isotope. Since the value of g for each resonance is unknown, it is assumed that half the resonances have one g and half the other. Only the 12 resonances below 10 ev are used for the averaging, including the one at "zero" energy found by Sturm.[5] These averages are given in Table III. It is difficult to appraise the error in these ratios; however, it is believed to be less than $\pm 0.5\times 10^{-4}$. It is seen that the values of the ratio are about the same for the two isotopes and are about $2\frac{1}{2}$ times larger than the estimated 1.45×10^{-4} derived from the "extreme compound nucleus" model. Specific information for comparison with the "cloudy crystal-ball" model is not available at this time.

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A Mixture of Central, Tensor, and Two-Particle Spin-Orbit Interactions for N^{14} and D^2

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The calculations of Elliott on the p-shell nuclei Li^6, Li^7, and B^{10} have been extended by the author to the N^{14} nucleus. Only the s^4p^{10} configuration was considered. The central and tensor potentials were taken to be of the Yukawa shape while spin-orbit potentials of both the Yukawa and Case-Pais forms were considered, the ranges being assumed equal. The conclusions are similar to those of Elliott.

The object of the deuteron work was to discover whether the conclusions drawn from the p-shell calculations were consistent with the deuteron data. Here the potentials were taken to be of the Yukawa form throughout with almost equal ranges. Only one calculation has been carried out, but it indicates that the data required by the p-shell nuclei are not inconsistent for the deuteron.

INTRODUCTION

It has been shown by Elliott[1] that a mixed central, tensor, and spin-orbit interaction of the form

$$V(\mathbf{r})=\tfrac{1}{3}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[V_cC_{12}f_0(r/r_0)+V_tS_{12}f(r/r_t)+V_sM_{12}f_s(r/r_0)],$$

with

$$C_{12}=1+\tfrac{1}{2}g(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-1);\quad S_{12}=[3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2]-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2);$$

$$M_{12}=3\hbar^{-1}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\mathbf{r}\times\mathbf{p}_{12});$$

$$f_0(z)=\frac{e^{-z}}{z};\qquad f_s(z)=-\frac{1}{z}\frac{d}{dz}\left(\frac{e^{-z}}{z}\right);$$

$$r_0=r_t=1.18\times 10^{-13}\ \text{cm},$$

may be used with a suitable value of g and of central, tensor, and spin-orbit depths V_c, V_t, and V_s to fit the ground-state total angular momentum, magnetic moment, and quadrupole moment of the p-shell nuclei Li^6, Li^7, and B^{10}, subject to the various assumptions which he makes.

TABLE I. Values of interaction parameters in N^{14}.

Case (1)	$r_0=1.18^{-}\times 10^{-13}$ cm	$2.28>g>0.63$	$x=-0.46+1.24g$ $y=-0.034-0.053g$
	$r_0=1.35^{-}\times 10^{-13}$ cm	$2.25>g>0.5$	$x=-0.32+1.40g$ $y=-0.024-0.043g$
Case (2)	$r_0=1.18^{-}\times 10^{-13}$ cm	$g>0$	$x=0.21+2.30g$ $y=-0.098-0.153g$
	$r_0=1.35^{-}\times 10^{-13}$ cm	$g>-0.12$	$x=0.55+2.90g$ $y=-0.112-0.190g$

[1] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).

N^{14} CALCULATION

Subject to these same assumptions, and restricted to the configuration s^4p^{10}, the author has shown that the total angular momentum, magnetic moment, and quadrupole moment of N^{14} may be achieved under the conditions set out in Table I, where x and y are defined by the relationships

$$x=V_t/V_c,\quad y=V_s/V_c.$$

In Case (1),

$$f_0(z)=\frac{e^{-z}}{z},\qquad f_s(z)=-\frac{1}{z}\frac{d}{dz}\left(\frac{e^{-z}}{z}\right),\qquad r_0=r_t.$$

In Case (2),

$$f_0(z)=f_s(z)=\frac{e^{-z}}{z},\qquad r_0=r_t.$$

It may be noted with regard to these calculations that, although the matrix elements of the tensor force are identical for the p^2 and p^{10} configurations, the matrix elements of the spin-orbit interaction for the two configurations are entirely different. These matrix elements were evaluated using the $\langle p^{10}|p^8,p^2\rangle$ fractional percentage coefficients of Elliott, Hope, and Jahn[2] and were checked independently by Hope[3] using his tensor-operator method.

For a symmetric-type interaction it has been found that a single configuration is inadequate for the binding energy of N^{14}. Consequently further calculations with the present model have not been undertaken. Work is in progress at Southampton on improving the wave functions. When this has been completed more detailed calculations will be possible.

DEUTERON CALCULATION

It was thought desirable to investigate whether, with an interaction of the above kind, it would be possible to fit the deuteron data. With $f_0(z)=f_s(z)=e^{-z}/z$, $r_0=1.35\times10^{-13}$ cm, $r_0/r_t=0.978$, the coupled differential equations for the deuteron $^3S_1$ and $^3D_1$ states were solved by the standard variation method and by a numerical method based on that of Hartree, the depths being adjusted to give the correct binding energy. Table II contains the results obtained. The constants with zero spin-orbit depth correspond to a result given by Feshbach and Schwinger[4] in a table of interpolated values. The deuteron P states were not investigated.

TABLE II. Deuteron data consistent with correct binding energy.

Central depth V_c(Mev)	Tensor depth V_t(Mev)	Spin-orbit depth V_s(Mev)	Quadrupole moment (10^{-27} cm^2)	Percentage D state
25.51	45.56	0	2.60	4.2
26.80	40.20	−2.68	2.81	4.9
27.91	39.07	−2.79	2.76[a]	4.8[a]
33.36	33.36	−3.34	2.56	4.1

[a] Interpolated values.

It can be seen that $x=1.4$, $y=-0.1$ is consistent with Case (1) in Table I and with the interpolated values of Table II.

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The Odd-Nucleon-Plus-Liquid-Drop-Model of Heavy Odd Nuclei*

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The odd-nucleon-plus-liquid-drop-model of the heavy odd nuclei is considered using the techniques developed by Racah in the construction of the nuclear ground states. The odd nucleon-core interaction is treated perturbation-wise, and the nuclear magnetic moments, quadrupole moments, and the energies of the excited states are investigated. The results are not wholly satisfactory; however, some indications of improvement may be seen.

I. INTRODUCTION

THE analogy of the atomic nucleus to a liquid drop has been successful in accounting for a considerable fraction of the experimental data regarding nuclei.[1–3] In particular this analogy has been successful in the fields of nuclear reactions, nuclear fission, and nuclear binding energies. An alternative approach, the shell model,[4] has been similarly successful in accounting for the properties of nuclear ground states, spins, parities, etc., and in explaining the "magic" behavior of certain nuclei containing closed shells of neutrons or protons. Neither model has, however, been completely successful in explaining nuclear moments. The main features of the variation of the nuclear magnetic moments with nuclear spin is correctly given by Schmidt limits; however, there are appreciable deviations from these predictions for several nuclei. The variation of the quadrupole moment with the number of odd nucleons is partially accounted for by the shell model. On the other hand, however, the shell model gives too small a value for the quadrupole moment of some nuclei. A spheroidal liquid drop model[5,6] is capable of giving the larger quadrupole moments required for these nuclei but tends to give quadrupole moments which increase monotonically

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