anthracene crystal, which would also produce a coincident particle for the second detector, is $C^{12}(n,n)3\alpha$ having a threshold of 7.88 Mev as computed from the isotopic mass values.[4] It can be shown that such processes may distort the neutron spectrum below 1.3 Mev.

Various spectral distributions obtained for Po-Be neutrons have been reported in the literature.[5–10] Of these spectra the results of Whitmore and Baker are in good agreement with our results. Location of intensity maxima as reported by Gursky *et al.* also agrees within experimental error with our results.

The authors wish to thank Dr. E. H. Krause for his interest and support and to express their appreciation to W. L. Myers for aid in performing the experiment.

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Some Possible Relationships between π-Meson Nucleon Scattering and π-Meson Production in Nucleon-Nucleon Collisions

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For a reaction such as $p+p\rightarrow\pi^{+}+n+p$, it is known that the interaction of the emitted neutron and proton will frequently result in the formation of a deuteron. An analogous effect is that of the interaction of the π meson with either the neutron or the proton. This interaction is known to be strong from studies of meson-nucleon scattering. Explicit calculations are made, which indicate that pronounced qualitative effects may indeed result from the meson-nucleon interaction. In particular, the $p+p\rightarrow\pi^{+}$ cross section is expected to be considerably larger than is the $n+p\rightarrow\pi^{+}$ cross section.

I. INTRODUCTION

As is well known, there is an implication from strong coupling meson theory[1] that the state of the meson-nucleon system which has an isotopic spin of $\frac{3}{2}$ and a spin of $\frac{3}{2}$ [to be designated as the $(\frac{3}{2},\frac{3}{2})$ state] should be one of strong interaction. An analysis accepting this possibility for the pion-nucleon scattering[2] made by Brueckner[3] has led to a very reasonable qualitative explanation of the magnitudes of these cross sections. A field-theoretic calculation by Chew[4] has led to results in general agreement with these suggestions.

Further implications of a strong interaction in the $(\frac{3}{2},\frac{3}{2})$, state of the meson-nucleon system have been suggested[5] for photomeson production. These have been in not unreasonable agreement with observed[6] angular distributions and magnitudes of the cross sections.

The modest successes of these suggestions would seem to indicate that it is worth seeking further implications of the hypothesized strong $(\frac{3}{2},\frac{3}{2})$ interaction. In this connection two suggestions have been made in respect to pion production in nucleon-nucleon collisions. The first of these[5] concerned the reactions

$$p+p\rightarrow\pi^{+}+d, \tag{A}$$

$$p+p\rightarrow\pi^{+}+n+p. \tag{A'}$$

To a first approximation, the angular distribution in the rest system and near the energetic threshold should be of the form

$$1+3\cos^2\theta,$$

where θ is the angle between the meson momentum vector and that of one of the incident protons. This is in rough agreement with measured cross sections.[7] The second suggestion in this connection[8] was that for the

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reactions

$$n+p\rightarrow\pi^{+}+n+n, \qquad \text{(B)}$$

$$n+p\rightarrow\pi^{-}+p+p, \qquad \text{(B}')$$

the $(\frac{3}{2},\frac{3}{2})$ state of the meson with respect to either nucleon is expected to be inhibited near energetic threshold because of angular momentum and parity considerations.[9] This might be expected to lead to qualitative differences between these and reactions (A) and (A′). Indeed, a study of pion production from the bombardment of deuterons by protons[10] suggests that the ratio of cross sections (A) and (A′) to the cross sections (B) or (B′) may be as great as 20:1 (see reference 8). In any case, the cross sections (B) and (B′) (these are expected by charge symmetry to be equal, except for Coulomb effects) are subject to direct measurement, although this does not seem to have been done as yet.

The above suggestions concerning the role played by the $(\frac{3}{2},\frac{3}{2})$ state in pion production by nucleon collisions were purely qualitative, there being no explicit calculations. The purpose of the present note is to present such a calculation much along the lines of that of Chew[4] for the pion-nucleon scattering. This analysis will fall into the general category known as the "Tamm-Dancoff" method and will be carried out along specific lines recently suggested.[11]

The results do, indeed, bear out the expected qualitative features, both as to the angular distributions and the magnitudes of the cross sections. The approximate cross sections are summarized in Eqs. (46) and Fig. 3.

II. CALCULATION OF THE SCATTERING AMPLITUDE

It is clearly impossible at present to make any calculation in meson field theory which can strictly be termed "quantitative." On the other hand, semiquantitative and qualitative field theoretic analyses have been of considerable use in the planning and analyzing of experiments. It is in the spirit of this latter category that the present work is presented.

Various perturbation calculations[12] of the nuclear production of mesons have been made. These have been in "order of magnitude" agreement with the observed cross sections and their dependence upon energy. Among the perturbation transitions is one which describes the emission of a meson by one nucleon, which is then "scattered" by the second nucleon before finding itself "free." We shall treat this "scattering" by the Tamm-Dancoff method along the lines used by Chew.[4] Aside from this, the calculation will be simply a third order (i.e., lowest order) perturbation calculation, except for our treatment of the nuclear force between the nucleons.

In the subsequent analysis we shall, for instance, (1) neglect multiple meson exchanges between the two nucleons; (2) treat all but the meson scattering in the $(\frac{3}{2},\frac{3}{2})$ state as a weak perturbation; (3) neglect radiative corrections; (4) treat the nucleons nonrelativistically and neglect their recoil except where this is most obviously unreasonable.

Our primary justification for such arbitrary omissions is that it seems difficult, if not impossible, to make a completely consistent calculation of any one of these effects. Consequently, we shall concentrate only on the "final scattering" of this emitted meson, which provides the motivation for the present work. Neglect of effects (1) and (3) above did not seem to change the qualitative features of the calculation in an analysis of the low-energy nuclear forces.[13] The approximation (2) would not seem to change Chew's conclusions in a qualitative manner. Finally, one can probably justify the neglect of nucleon recoil in a theory which cuts off momentum space integrals at sufficiently low values.

A. Formulation of the Problem

We suppose the Hamiltonian describing the meson-nucleonic system to be

$$H=H_0+H', \qquad (1)$$

where H_0 is the energy of the noninteracting mesons and nucleons and

$$H'=h_1+h_2, \qquad (2)$$

with

$$h_1=\frac{g}{2M}[\boldsymbol{\sigma}^{(1)}\cdot\nabla_1\boldsymbol{\phi}(\mathbf{z}_1)\cdot\boldsymbol{\tau}^{(1)}],$$
$$h_2=\frac{g}{2M}[\boldsymbol{\sigma}^{(2)}\cdot\nabla_2\boldsymbol{\phi}(\mathbf{z}_2)\cdot\boldsymbol{\tau}^{(2)}]. \qquad (3)$$

M is the nucleonic mass, $\mathbf{z}_1$ and $\mathbf{z}_2$ are the space coordinates of the two nucelons and the $\boldsymbol{\sigma}$'s and $\boldsymbol{\tau}$'s are their respective spin and isotopic spin operators. We may suppose Eqs. (3) to have been derived from the pseudoscalar coupling term by the Dyson[14] transformation, if we care to consider the higher-order terms as negligible.[15]

The process to be considered is that in which the two nucleons with respective momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ collide to produce a meson with momentum $\mathbf{q}$, leaving the two nucleons with final momenta $\mathbf{p}_1'$, and $\mathbf{p}_2'$. It will be convenient to consider the process in the rest system:

$$\mathbf{p}_1+\mathbf{p}_2=0, \quad \text{or} \quad \mathbf{p}_1\equiv\mathbf{p}=-\mathbf{p}_2, \qquad (4)$$

$$\mathbf{p}_1'+\mathbf{p}_2'+\mathbf{q}=0. \qquad (5)$$

[9] This is because the final nucleons are expected to be primarily in a ^1S state and the meson in a P state with respect to these. Consequently, the initial state must be an admixture of $^3S_1$ and $^3D_1$, or an isotopic spin zero state. However a final state of total isotopic spin zero cannot be formed if the meson forms a state of isotopic spin $\frac{3}{2}$ with either nucleon.

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[11] K. Brueckner and K. Watson, Phys. Rev. **90**, 699 (1953).

[12] K. Brueckner, Phys. Rev. **82**, 598 (1951).

[13] K. Brueckner and K. Watson, Phys. Rev. **92**, 1023 (1953).

[14] F. J. Dyson, Phys. Rev. **73**, 929 (1948).

[15] See, for instance, Brueckner, Gell-Mann, and Goldberger, Phys. Rev. **90**, 476 (1953).

The relative momentum of the two nucleons in the final state is

$$\mathbf{p}'=\tfrac{1}{2}(\mathbf{p}_1'-\mathbf{p}_2'). \quad (6)$$

$\mathbf{q}$ will be taken as the relative momentum of the meson with respect to the center of mass of the two nucleons. (We shall consistently drop terms of relative order $\mu/2M$, where μ is the rest mass of the meson.)

The total energy of the system is

$$E_a=p^2/M, \quad (7)$$

and we shall suppose this to be sufficiently near the energetic threshold that terms of relative order $(E_a-\mu c^2)/\mu c^2$ can be neglected.

Following the notation of Brueckner and Watson[11] we define

$$a\equiv E_a+i\eta-H_0, \quad (8)$$

where η is an infinitesimal positive parameter. The part of the Møller wave matrix which describes meson production is

$$\Omega_p=\Omega_s(1/a)H', \quad (9)$$

where Ω_s is diagonal in particle occupation numbers and H' produces a meson [see Eq. (28) of reference 11]. We may describe Eq. (9) by saying that H' produces the meson, after which Ω_s scatters the meson and the two nucleons into the final observed state. Equation (9) is formally exact if Ω_s satisfies the Lippmann-Schwinger equation:

$$\Omega_s=1+(1/a)\mathcal{V}\Omega_s, \quad (10)$$

where $\mathcal{V}$ is the interaction potential between the three particles.

There will be contained in $\mathcal{V}$ the nuclear force interaction V_N between the two nucleons.[16] We define the remainder of $\mathcal{V}$ to be v by the equation

$$\mathcal{V}\equiv V_N+v. \quad (11)$$

By means of a little algebraic manipulation we can rewrite Eq. (9) as

$$\Omega_p=\frac{1}{a-\mathcal{V}}H'=\frac{1}{a}w^{(-)\dagger}\left[1+v\frac{1}{a-V_N-v}\right]H'. \quad (12)$$

$w^{(-)\dagger}$ satisfies the Lippmann-Schwinger equation:

$$w^{(-)\dagger}=1+w^{(-)\dagger}V_N\,1/a, \quad (13)$$

and so describes the scattering of the two nucleons in the final state. Indeed, if we define the final state nuclear wave function to be[17]

$$\phi^{(-)}=w^{(-)}\chi_{p'},$$

with

$$\chi_{p'}=(2\pi)^{-\frac{3}{2}}\exp\{i\mathbf{p}'\cdot(\mathbf{z}_1-\mathbf{z}_2)\}, \quad (14)$$

then $\phi^{(-)}$ satisfies the Schrödinger equation (in a momentum representation):

$$\left[\frac{l^2}{M}-\frac{p'^2}{M}\right]\phi^{(-)}(l)=-\int V_N(\mathbf{l},\mathbf{l}')\phi^{(-)}(l')d^3l'. \quad (15)$$

We designate the plane wave function for the relative motion of the incoming nucleons and the outgoing meson by χ_p and χ_q, respectively. The coefficient of $1/a$ in Eq. (12) is the transition matrix T. For our process this is then:

$$T=\left(\phi^{(-)}\chi_q,\left[1+v\frac{1}{a-V_N-v}\right]H'\chi_p\right). \quad (16)$$

Although this has been derived only when $\phi^{(-)}$ describes a scattering state of the two nucleons, the result is easily seen to hold when they are bound to form a deuteron.[18] In this case, $\phi^{(-)}$ is just the deuteron wave function.

To simplify Eq. (16), we consider the term

$$\begin{aligned}(\phi^{(-)}\chi_q,H'\chi_p)&=\phi^{(-)}(\mathbf{p}\pm\mathbf{q})(\mathbf{q},\mathbf{p}\pm\mathbf{q}\,|\,H'\,|\,\mathbf{p})\\&\simeq\phi^{(-)}(\mathbf{p})(\mathbf{q},\mathbf{p}\pm\mathbf{q}\,|\,H'\,|\,\mathbf{p}).\end{aligned} \quad (17)$$

Now for $p'^2/M\ll p^2/M$, it is reasonable to neglect the l' in V_N in Eq. (15)[19] and to write that equation as

$$\begin{aligned}\frac{p^2}{M}\phi(p)&\simeq-V_N(p)\int\phi(l')d^3l'\\&=-V_N(p)(2\pi)^{\frac{3}{2}}\Phi_0.\end{aligned} \quad (18)$$

where Φ_0 is the value of ϕ in a coordinate representation evaluated for $\mathbf{z}_1=\mathbf{z}_2$. Then,

$$\begin{aligned}\phi(p)&\simeq(2\pi)^{\frac{3}{2}}\Phi_0\left[-V_N\bigg/\frac{p^2}{M}\right]\\&\simeq(2\pi)^{\frac{3}{2}}\Phi_0V_N(1/a).\end{aligned} \quad (19)$$

The last step is seen to be approximately valid near threshold when substituted into Eq. (17). A similar argument for the remainder of Eq. (16) leads to[19]

$$T\simeq(2\pi)^{\frac{3}{2}}\Phi_0\left(\chi_{p'}\chi_q,\;\left[V_N\frac{1}{a}+v\frac{1}{a-V_N-v}\right]H'\chi_p\right), \quad (20)$$

where p' is considered to be negligibly small. The values of Φ_0 appropriate to our problem have been considered previously in detail.[19]

The potential v in Eq. (20) will be evaluated to order

[16] More specifically, we may consider V_N to be that part of $\mathcal{V}$ which introduces momentum transfers between the nucleons, but is diagonal in the momentum of the meson.

[17] We suppose the fact that $w^{(-)\dagger}$ is to be operating on the wave function of a meson of momentum $\mathbf{q}$ to have been taken into account in eliminating the meson field variables from H_0 in $w^{(-)\dagger}$.

[18] See N. C. Francis and K. M. Watson, Phys. Rev. **93**, 313 (1954). Here the same formal problem was considered in connection with the deuteron stripping reaction.

[19] The separation of the final state interaction of the two nucleons has been much discussed [see, for instance, K. M. Watson, Phys. Rev. **88**, 1163 (1952)]. Our purpose at present is not to repeat the detailed physical arguments, but to show how this arises in a consistent field-theoretic treatment of the problem.

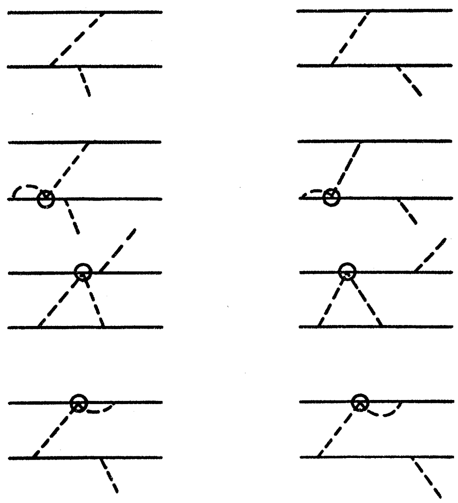

FIG. 1. Some typical processes arising in connection with the first term in expression (23). Heavy lines represent nucleons, dotted lines mesons. Time is considered to be increasing to the right.

g^2 only. From reference 11 and Eq. (3), we have

$$v=v^{(1)}+v^{(2)}+v^{(3)}+v^{(4)}, \tag{21}$$

where the superscripts (+) or (−) designate the matrix elements of h which create or absorb one meson, respectively,

$$\begin{aligned} v^{(1)}&=h_1{}^{(-)}\frac{1}{a}h_2{}^{(+)}+h_2{}^{(-)}\frac{1}{a}h_1{}^{(+)},\\ v^{(2)}&=h_1{}^{(-)}\frac{1}{a}h_1{}^{(+)}+h_2{}^{(-)}\frac{1}{a}h_2{}^{(+)},\\ v^{(3)}&=h_1{}^{(+)}\frac{1}{a}h_1{}^{(-)}+h_2{}^{(+)}\frac{1}{a}h_2{}^{(-)},\\ v^{(4)}&=h_1{}^{(+)}\frac{1}{a}h_2{}^{(-)}+h_2{}^{(+)}\frac{1}{a}h_1{}^{(-)}. \end{aligned} \tag{22}$$

We have already included in V_N that part of $v^{(1)}$ which refers to emission and absorption of the *same* meson and so must omit this from the definition of $v^{(1)}$ above.

In accordance with the approximation (1) described at the beginning of this section we shall neglect the V_N in the denominator of Eq. (20). That in the numerator will be included only to order g^2, and so is obtainable from the expression written for $v^{(1)}$ in Eq. (22). A further consequence of our assumption that many meson exchanges between the nucleons are to be neglected is that $v^{(1)}$ and $v^{(4)}$ may be treated as small perturbations. Thus we must consider T [Eq. (20)]:

$$[v^{(1)}+v^{(4)}]\frac{1}{a-v^{(2)}-v^{(3)}}+[v^{(2)}+v^{(3)}]\frac{1}{a-v^{(2)}-v^{(3)}}. \tag{23}$$

Now, $v^{(2)}+v^{(3)}$ is the interaction for scattering a meson by either of the two nucleons (except for some renormalization effects). We may thus write

$$\frac{1}{a-v^{(2)}-v^{(3)}}=\frac{1}{a}+\frac{1}{a}t_s\frac{1}{a},$$

where t_s is the scattering matrix for the meson from the two nucleons. As we are to neglect multiple scattering,[20] t_s is the sum of amplitudes for scattering from the individual nucleons. The types of transitions resulting from the first term in expression (23) are indicated in Fig. 1. (The interaction t_s is represented by an intersecting meson and nucleon line with a circle drawn through the point of intersection). The corrections involving t_s are seen to involve either multiple meson exchanges between the nucleons or radiative corrections to the upper diagram, both of which we are ignoring.

We also note that $v^{(3)}$ contributes to meson-nucleon scattering only in the $I=\frac{1}{2}$, $J=\frac{1}{2}$ [i.e., $(\frac{1}{2},\frac{1}{2})$] state and so may be treated as a perturbation by the assumptions made at the beginning of this section. Then the second term in expression (23) is

$$v^{(3)}\frac{1}{a-v^{(2)}}+v^{(2)}\frac{1}{a-v^{(2)}}.$$

Now the $v^{(2)}$ in the denominator of the first term may be neglected for the same reason: this term is nonvanishing only in the $(\frac{1}{2},\frac{1}{2})$ scattering state. But this state is to be treated by perturbation theory, so the $v^{(2)}$ can be neglected.

We finally summarize the above arguments by rewriting Eq. (20) as

$$T=(2\pi)^{\frac{3}{2}}\Phi_0\bigg(\chi_{p'}\chi_q,\bigg[(V_N+v^{(1)}+v^{(3)}+v^{(4)})\frac{1}{a}+v^{(2)}\frac{1}{a-v^{(2)}}\bigg]H'\chi_p\bigg). \tag{24}$$

Were we to drop the $v^{(2)}$ in the denominator of the last last term we would arrive at the perturbation expression used by Brueckner,[12] except that he did not give a field theoretic justification for his use of Φ_0. The term $v^{(2)}[1/(a-v^{(2)})]$ may be written as $t(1/a)$, where

$$t=v^{(2)}+v^{(2)}(1/a)t \tag{25}$$

is the Lippmann-Schwinger integral equation for the scattering matrix t. Equation (25) is to be resolved into four equations for the four substates of spin and isotopic spin for the scattering of a meson from either nucleon. By assumption (2) we set $t=v^{(2)}$ for all but the $(\frac{3}{2},\frac{3}{2})$ state.

[20] K. Watson, Phys. Rev. 89, 575 (1953).

Equation (24) is then expressed as

$$T=(2\pi)^{\frac{3}{2}}\Phi_0\bigg(\chi_{p'}\chi_q,\times[V_N+v^{(1)}+v^{(3)}+v^{(4)}+t]\frac{1}{a}H'\chi_p\bigg). \quad (26)$$

It must be admitted that it is not easy to justify neglecting the terms which were omitted in obtaining Eq. (26) from Eq. (20). However, we are interested in what is essentially a qualitative feature of the cross sections. Equation (26) permits us to develop this point [the effect of the scattering of the emitted meson in the $(\frac{3}{2},\frac{3}{2})$ state] without obscuring the result with questionable and detailed calculations of correction terms. Whether or not these may predominate over the effects included in our calculations may, perhaps, best be answered by experiment.

B. Evaluation of the Matrix t

The evaluation of Eq. (26) is the same as that of Brueckner[12] except for the treatment of t as the solution to Eq. (25). We shall consequently discuss only the determination of t.

The potential $v^{(2)}$ is

$$v^{(2)}=v_1^{(2)}+v_2^{(2)}, \quad (27)$$

where $v_1^{(2)}$ is the interaction for scattering the meson by nucleon "1", etc. As we are neglecting multiple meson exchanges, we write

$$t=t_1+t_2, \quad (28)$$

where

$$t_1=v_1^{(2)}+v_1^{(2)}(1/a)t_1, \quad (29)$$

with a corresponding equation for t_2 in terms of $v_2^{(2)}$.

The potentials $v_1^{(2)}$ and $v_2^{(2)}$ have been given by Brueckner and Watson[13] as

$$(\mathbf{k}'|v_1^{(2)}|\mathbf{k})=\lambda\frac{k'k}{(\omega_{k'}\omega_k)^{\frac{1}{2}}}[E_a+i\eta-\omega_{k'}-\omega_k]^{-1}\times\{E_{\frac{3}{2}}F_{\frac{3}{2}}-\tfrac{1}{2}E_{\frac{3}{2}}F_{\frac{1}{2}}-\tfrac{1}{2}E_{\frac{1}{2}}F_{\frac{3}{2}}+\tfrac{1}{4}E_{\frac{1}{2}}F_{\frac{1}{2}}\}, \quad (30)$$

etc. Here $\omega_k=[\mu^2+k^2]^2$, etc., the E_I's and F_J's are projection operators for the substates of isotopic spin and angular momentum.[13] Also

$$\lambda\equiv\left(\frac{g^2}{4\pi}\right)\frac{1}{3\pi}\frac{1}{M^2}. \quad (31)$$

We can express the scattering matrices t_1 and t_2 as

$$t_1=t_1(\tfrac{3}{2}\tfrac{3}{2})E_{\frac{3}{2}}F_{\frac{3}{2}}+t_1(\tfrac{3}{2}\tfrac{1}{2})E_{\frac{3}{2}}F_{\frac{1}{2}}+t_1(\tfrac{1}{2}\tfrac{3}{2})E_{\frac{1}{2}}F_{\frac{3}{2}}+t_1(\tfrac{1}{2}\tfrac{1}{2})R_{\frac{1}{2}}F_{\frac{1}{2}}, \quad (32)$$

with a similar expression for t_2, the scattering from nucleon "2". The integral equation (29) for $t_1(\frac{3}{2}\frac{3}{2})$ is (the nucleon recoil energy is dropped from this equation):

$$(k'|t_1(\tfrac{3}{2}\tfrac{3}{2})|k^0)=\lambda\frac{k'k^0}{(\omega_{k'}\omega_{k^0})^{\frac{1}{2}}}[E_a-\omega_{k'}-\omega_{k^0}]^{-1}+\lambda\int\frac{k'k}{(\omega_{k'}\omega_k)^{\frac{1}{2}}}\frac{k^2dk}{[E_a-\omega_{k'}-\omega_k][E_a+i\eta-\omega_k]}\times(k|t_1(\tfrac{3}{2}\tfrac{3}{2})|k^0), \quad (33)$$

using the expression (30) for the potential. Before attempting to solve this equation we observe the approximate magnitudes of the momentum variables. Reference to Fig. 2 indicates that the initial meson momentum $\mathbf{k}^0$ is

$$\mathbf{k}^0\simeq\pm\mathbf{p}, \quad (34)$$

the initial nucleon momentum, since the nucleons are brought to rest in a first approximation by the event under consideration.

The final meson momentum $\mathbf{k}'$ is just:

$$\mathbf{k}'=\mathbf{q}. \quad (35)$$

Thus,

$$\omega_{k^0}\simeq p\simeq(M\mu)^{\frac{1}{2}}. \quad (36)$$

The integral in Eq. (33) is assumed to be cut off at a value

$$k=k_c\simeq M. \quad (37)$$

We now attempt to solve Eq. (33) with a trial function of the form

$$(k'|t_1(\tfrac{3}{2}\tfrac{3}{2})|k^0)=-\lambda\Delta\frac{k'k^0}{(\omega_{k'}\omega_{k^0})^{\frac{1}{2}}}\times\begin{cases}\dfrac{1}{\omega_{k^0}} & \text{for } \omega_{k^0}>\omega_k\\[2mm] \dfrac{1}{\omega_k} & \text{for } \omega_{k^0}<\omega_k\end{cases} \quad (38)$$

where Δ is a constant to be determined. We substitute this into Eq. (33) and evaluate the integral, keeping in mind the magnitudes of the parameters of Eqs. (34)–(37). If we keep only the leading term proportional to k_c in the integral [this implies an error of the order of 20 percent according to Eq. (37)], we obtain the following algebraic equation for Δ:

$$\Delta=1+\Delta\lambda\omega_{k^0}k_c, \quad (39)$$

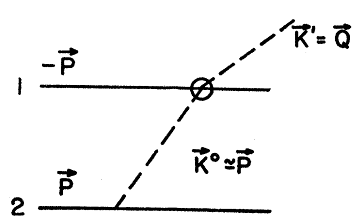

FIG. 2. Diagrammatic representation of the term $t_1(1/a)h_2^{(+)}$ in the transition operator T [Eq. (26)]. The scattering matrix t_1 is represented by a circle. Momenta are indicated, the final nucleons being nearly brought to rest by the event.

or

$$\Delta=\left[1-\frac{g^2}{4\pi}\frac{1}{3\pi}\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\frac{k_c}{M}\right]^{-1}, \tag{40}$$

by Eqs. (31) and (36). In evaluating the integral in Eq. (33) we have dropped an imaginary term which is negligible for our purposes, but which prevents Eq. (40) from becoming infinite.

We finally obtain[21]

$$(q|t_1(\tfrac{3}{2}\tfrac{3}{2})|p)\simeq-\left[\frac{g^2}{4\pi}\frac{1}{3\pi}\frac{q}{M^2\mu}\right]\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\Delta. \tag{41}$$

The other submatrices t of Eq. (32) are easily obtained in first Born approximation from Eq. (30). The evaluation of the remainder of Eq. (26) has been given several times in the literature[12] and will not be repeated here.

III. THE CROSS SECTIONS

The differential cross sections are obtained from Eq. (26) as

$$d\sigma=(2\pi)^4\frac{dJ}{v_r}S|T|^2. \tag{42}$$

Here S is the appropriate sum and average over final and initial spin substates and v_r is the relative velocity of the colliding nucleons; dJ is the volume in momentum space available to the particles in the final state. For a final state containing a deuteron and a pion of momentum $\mathbf{q}$:

$$dJ\simeq q\mu d\Omega, \tag{43}$$

where $d\Omega$ is the solid angle into which $\mathbf{q}$ is directed. For a final state in which the nucleons are not bound,

$$dJ\simeq\frac{(\mu M)^{\frac{3}{2}}}{\sqrt{2}}[T_k(T_0-T_k)]^{\frac{1}{2}}dT_k d\Omega d\Omega_{p'}. \tag{44}$$

In this equation T_k is the kinetic energy of the meson, T_0 is its maximum energy, and $d\Omega_{p'}$ is the solid angle into which the nucleonic relative momentum $\mathbf{p}'$ is directed.

We shall here present the cross sections in the Marshak-Foldy[22] approximation. This involves dropping terms with somewhat larger energy denominators than those which are kept [according to Brueckner's arguments in reference 12 this may involve errors of the order of 20 percent in T as defined by Eq. (26)]. This approximation is reasonable for the magnitudes of the cross sections, but is dangerous for a discussion of angular distributions. In the Appendix we present the complete results derived from Eq. (26). The evaluation of the final state nuclear wave function Φ_0 has been discussed in reference 18.

[21] This is just the result which would also have been obtained had we used the variational method used by Chew (reference 4) in solving Eq. (33).

[22] R. Marshak and L. Foldy, Phys. Rev. **75**, 1493 (1949). A discussion of the relation of this to field theory has been given by Brueckner (reference 12).

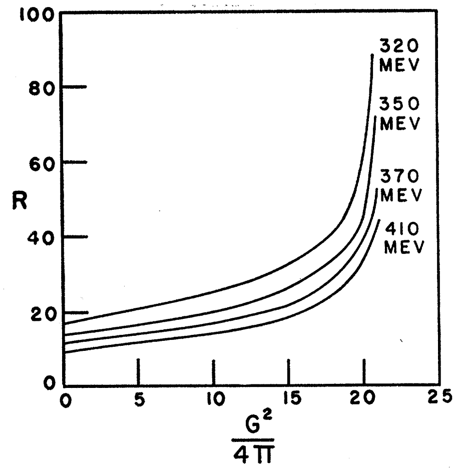

FIG. 3. The ratio R of Eq. (47) is plotted against the coupling constant for several values of the nucleon bombarding energy (in the laboratory system).

If we define

$$\sigma_0\equiv2\pi\left(\frac{g^2}{4\pi}\right)^3\left(\frac{\mu}{M}\right)^{\frac{3}{2}}\frac{1}{M^2}, \tag{45}$$

the total cross sections are approximately[23]

$$\sigma[p+p\rightarrow\pi^++d]=\sigma_0\left[\frac{\gamma/M}{1-r_0\gamma}\right]\left(\frac{q}{\mu}\right)^3\left[1+\frac{4}{9}\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\Delta\right]^2,$$

$$\sigma[p+p\rightarrow\pi^++n+p]=\frac{3\sqrt{2}}{8}\sigma_0\left[\frac{T_0^2}{\mu M}\right]\left(\frac{M}{\mu}\right)^{\frac{1}{2}}\left[1+\frac{4}{9}\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\Delta\right]^2, \tag{46}$$

$$\sigma[n+p\rightarrow\pi^++n+n]=\frac{3\sqrt{2}}{16}\sigma_0\left[\frac{T_0^2}{\mu M}\right]\left(\frac{M}{\mu}\right)^{\frac{1}{2}}.$$

Here $\gamma=(M\times\text{deuteron binding energy})^{\frac{1}{2}}$ and r_0 is the ^3S-state effective range for low-energy $n-p$ scattering. Δ is given by Eq. (40) and represents the effect of the scattering of the meson in the $(\frac{3}{2}\frac{3}{2})$ state as indicated in Fig. 2.

The energy spectrum of the mesons is given in reference 19. The angular distribution of the mesons is, in the Marshak-Foldy approximation from which Eq. (46) has been obtained, $1+3\cos^2\theta$, where θ is the angle between $\mathbf{q}$ and $\mathbf{p}$. This angular distribution is not to be taken seriously for the $n+p\rightarrow\pi^++n+n$ process, as is

[23] The T_0^2 energy dependence (rather than a T_0^3 dependence) for the last two cross sections in Eq. (46) is due to the strong interaction between the outgoing nucleons. The differential cross section to be integrated has the form of Eq. (47) of reference 19. Equation (46) represents a sufficient approximation to this integral. A common normalization factor for the three Eqs. (46) has also been approximated. That is, the evaluation of Φ_0 in Eq. (26) has been done as in reference 19 with the $[f(r)]^{-1}$ of Eq. (44) in reference 19 replaced by the "impact parameter" $(M\mu)^{\frac{1}{2}}$.

indicated in the Appendix. However, if Δ is large the angular distribution is of this form on the basis of symmetry arguments only.[5]

The ratio,

$$R=\frac{\sigma[p+p\rightarrow\pi^{+}+d]+\sigma[p+p\rightarrow\pi^{+}+n+p]}{\rho[n+p\rightarrow\pi^{+}+n+n]}, \quad (47)$$

is plotted in Fig. 3 for several values of the total energy in the (laboratory system) as a function of $g^2/4\pi$. For a cutoff $k_c=M$, both the meson-nucleon scattering and the low energy nuclear forces are fitted by $g^2/4\pi\simeq 15-18$. Reference to Fig. 3 indicates that for an energy of 340 Mev (the energy of the Berkeley cyclotron) the ratio R may be of the order of 20 or 30 which is reasonable.[8]

Since the R *versus* $g^2/4\pi$ curve is rather steep in this region, it is difficult to accept the numerical values too literally. On the other hand, observation of an experimental ratio of R which is clearly larger than would be expected on the basis of deuteron formation alone[24] might be interpreted as giving an indication that the strong meson-nucleon interaction in the $(\frac{3}{2},\frac{3}{2})$, state does play an important role in the $p+p\rightarrow\pi^{+}$ reaction. A measurement of the energy variation of R might also provide a reasonable test of the present theory, since this predicts that R should vary only with the relative probability for deuteron formation at various energies near threshold.

IV. CONCLUSIONS

Although our calculations have depended on the rather specific form of meson field theory, it is likely that the role played by the meson scattering in the final state has a more general validity. Indeed, this effect seems to be analogous to the interaction of the neutron and the proton in the reaction,

$$p+p\rightarrow\pi^{+}+n+p,$$

by which a deuteron is formed. This is to a certain extent suggested by the appearance of t in Eq. (26). We have not, however, been able to find a satisfactory general formulation of this effect and so present the calculation given above.

Rough estimates of the energy dependence of Δ seem to indicate Δ should increase with energy and thus may be larger than its value at the energetic threshold, as given by Eq. (40).

[24] This effect is approximately indicated in Fig. 3 by the value of R for $g^2/4\pi=0$.

The differential cross section for the reactions,

$$n+p\rightarrow\pi^{0}+n+p$$
$$\rightarrow\pi^{0}+d,$$

need not be calculated, since they can be obtained from Eqs. (46) using charge-independence arguments.[25]

APPENDIX

The angular distribution of the meson as obtained from Eq. (26) are of the form

$$A+B\cos^2\theta. \quad (A\text{–}1)$$

For the reactions $p+p\rightarrow\pi^{+}$:

$$A=\left[1+\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\left(\frac{4}{9}\Delta+\frac{5}{9}\right)\right]^2,$$

$$B=3\left[1+\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\left(\frac{4}{9}\Delta+\frac{1}{18}\right)\right]\times\left[1+\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\left(\frac{4}{9}\Delta-\frac{17}{18}\right)\right]. \quad (A\text{–}2)$$

For the reactions $n+p\rightarrow\pi^{+}$:

$$A=\left[1-\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\right]^2,$$
$$B=3\left[1-\frac{4}{9}\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\right]\left[1+\frac{2}{3}\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\right]. \quad (A\text{–}3)$$

To obtain the approximation of Eqs. (46), we replace Eq. (A–2) by

$$\tfrac{1}{3}B=A=\left[1+\frac{4}{9}\Delta\left(\frac{\mu}{M}\right)^{\frac{1}{2}}\right]^2. \quad (A\text{–}4)$$

Equations (A–3) should be replaced by

$$A=1,\quad B=3. \quad (A\text{–}5)$$

The normalization of the differential cross sections can easily be obtained by comparison of these equations with Eqs. (46).

Note added in proof.—J. L. Gammel [Phys. Rev. (to be published)] has numerically integrated Eq. (33) on the Los Alamos MANIAC to find R [Eq. (47)]. With $(g^2/4\pi)$ chosen to fit the pion-nucleon scattering, he finds that $R\simeq 18$ for energies less than 380 Mev and increases appreciably for higher energies. We are indebted to Dr. Gammel for informing us of his work.

[25] K. Watson and K. Brueckner, Phys. Rev. **83**, 1 (1951). See Eq. (21) of this reference.