. Calculated and measured zenith angle dependence of extensive air showers at mountain altitudes.

assumption (3) certainly exist, but merely change slightly the effective directional sensitivity of the detector, to which the calculated angular distribution is quite insensitive (see Table I).

The computed angular distributions of the shower axes are represented by the solid curves in Figs. 2 and 3. Since the shower rate is not known for atmospheric depths greater than 76 cm, the sea level calculation had to be based upon an extrapolation of the altitude curve to greater depths. This was done by extending the lower part of the curve of Fig. 1, as a straight line on the semilogarithmic graph.

Experimental curves of the directional distribution, obtained by Deutschmann,[2] Daudin,[1] Williams,[3] and McKay and Brown[4] are also plotted in Figs. 2 and 3. The distributions deduced from the altitude curve are similar to those measured by Deutschmann, and by McKay and Brown, at sea level and at mountain altitude, respectively. The distribution found by Daudin is slightly steeper, and the distribution of Williams is considerably steeper, than the calculated curve.

EXPERIMENTAL RESULTS

Measurements of the shower rate of horizontal and vertical cylindrical counters were carried out at Climax, Colorado,[6] (49.5 cm of Hg) and at New Haven, Connecticut (76 cm of Hg). The arrangement at sea level is shown in Fig. 4. The geometry at Climax was similar, except that smaller counters were used and the counters at D were located at D' in Fig. 4. To minimize the effects of surrounding materials, the counters were placed under a light canvas tarpaulin at Climax. At New Haven they were placed in the open air on the flat roof of the Sloane Physics Laboratory, encased in light plastic bags. To avoid effects of inequalities of counter construction, vertical and horizontal counters were alternated periodically. The results are given in Table II. Table III shows values of R computed for assumed angular distributions of the form x^n ($x \equiv \cos\theta$). In Table IV, the measured horizontal-to-vertical counting ratios (R) are compared with values computed from the angular distributions of Figs. 2 and 3. In these computations, scattering of shower particles in the air was neglected, and all particles were assumed to move parallel to the shower axis. Details of computing these ratios are given in Appendix II.

The measured and computed horizontal-vertical ratios both indicate a steeper angular distribution at sea level than at mountain altitude. The measured ratios are lower than the ratios computed from the angular variation derived from the altitude curve, by about the same amount at each altitude. Before attributing significance to this difference, however, the following effects must be considered: (1) scattering of shower particles about the direction of their axis, (2) absorption and multiplication of particles in the counter walls, (3) backscattering from material below the counters.

Approximate calculations have been made of the effect of each of these factors on the value of (R). The

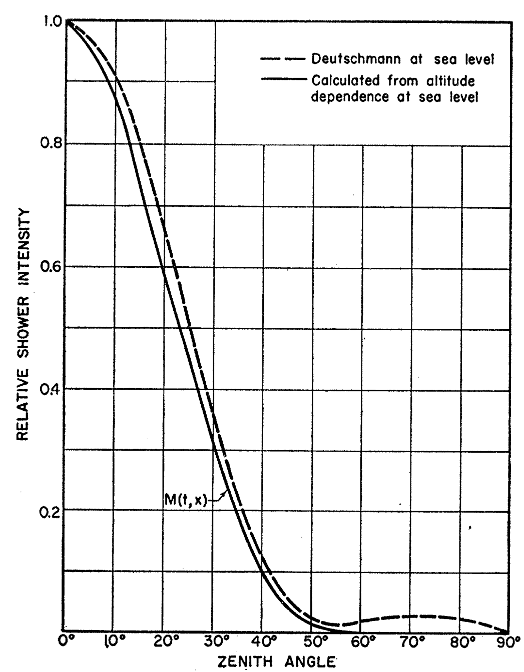

FIG. 3. Calculated and measured zenith angle dependence of extensive air showers at sea level.

numerical results of Richards, Roberg, and Nordheim,[15] on the number and angular distribution of low energy cascade electrons, were used. The calculations indicate a reduction in R of about 7 percent due to scattering in air, and a further reduction of 2 percent due to multiplication and absorption in counter walls. A very crude estimate of the effect of backscattering from the ground suggests a reduction in R of another 2 percent. The values of R, after these approximate corrections have been applied, are given in Table IV.

SUMMARY

The angular variation of the axes of extensive air showers incident upon Geiger counter detectors in the lower atmosphere has been computed in detail from the altitude dependence. This variation may be approximated by the function $\cos^{5.5}\theta$ at 3500 meters elevation and by $\cos^{7.5}\theta$ at sea level. An experimental test of these distributions, by comparing the rates of horizontally and vertically oriented counters, shows the predicted steepening of the angular distribution with increasing atmospheric depth. The measured horizontal-vertical counting ratios (R) are in fair agreement with the values calculated from the altitude dependence, after corrections for secondary effects, such as scattering of shower particles, have been considered. The results are consistent with the cloud chamber observations of McKay and Brown at 3250 meters, and with those of Deutschmann at sea level. They also agree well with results[16] recently obtained at 2000 m by Cresti, Loria, and Zago, who find a distribution of the form $\cos^n\theta$ with n in the range of 4 to 5, for the shower particles. There is fairly good agreement with the results of Bassi *et al.*[7] at 2000 m, although at sea level the angular distribution found here is steeper.

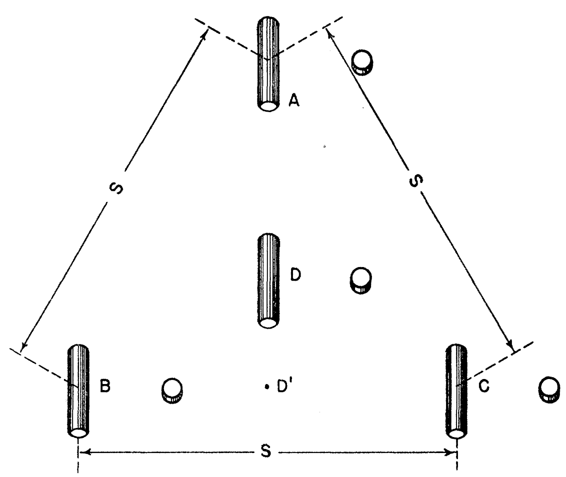

FIG. 4. Vertical view of coincidence counter arrangement used to test zenith angle dependence of air showers at sea level. Coincidence rates ABC and $ABCD$ of the horizontal counters were compared with the corresponding rates for the vertical counters. At sea level, $S=4$ meters. At 3500 m, $S=2.8$ meters, and the counter at D was at D'.

[15] J. A. Richards and L. W. Nordheim, Phys. Rev. **74**, 1106 (1948); J. Roberg and L. W. Nordheim, Phys. Rev. **75**, 444 (1949).

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TABLE II. Counting rates of arrangement of Fig. 4. The counters used at 3500 meters had active dimensions of 2.5 cm ×32 cm. Those used at sea level had dimensions of 5 cm×100 cm.

Coincidence combination	Vertical rate (hr^{-1})	Horizontal rate (hr^{-1})	Horizontal-to-vertical ratio (R)
3500 meters elevation			
$D'BC$	3.47±0.12	7.37±0.18	2.13
ABC	3.35±0.12	7.11±0.18	2.12
$ABCD'$	2.15±0.11	4.63±0.15	2.15
Sea level			
ABC	2.77±0.07	7.38±0.12	2.71
$ABCD$	1.78±0.06	4.83±0.10	2.66

TABLE III. Computed values of the ratio (R) of horizontal to vertical counting rates of cylindrical counters at atmospheric pressures of 50-cm Hg and at sea level. The counters at sea level have a ratio of 0.038 between their minimum and lateral cross sections. For the counters at 50-cm Hg, the ratio is 0.058.

$M(x,t)$	R(50 cm Hg)	R(sea level)
x^3	1.86	1.89
x^5	2.33	2.39
x^7	2.76	2.82
x^9	3.16	3.23

TABLE IV. Comparison of computed and measured values of R.

R, computed from angular distribution deduced from altitude curve	2.56	3.15
Computed R, after correcting for scattering and counter walls	2.31	2.84
Measured value of R	2.13±0.07	2.68±0.07

The absence of a marked discrepancy between the calculated and measured values of R suggests that decay processes do not greatly affect the angular distribution of air showers in the lower atmosphere.

APPENDIX I. DEDUCTION OF ANGULAR VARIATION FROM ALTITUDE DEPENDENCE

Let: t be the atmospheric depth measured along the vertical direction; x represent $\cos\theta$, where θ is the angle between the axis of a shower and the zenith; φ represent the azimuthal angle of the shower axis; β represent the angle between the direction of the shower axis and the direction of the axis of the horizontal counters; $N(t,x)$ be the counting rate per steradian of the shower detector at atmospheric depth t, averaged over all azimuths (φ) at constant zenith angle θ; $\rho(t)$ be the air density at depth t; and $f(x)$ represent the directional sensitivity of the detector, which is assumed the same at all altitudes. Then

$$N(t,x)=N(t/x,1)f(x)[\rho(t)/\rho(t/x)]^{2(\gamma-1)-0.2}f(x). \quad (1)$$

The factor involving the air density can be deduced from assumptions 1 to 6, by reasoning given elsewhere.[17] Taking the values given in assumptions 5 and 6, using the empirical expression $\rho \propto t^{0.9}$, one finds this factor is $x^{0.72}$. This was rounded to $x^{0.7}$.

It was found that $x^{0.5}$ is a good empirical approximation for $f(x)$ for the arrangement of Fig. 4, for zenith angles smaller than 60 degrees. The deduced angular dependence at 50 cm of Hg is in any case very insensitive to inaccuracy in this function. (See Table I.) Thus,

$$N(t,x) = N(t/x,\, 1)x^{1.2}. \quad (2)$$

Let $C(t)$ be counting rate of the shower detector at depth t. Then,

$$C(t) = 2\pi \int_0^1 N(t/x,\, 1)x^{1.2}dx. \quad (3)$$

The expression for $C(t)$ can be integrated by parts, following the standard procedure of the gross transformation.

One then obtains

$$N(t,1) = [2.2C(t) - tC'(t)]/2\pi. \quad (4)$$

Finally, from Eq. (2):

$$N(t,x) = N(t/x,\, 1)x^{1.2} = [2.2C(t/x) - (t/x)C'(t/x)]x^{1.2}/2\pi. \quad (5)$$

To obtain the zenith angle dependence of the incident showers, we must divide $N(t,x)$ by the directional sensitivity $(x^{0.5})$ of the detector, to get the zenith angle dependence:

$$M(t,x) = [2.2C(t/x) - (t/x)C'(t/x)]x^{0.7}/2\pi. \quad (6)$$

APPENDIX II. CALCULATION OF RELATIVE COUNTING RATES OF DIFFERENT SHAPED COUNTERS, FROM THE ZENITH ANGLE DEPENDENCE

A. General Method

The shower rate per steradian detected by an isotropic detector has been calculated in Appendix I, as $M(t,x)$. If the particles in the atmosphere are not scattered, the sensitivity of a set of counters to showers from a single direction will depend upon (1) the projected area $A(x,\varphi)$ of the counters on the plane perpendicular to the shower axis, and (2) the geometrical configuration and spacing of the projections of the counters upon that plane. Now, from assumption (5), the showers counted will vary as $A^\gamma(\gamma = 1.5)$. The effect of the change in counter spacing with zenith angle is very small for the geometry used, and it will be neglected. Hence, the total counting rate is obtained as

$$C(t) = \int_{x=0}^1 \int_{\varphi=0}^{2\pi} M(t,x)\left[\frac{A(x,\varphi)}{A(1,0)}\right]^{1.5} d\varphi dx.$$

[17] A. Daudin and J. Daudin, J. phys. radium **10**, 394 (1950). Compare also reference 11.

B. Horizontal Cylinders—Counting Rate, $H(t)$

$$A(x,\varphi)/A(1,0) = \sin\beta = (1-\cos^2\beta)^{\frac{1}{2}} = (1-\sin^2\theta \sin^2\varphi)^{\frac{1}{2}},$$

and

$$H(t) = \int_0^1 M(t,x) \int_0^{2\pi} [1-(1-x^2)\sin^2\varphi]^{0.75} d\varphi dx.$$

Expanding by binomial theorem and letting $(1-x^2) \equiv y^2$:

$$(1-y^2\sin^2\varphi)^{0.75} = 1 - 0.75(y^2\sin^2\varphi) - (0.75)(0.25)(y^4\sin^4\varphi)/2 - (0.75)(0.25)(1.25)(y^6\sin^6\varphi)/6 - \cdots;$$

$$\int_0^{2\pi} (1-y^2\sin^2\varphi)^{0.75}d\varphi = 2\pi[1-(3/8)y^2-(9/256)y^4 - (25/2048)y^6 - \cdots] = 2\pi f(x).$$

So,

$$H(t) = 2\pi \int_0^1 M(t,x)f(x)dx.$$

This equation can be integrated numerically. If the directional distribution $M(t,x)$ is some power α of $\cos\theta$, then,

$$H(t) = 2\pi \int_0^1 x^\alpha[1-(3/8)(1-x^2)-(9/256)(1-x^2)^2 - (25/2048)(1-x^2)^3 \cdots]dx$$

$$= 2\pi\left[\frac{1}{\alpha+1} - \frac{3}{4(\alpha+1)(\alpha+3)} - \frac{9}{32(\alpha+1)(\alpha+3)(\alpha+5)} - \frac{75}{128(\alpha+1)(\alpha+3)(\alpha+5)(\alpha+7)} - \cdots\right].$$

C. Vertical Cylinders—Counting Rate, $V(t)$

Let A be the cross-sectional area of the counter when cut by a plane containing the axis of the cylinder. Let A_e be the cross-sectional area of the counter when cut by plane perpendicular to the axis of the cylinder. The projected area of a vertical counter upon a plane perpendicular to a direction with zenith angle θ is given by

$$A\sin\theta + A_e\cos\theta.$$

So,

$$A(x,\varphi)/A = \sin\theta + (A_e/A)\cos\theta.$$

For the counters used at Climax, $A_e/A = 0.058$. (For the counters used at New Haven, $A_e/A = 0.038$.) So,

$$V(t) = 2\pi \int^1 M(t,x)(\sin\theta + 0.058\cos\theta)^{1.5}dx.$$

This equation can be integrated numerically without difficulty.