sideration of the energy separation of conversion electron lines, after the lines are assigned to the same activity from decay rate and excitation curve data.

More definitive experiments are needed, studying each activity separately and in more detail, particularly for the activities belonging to nuclei of mass less than 193.

We are indebted to many people for their generous help in this problem. In particular, we wish to thank W. F. Goodell and the operating crew of the Nevis cyclotron who made the bombardments for the early stages of this work; E. P. Tomlinson who made his high-resolution variable-field spectrometer available to us; W. M. Preston and the crew of the Harvard cyclotron for their hospitality and help in carrying out most of the bombardments; the crew of the Brookhaven 60-inch cyclotron for their irradiation of Pt for some subsidiary experiments; M. Slavin of Brookhaven for his help in running some of the photometry measurements; and M. Goldhaber for helpful discussions.

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Some Regularities in the Nuclear Level Spacings of Hg, Au, and Pt*

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An attempt is made to determine any systematic behavior of nuclear energy levels in a number of heavy elements, as the number of protons (for a fixed neutron number) or neutrons (for a fixed proton number) is changed. Certain regularities in the movement of levels are pointed out and discussed. The occurrence of "pure" $M1$ and mixed $M1+E2$ transitions (in odd-A nuclei) as related to the type of odd particle and the change in orbital angular momentum is summarized. Empirical evidence for L subshell conversion regularities for $M4$ and $E3$ multipole orders is given. An extension of this work is suggested.

I. THE DECAY SCHEMES AND LEVEL MOVEMENTS

THE recent studies of new isotopes of Hg and their daughter activities[1–3] make it possible to look for regularities of level movements in this region of the periodic table. Certain regularities in the Te-Xe-Ba region have been pointed out by Goldhaber and Hill.[4]

We should first remark on the validity of the spin assignments in the previous article.[3] Experimentally only the multipolarity of the radiations have been determined and, in principle, one must have a direct measurement of the spin of at least one state of a nucleus in order to be able to give the spins of the other levels. Since the intensity measurements are not very precise and since no coincidence measurements were done for most of the isotopes, we do not have a good knowledge of the branching ratios of the K captures and, therefore, it is hard to deduce the spins uniquely from the directly measured spin of the last stable isotope in the relatively long chains which have been investigated. We, therefore, have to make use of some general arguments of similarity and of more or less well-established regularities, such as the assignments $i_{13/2}\rightarrow f_{5/2}$ to the $M4$ transitions for odd-neutron nuclei near the end of the 126 shell. This, of course, is not the rigorous way of treating the experimental data which are supposed to prove the similarity in decay schemes. Strictly speaking, our discussion shows only the internal consistency of the regularities mentioned, suggesting that we are not too far from the truth.

The similarity in the decay schemes of the Au and Hg isotopes studied is outstanding (see the previous article). It is best seen when plotting the relative separation of pairs of levels j_1 and j_2 as a function of N or Z. We see (Fig. 1) that the separation of the odd neutron states

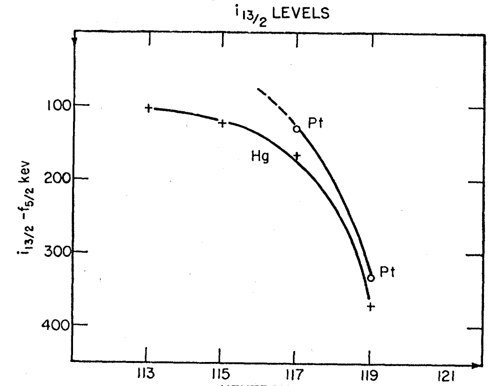

Fig. 1. Separation of $i_{13/2}$ and $f_{5/2}$ levels as a function of neutron number for Hg and Pt.

* Work supported by the U. S. Atomic Energy Commission.
[1] Douglas, Foster, and Thompson, Revs. Modern Phys. **25**, 469 (1953).
[2] J. H. Moon and A. L. Thompson, Phys. Rev. **83**, 892 (1951).
[3] Gillon, Gopalakrishnan, de-Shalit, and Mihelich, preceding paper [Phys. Rev. **93**, 124 (1953)].
[4] M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24**, 179 (1952).

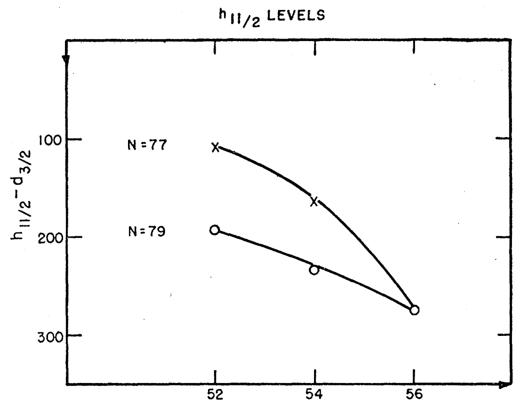

FIG. 2. Separation of $h_{11/2}$ and $d_{3/2}$ levels as a function of proton number, with the neutron number fixed at 77 or 79.

$f_{5/2}$ and $i_{13/2}$ increases as the number of neutrons is increased and that the increase is quite steep near the end of the shell. This behavior is completely analogous to the behavior of the $h_{11/2}-d_{3/2}$ neutron states near the end of the 82 shell.[4] The similarity is more striking in that an addition of two protons (going from Pt to Hg) increases the separation of the $i_{13/2}-f_{5/2}$ odd-neutron states in the same manner as the addition of two protons (change from Te to Xe to Ba) increases the separation of the $h_{11/2}-d_{3/2}$ neutron states (Fig. 2). The last effect has its complete analog in the odd-proton states $h_{11/2}-d_{3/2}$ and their relative movements as pairs of neutrons are added (Fig. 3). It is perhaps worth noting here that despite the paucity of odd-proton isomers between $Z=50$ and $Z=82$, it is not surprising that metastable $h_{11/2}$ states have appeared. For lower proton number, i.e., the rare earth region, the low-lying first excited states of the even-even core tend to destroy the metastability of the expected single-particle levels. But as the magic number 82 is approached, the energy of the even-even first excited

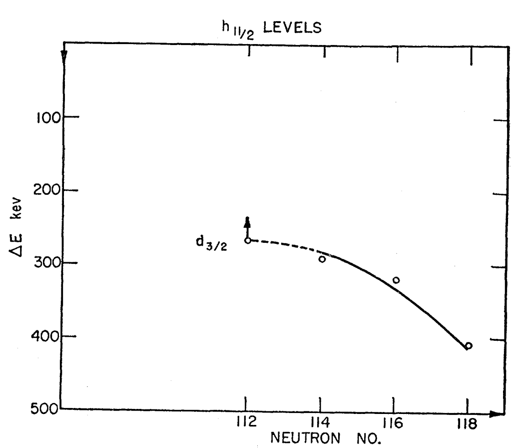

FIG. 3. Separation of the $h_{11/2}$ and $d_{3/2}$ levels in odd Au isomers as a function of neutron number.

states increase to several hundred kev so that low-lying single-particle isomeric levels are possible.

The behavior of the $d_{3/2}-s_{1/2}$ separation for a fixed proton number under the addition of pairs of neutrons (Fig. 4) is similar to the behavior of the same separation for a fixed neutron number under the addition of pairs of protons (see Fig. 79 of reference 2).

The interpretation of these movements always suffers from the lack of definite information as to whether the transitions studied are due to "holes" or "particles." More precisely, if a transition between, say, a $g_{7/2}$ state and a $d_{5/2}$ state is observed in a nucleus which contains 7 particles outside the closed shell of 50, it is hard to say whether the configurations involved are $g_{7/2}{}^7-d_{5/2}g_{7/2}{}^6$ (particle transition) or $d_{5/2}{}^6g_{7/2}-d_{5/2}{}^5g_{7/2}{}^2$ (hole transition). Clearly a clarification of this ambiguity is essential for an interpretation of the facts. Thus if one ignores the interaction between the nucleons (assuming that the approximate central field takes care of most

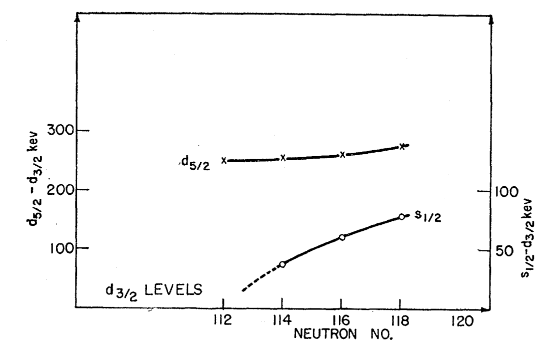

FIG. 4. Separation of $d_{5/2}$ and $d_{3/2}$ levels in odd Au isomers as a function of neutron number (left-hand ordinate scale); Separation of $s_{1/2}$ and $d_{3/2}$ levels for the same isotopes (right-hand ordinate scale).

of it), one gets for a "particle transition":

$$\Delta E_p = E(j_1{}^{2m+1}j_2{}^{2n}) - E(j_1{}^{2m}j_2{}^{2n+1}) = E(j_1) - E(j_2),$$

and for a "hole transition" (which is always associated with breaking one pair and creating another):

$$\Delta E_h = E(j_1{}^{2m+1}j_2{}^{2n}) - E(j_1{}^{2m+2}j_2{}^{2n-1}) = -E(j_1) + E(j_2) = -\Delta E_p,$$

where $E(j^k)$ is the energy of k nucleons in the state j, etc. One thus sees that, depending on whether the states differ by a particle or a hole, one state or the other will be the ground state. The introduction of an extra interaction between the nucleons, such as the pairing energy, complicates the relations somewhat, but a difference of essentially the same origin remains between a hole and a particle transition.

If in the above expressions j_2 is a lower (more tightly bound) state than j_1, then, ignoring as we did the interaction between the nucleons, we can expect a movement of the levels j_1 and j_2 relative to each other of the form shown in Fig. 5. The break in the line occurs at the value $n=n_2$= the maximum number of nucleons which one can put in the state j_2. In reality "the state j_2" is probably a mixture of the states $(j_1^{2m}j_2^{2n+1})$ and $(j_1^{2m+2}j_2^{2n-1})$ and perhaps some others. The configuration interaction which is responsible for this mixture will usually shift the energy levels considerably and it is hard to predict which will be shifted more and which less. From the experimental results on the movement of the $i_{13/2}$ levels relative to the $f_{5/2}$ levels or of $h_{11/2}$ relative to the $d_{3/2}$ levels, one may conclude that the high-spin states ($i_{13/2}$ and $h_{11/2}$) are more affected by the configuration interaction than the low-spin ones ($f_{5/2}$ and $d_{3/2}$), which conclusion is perhaps not unreasonable.

There is another point regarding the movement of levels which is worth emphasizing. It is concerned with the movement of the spin doublets. In the Hg and the Pt nuclei we can follow the movement of the $p_{3/2}$ and $p_{1/2}$ states, and in the Au we can follow the $d_{5/2}$ and the $d_{3/2}$ states. Although we do not see both the $p_{3/2}$ and $p_{1/2}$ states in all the Hg's and the Pt's, Fig. 6 suggests that their crossing occurs rather sharply, since $p_{3/2}$ is the lowest state in Hg^{195} and Hg^{193}. It seems to be too early to make any speculations about this. The $d_{5/2}-d_{3/2}$ separation in the Au's is remarkably constant over this large range (Fig. 4). This is particularly interesting in view of the attempt[5] to explain the spin-orbit interaction in terms of second-order effects of tensor forces. A core in a non-S state should be assumed in order to get nonvanishing results, and one might expect that a change in the number of nucleons would change the different modes of excitation of the core, and therefore the spin doublet separation appreciably.

It has been pointed out already[6,7] that the matrix elements for the $M4$ transitions increase as one approaches magic numbers. A similar effect has been observed in β-decay matrix elements[8] and was interpreted as due to a "purification" of the single-nucleon wave function as one approaches a magic number. Although our present results suffer from a lack of an accurate knowledge of the K capture branching from the isomeric states in the Hg's, as do the data on Pt and Pb,[4] it is interesting to note that they too suggest this general trend (see Table I).

However, one should be careful with the interpretation of such results. The values of $|M|^2$ were obtained by comparing the observed half-lives (corrected for conversion: $\tau_\gamma = T_{1/2}(1+\alpha_{\text{tot}})/\ln 2$) with those deduced from the Weisskopf[9] formula for single-nucleon transitions. If some corrections which should probably be applied to the Weisskopf formula turn out to be energy dependent, one cannot say whether the trend observed

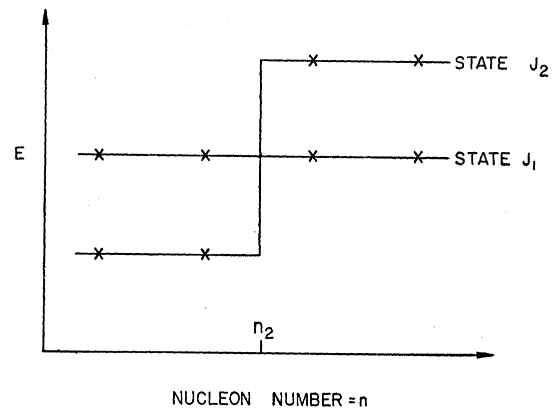

FIG. 5. The theoretical relative movement of two single-particle levels j_1 and j_2 (odd-A nuclei), as nucleons are added. The break occurs at the number of nucleons $n_2=2j_2+1$, the maximum number of nucleons one can put into the state j_2.

in $|M|^2$ is due to real nuclear effects or to a continuous change in our standard with which we compare the observed half-lives, since the energies associated with the transitions considered also show a very regular trend. The existence of similar regularities in β^- decay, however, supports the assertion that at least part of the effect in the $M4$ transitions is of a real nuclear structure origin.

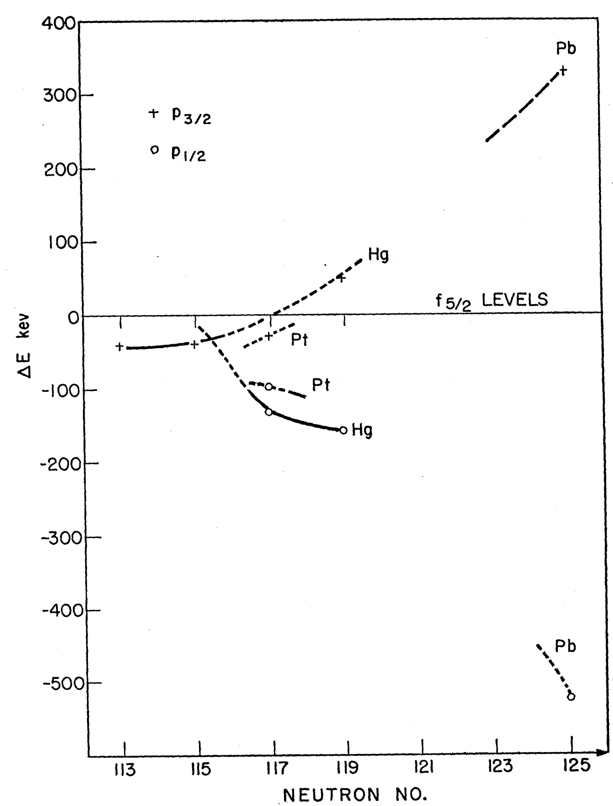

FIG. 6. The separation of the $p_{3/2}-f_{5/2}$ and $p_{1/2}-f_{5/2}$ levels as a function of neutron number for Pb, Hg, and Pt.

[5] A. Feingold, Ph.D. thesis, Princeton University, 1952 (unpublished).
[6] S. A. Moszkowski, Phys. Rev. **89**, 474 (1953).
[7] K. Gottfried, thesis, McGill University, 1953 (unpublished).
[8] A. de-Shalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953).
[9] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

TABLE I. The calculations for $|M|^2$ for some $M4$ transitions in heavy nuclei.

	E Mev	$T_{\frac{1}{2}}$	% via I.T.	α_K	K/L	α_{tot} [a]	$\tau_{\gamma\,exp}$ (sec)	$\lvert M\rvert^2$
$_{82}Pb_{125}{}^{207}$	1.063	0.9 sec	100	0.103	5.2	0.13	1.47	1.54
$_{80}Hg_{119}{}^{199}$	0.368	44 min	100	4.4	1.6	8.0	3.4×10^4	1.00
$_{80}Hg_{117}{}^{197}$	0.165	23 hr	97	87	0.45	370	4.7×10^7	1.02
$_{80}Hg_{115}{}^{195}$	0.122	38 hr	48	275	0.2	2062	8.3×10^8	0.88
$_{80}Hg_{113}{}^{193}$	0.101	12 hr	20 ± 10	560	0.12	6608	2.06×10^9	
$_{78}Pt_{119}{}^{197}$	0.337	80 min	100	5.2	1.3	10.4	7.8×10^4	0.98
$_{78}Pt_{117}{}^{195}$	0.129	3.8 d	100	200	0.25	1575	5.8×10^8	0.77

[a] $\alpha_{tot}=\alpha_K(1+1.3L/K)$; the factor 1.3 is to take into account M conversion.

II. THE MIXING OF $E2$ and $M1$ RADIATIONS

It has become evident that measurements of the relative conversion of the three L subshells is a useful tool for the analyzing of the multipolarity of gamma-ray transitions. In particular, both empirical[10,11] and theoretical[12] evidence shows that $M1$ and $E2$ transitions convert entirely differently in the L subshells for heavy nuclei. One is now able to more or less reliably analyze the amount of mixing of $E2$ with $M1$ radiation. Since several $M1$ and $M1+E2$ transitions are reported in the previous work, it may be useful to make a brief summary of these transitions, along with others previously reported. It is worth pointing out that on the single-particle model, the only magnetic dipole transitions "allowed" are those between the state $j=l\pm1/2$ to $j'=l\mp1/2$ (i.e., the same l). Thus, the nuclear decay from the state $f_{5/2}$ to the state $p_{3/2}$ should proceed via electric quadrupole rather than magnetic dipole, on a strict one-particle model.

It has been pointed out by Sachs and Ross[13] that the existence of $M1$ transitions with $\Delta l=2$ gives strong evidence for a departure from additivity of the intrinsic nucleon moments, and thus imposes certain restrictions on the possible forms of the interaction moment. It is therefore interesting to check whether or not these transitions behave like the $M1$'s for which $\Delta l=0$.

TABLE II. Relatively pure and mixed $M1$ radiation in odd-A nuclei. Energy (kev) in parentheses. In square brackets is given the ratio of magnetic γ to electric γ intensities, as deduced from L conversion ratios.[a]

	Odd proton		Odd neutron	
	$\Delta l=0$	$\Delta l=2$	$\Delta l=0$	$\Delta l=2$
"Pure" $M1$	Li^7(477) Au^{191}(253) Au^{193}(259) Au^{195}(262) Au^{197}(279)	Cs^{133}(81) Cs^{135}(247) Pm^{147}(91)	Pt^{195}(99) Hg^{199}(209) Fe^{57}(14)	Hg^{193}(39.0) Hg^{195}(36.7) Hg^{199}(51) Pt^{195}(29) Te^{121}(213) Te^{123}(159) Te^{125}(35) Xe^{131}(80) Sn^{117}(159) Sn^{119}(24)
Mixed $M1+E2$	Ir^{191}(129) [5:1]	Au^{197}(77.4) [7:1] Au^{195}(61.4) [5:1] Au^{193}(37.9) [?] Ir^{191}(82.0) [1.5:1]		

[a] *Note added in proof.*—The transitions listed are those for which the experimental data are good enough to indicate the possibility of mixing *and* for which the change in orbital momentum has been specified.

In Table II we have listed various $M1$ transitions in odd-A nuclei which are "pure" or mixed (with $E2$) classified with respect to the odd particle making the "jump" and the change in orbital momentum (i.e., $\Delta l=0$ or 2). The "pureness" of the $M1$ is a relative quantity; however, the transitions so listed should have less than 5 percent $E2$ admixture. It can be seen that the pure $M1$ γ rays in odd-Z nuclei are not predominantly associated with either $\Delta l=0$ or 2. All observed mixed low-energy transitions are for odd Z and, with one exception, are characterized by $\Delta l=2$. The relative intensities of $M1$ and $E2$ radiation in all these mixed transitions are of roughly the same order of magnitude (the ratio of electric γ rays to magnetic γ rays being ≥0.14). We may thus conclude that $\Delta l=2$ $M1$'s are somewhat slower than $\Delta l=0$ ones.[14] For odd-neutron nuclei all listed transitions are "pure" $M1$; this might indicate that $E2$ transitions for neutrons are slower than for protons, since otherwise we could have expected them to compete favorably with the $M1$ transition.

It is evident that this table should be extended by measurements in other regions of the periodic table. If the regularities just mentioned here persist, then we shall have another indication as to the extent to which a single-particle wave function can describe the low-lying states of the odd-even nuclei.

III. L-SHELL CONVERSION SYSTEMATICS

Below are tabulated some additional empirical data on L-shell conversion. Considerable evidence supporting the calculations of Gellman, Griffith, and Stanley[12] is available for $M1$ and $E2$ transitions and will not be repeated here. For $M4$ and $E3$ transitions new and useful data have been obtained and are listed in Table III.

It will be noted that the increase of $\beta_{L_{III}}/\beta_{L_I}$ for $M4$ transitions as Z^2/E increases is predicted by the theoretical calculation of Tralli and Lowen.[12] However, most of these data are for transitions in nuclei of about the same atomic number, so that what is actually shown is the trend with energy.

[10] J. W. Mihelich, Phys. Rev. **87**, 646 (1952).

[11] J. B. Swan and R. D. Hill, Australian J. Phys. (to be published).

[12] Gellman, Griffith, and Stanley, Phys. Rev. **85**, 944 (1952).

[13] R. G. Sachs and M. Ross, Phys. Rev. **84**, 379 (1951).

[14] R. L. Graham and R. E. Bell, Can. J. Phys. **31**, 377 (1953).

For $E3$ transitions, the L_{II}/L_{III} ratio increases with increasing energy, the same behavior having been noted for $E2$ transitions.[10]

Finally, we wish to point out further evidence for a Z dependence of K/L ratios for $M1$ transitions. As mentioned by Goldhaber and Sunyar[15] and Swan and Hill,[11] for high Z, K/L ratios for $M1$ transitions are less than for low Z. For the $M1$ transitions in Au ($Z=79$, $E=252$ to 279 kev),[3] K/L is $\simeq 5$, whereas for low Z and the same Z^2/E one could expect a ratio of ~ 8.

Suggestions for Future Work

The work presented here and in the previous paper is far from complete. The main purpose was to show a possible way for a systematic study of nuclear spectra and the sort of results one may expect. Because of the rather small number of transitions usually observed in a single nucleus, it seems more promising to look for regularities in a whole series of similar nuclei and the method described here is most adequate for this purpose. Provided the separation of the element $Z+1$ from the element Z can be done with very little carrier (or better with no carrier at all), this method presents no exceptional difficulties and can certainly be improved once a systematic search is undertaken.

An important problem which did not get its final answer in the present work is that of the relative movement of the spin-doublet states $p_{3/2}$ and $p_{1/2}$. Their crossing at a certain neutron number seems to be well established, but their movement to the left of the crossing point (smaller neutron number) is not yet clear. (See Fig. 6.) A complete picture might cast some light on the origin of the spin-orbit interaction in nuclei or at least exclude some otherwise possible approaches. It would seem that the study of the neutron-deficient isotopes of Pb (produced by high-energy bombardment of Tl), as well as those of Pt (produced by the (p, xn) reaction on Ir) would contribute important data to the clarification of this point as well as others.

[15] M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951).

TABLE III. Experimental relative intensity of L-subshell conversion for $M4$ and $E3$ transitions.

$M4$	Nucleus	T(kev)	Z^2/E	L_I	L_{II}	L_{III}	Reference
	Ba^{137m}	661	5	1	...	~ 0.12	a
	Sn^{117m}	155	16	1	...	~ 1	b
	Te^{125m}	109	25	1	...	~ 1	b
	Te^{123m}	88.5	31	1	...	~ 2	b
	Hg^{197m}	165	39	1	<0.1	1.5	c
	Pt^{193m}	136	45	1	...	2	d
	Hg^{195m}	122	53	1	...	2	e
	Hg^{193m}	101	63	1	...	3.5	e
$E3$	Nucleus	T(kev)		L_I	L_{II}	L_{III}	Reference
	Au^{193m}	32		...	0.67	1	e
	Au^{195m}	57		...	1.1	1	e
	Dy^{165m}	108		...	1.5	1	b, f
	Au^{197m}	130		<0.1	2.5	1	e

[a] W. Kinney (private communication from G. N. Glasoe).
[b] See reference 10.
[c] J. W. Mihelich and A. de-Shalit, Phys. Rev. **91**, 78 (1953).
[d] Swan, Portnoy, and Hill, Phys. Rev. **90**, 257 (1953).
[e] See reference 3.
[f] Jordan, Cork, and Burson, Phys. Rev. **91**, 497 (1953).