Slow Neutron Resonances in Europium*

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The total cross section of europium has been measured over a range of neutron energy from 0.08 to 58 ev. Resonances occur at 0.327, 0.461, 1.055, 1.76, 2.46 2.73, 3.35, 3.84, 6.25, 7.36, and 8.98 ev. Between 10- and 20-ev resonances were found at 10.6, 11.8, 12.8, 15.1, and ~19.5; however, it is probable that several additional resonances were present but not observed because of insufficient instrument resolution. Many resonances are present above 20 ev which could not be resolved. The strength, $\sigma_0\Gamma^2$, and various other Breit-Wigner parameters have been obtained for the resonances below 10 ev. The ratio of the average reduced neutron width to the average level spacing, Γ_n^0/D, was found to be 3.5×10^{-4} for Eu^{151} and 3.3×10^{-4} for Eu^{153}.

I. INTRODUCTION

DURING the past several months there has been in progress at Brookhaven National Laboratory a program for measuring the total cross sections of the rare earth elements.[1] The measurements include the range of neutron energies from about 0.1 to 50 ev. One purpose of this survey is to obtain a knowledge of the resonance structure of these interesting nuclides and to attempt to correlate this structure with existing nuclear theory. One can compare for example, the average spacing and strengths of resonances with theoretical predictions based on specific nuclear models. The rare earth region is particularly sensitive for comparison with the so-called "cloudy crystal-ball" model of Feshbach, Porter, and Weisskopf.[2]

Europium has two stable isotopes, Eu^{151} and Eu^{153}, having abundances[3] of 47.77 percent and 52.23 percent, respectively. Both are odd-even isotopes, and experience indicates that odd-even isotopes in this region of the nuclear periodic table are characterized by relatively closely spaced slow-neutron resonances of moderate strength. Previous measurements[4,5] of the europium cross section disclosed a complicated resonance structure including one resonance in[4] Eu^{151} which lies at approximately "zero" energy.[5] There appeared to be several additional resonances below 5 ev, with evidence for one of these at approximately 3.3 ev belonging[6] to Eu^{151}. The earlier measurements were made with insufficient instrument resolution to permit the details of the structure to be observed.

II. EXPERIMENTAL DETAILS

The total cross section measurements were made with the BNL crystal spectrometer, which has been described in detail previously.[7] The crystals used for the monochromator were as follows: 0.08 to 0.2 ev, NaCl (220); 0.1 to 0.4 ev, NaCl (240); 0.3 to 58 ev, Be ($12\bar{3}1$).

Three europium samples from two independent sources were used in the measurements. Samples 0.0509 and 0.0117 g/cm² in thickness were prepared from Johnson, Matthey, and Company, Ltd. "Specpure" Europium Oxide. Spectroscopic analysis of this material disclosed no significant impurities. A third sample 0.1747 g/cm² in thickness was prepared from europium oxide obtained from Research Chemicals, Inc. This oxide had a nominal purity of better than 99.8 percent. Before preparing the samples it was necessary to bake the oxides for several hours at approximately 800°C to remove water. The material for each sample was then placed in a 5-ml volumetric flask for weighing and processing. After being weighed the oxide was converted to the nitrate by dissolving it in nitric acid; hydrogen in the sample (the water of crystallization of the nitrate) was displaced by deuterium by means of several stages of dissolving it in D_2O and evaporating it to dryness; and finally the "deuterated" nitrate was dissolved in D_2O and diluted to standard volume. In this manner the concentration of the solution was accurately determined. For the measurement, an aliquot was transferred to a precision quartz cell. Transmission measurements were made against an identical cell filled with D_2O. It is estimated that the error in determining the thickness of each sample was approximately ±1 percent. A small correction to the observed transmission was needed in the case of the 0.1747-g/cm² sample to correct for the presence of the $(NO_3)_3$ radical in the solution.

III. RESULTS

Total Cross Section

The observed total cross section of europium (uncorrected for the effects of instrument resolution or Doppler broadening) is shown in Fig. 1. The resonances which were observed are listed in Table I. In addition

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[1] Foote, Landon, and Sailor, Phys. Rev. **92**, 656 (1953); Sailor, Landon, and Foote, Phys. Rev. **91**, 450 (1953); Foote, Sailor, and Landon, Phys. Rev. **90**, 362 (1953); Sailor, Foote, and Landon, Phys. Rev. **89**, 904 (1953).
[2] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953); Nuclear Development Associates, Inc., Report NYO-3076, 1953 (unpublished).
[3] D. C. Hess, Jr., Phys. Rev. **74**, 773 (1948).
[4] Borst, Ulrich, Osborne, and Hasbrouck, Phys. Rev. **70**, 557 (1946).
[5] W. J. Sturm, Phys. Rev. **71**, 757 (1947).
[6] M. Goldhaber (unpublished). See reference 20 in W. J. Sturm, Phys. Rev. **71**, 757 (1947).
[7] L. B. Borst and V. L. Sailor, Rev. Sci. Instr. **24**, 141 (1953).

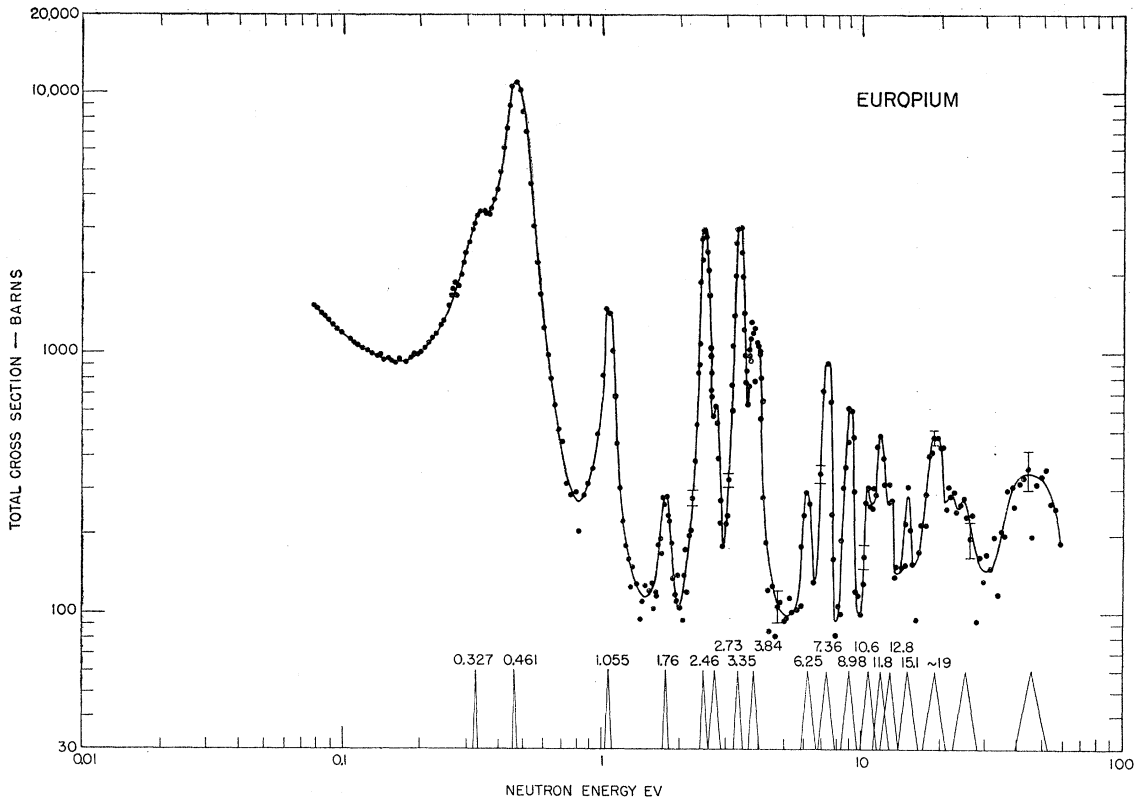

FIG. 1. Total cross section of europium. The location of each resonance is indicated by the triangle and the value of E_0. The width of the triangle represents the width of the instrument resolution function at that energy. Uncertainties due to counting statistics are, in general, smaller than the plotted points, except in the regions where typical errors are shown.

to those listed in the table it is probable that several more resonances occur between 10 and 20 ev which could not be observed because of insufficient resolution. Above 20 ev many resonances are present but are unresolved.

The resonances at 1.76 and 2.73 ev are markedly weak, and at first sight would appear to be due to impurities. It is possible in principle to identify impurities by means of the resonance spectrum, because the line structure of each element is unique. The most probable impurities in europium would be other rare earth elements, which are generally difficult to detect by ordinary chemical or spectroscopic means. However, the cross sections of all of the rare earths, with the exception of terbium,[1] have been carefully studied below 10 ev; and it is found that no rare earth impurity (with possible exception of Tb) can account for the two lines in question.[8] In fact, a search among all those elements for which the resonance structure has been determined[9] (the majority of the 92 elements), discloses no impurities which could reasonably account for the 1.76- and 2.73-ev resonances. Probably both of these resonances belong to europium. Further evidence in regard to the 2.73-ev line is obtained from activation measurements which indicate that it belongs to Eu^{151}, although the identification is not certain.

In an independent study,[10] Wood has determined which of the resonances below 10 ev belong to Eu^{151}. The assignment was made by measuring the amount of 9-hour Eu^{152} activity produced by the irradiation of a Eu sample with monochromatic neutrons. The isotopic assignments of Wood are listed in Table I.

Analysis of Resonances

Except at very low energies the experimentally observed shape of a neutron resonance is not the same as the theoretical shape because of the effects of instrument resolution and Doppler broadening. With the BNL crystal spectrometer these distortions are appreciable at 1 ev and become increasingly more severe at higher energies. Several methods are available for obtaining various combinations of the "true" Breit-Wigner parameters of a resonance. These are as follows: (1) A direct fit of the experimental curve to a theore-

[8] Dysprosium, for example, has strong resonances at 1.7 and 2.7 ev, but an additional very strong resonance at 5.5 ev. If the 1.76- and 2.73-ev resonances in the europium sample were due to a small dysprosium impurity, then the resonance at 5.5 ev would also have been observed.

[9] *Neutron Cross Sections*, U. S. Atomic Energy Commission Report, AECU-2040 (U. S. Government Printing Office, Washington, D. C., 1952).

[10] R. E. Wood (to be published).

TABLE I. Neutron resonances in europium. Entries containing σ_0 are isotopic values. Parameters obtained directly from analysis are in normal type, while values derived from the assumption that $\Gamma_t=0.09$ ev are in italics. The "reduced width", $\Gamma_n^0\equiv\Gamma_n/E_0^{\frac{1}{2}}$, was computed by assuming $g=\frac{1}{2}$. The various methods for analysis are described in the text.

	E_0 (ev)	Isotope[a]	$\sigma_0\Gamma^2$ barn ev^2	$\sigma_0\Gamma$ barn ev	$10^3 g\Gamma_n$ ev	$10^3\Gamma_n^0$ ev	Method of analysis
1	−0.011[b]	151[c]	...	...	...	...	...
2	+0.327±0.001	151	30.0±1.5	347±14	0.044	0.15	1
3	0.461±0.001	151	213±10	2230±60	0.396	1.11	1
4	1.055±0.005	151	35±6	330±50	0.136	0.25	1
5	1.76 ±0.02	Not 151	*4.6±1.5*	53±14	0.034	0.06	4
6	2.46 ±0.01	Not 151	86±10	*960±120*	0.90	1.20	3, 4
7	2.73 ±0.05	151(?)	*3.8±1.5*	42±10	0.04	0.05	4
8	3.35 ±0.01	151	113±12	1340±140	1.72	1.80	4
9	3.84 ±0.02	Not 151	73±16	*800±200*	1.19	1.26	4
10	6.25 ±0.04	Not 151(?)	*16±4*	184±20	0.44	0.37	4
11	7.36 ±0.03	151	126±15	*1380±140*	3.91	2.76	4
12	8.98 ±0.05	Not 151	95±20	*1050±300*	3.63	2.53	4
13	10.6 ±0.2	...	...	...	...	...	...
14	11.8 ±0.1						
15	12.8 ±0.3						
16	15.1 ±0.2						
17	~19.5						

[a] Isotopic assignments of R. E. Wood (see reference 10).
[b] See reference 5.
[c] See reference 4.

tical one-level formula when resolution and Doppler broadening are small enough to be neglected; (2) A point-by-point fit of the experimental curve to the theoretical formula with corrections included for resolution and Doppler broadening;[11] (3) Direct fit of the "wings" of the resonance (in the region of the curve where the effects of resolution and Doppler broadening are negligible) to the theoretical formula; and (4) The "area analysis" method as refined by Melkonian *et al.*[12]

The choice of the method of analysis to be applied to a particular resonance depends on many practical considerations; e.g., the proximity of other resonances and the resolution width at E_0. In the case of the europium spectrum the method selected for the analysis of each resonance is noted in Table I.

The resonances at 0.327 and 0.461 ev lie at a low enough energy so that the effects of instrument resolution and Doppler broadening are negligible. Therefore, the Breit-Wigner formula can be fitted directly to these resonances [(i.e., method (1) above)]. Note, however, that they lie so close together that it is necessary to analyze them simultaneously. In carrying out this analysis it was assumed that resonant scattering is negligible and that all other resonances are far enough separated so as to contribute a background cross section which is essentially constant over the energy range under consideration. It was further assumed that each resonance satisfies a Breit-Wigner single-level formula and that the resultant cross section can be obtained by adding the individual cross sections. This implies that the resonances do not mutually fulfill the rather stringent requirements of the many-level formula in regard to identity of the entrance and exit channels and hence do not exhibit interference effects.

The curve fitting was accomplished by using the method of least squares to solve for E_0, Γ, and σ_0 for each resonance plus the constant background contribution from other resonances. The final solution involved a seventh-order determinant. The results of the analysis are presented in Table II, and the individual and combined theoretical curves are shown with the experimental points in Fig. 2. It is believed that the excellent

TABLE II. Precise Breit-Wigner parameters for three of the europium resonances. These results were derived by fitting the experimental points to the single-level formula by the method of least squares. The entry $F\sigma_0$ is the total cross section at resonance for the element. All other entries are corrected for the isotopic abundance F.

	No. 2	No. 3	No. 4
E_0	0.327±0.001	0.461±0.001	1.055±0.005
Isotope[a]	151	151	151
$F\sigma_0$(barns)	1930±50	11 100±100	1500±70
σ_0(barns)	4040±100	23 230±200	3100±150
Γ_t(ev)	0.086±0.002	0.096±0.002	0.105±0.010
$g\Gamma_n$(ev)	$(0.044\pm0.002)\times10^{-3}$	$(0.395\pm0.010)\times10^{-3}$	$(0.14\pm0.02)\times10^{-3}$
$g\Gamma_n/\Gamma$	$(0.51\pm0.02)\times10^{-3}$	$(4.1\pm0.2)\times10^{-3}$	$(1.3\pm0.2)\times10^{-3}$

[a] Isotopic assignments of R. E. Wood (see reference 10).

[11] V. L. Sailor, Phys. Rev. **91**, 53 (1953).
[12] Melkonian, Havens, and Rainwater, Phys. Rev. **92**, 702 (1953).

agreement between the calculated and experimental curves justifies the assumption of the applicability of the single-level formula; and, that if interference effects were present, a marked misfit would have resulted in the region between the two resonances.

The 1.055-ev resonance was analyzed by method (1) after correcting for the contributions of the 0.327- and 0.461-ev resonances. The results of this analysis are less accurate because resolution and Doppler broadening, though small, are appreciable. To reduce the effects of resolution and Doppler broadening, points near the center of the resonance were not used in the analysis because the smearing effects are largest at the center. The parameters for the 1.055-ev resonance are also listed in Table II.

Less precise results were obtained for the resonances at higher energies because of the close spacing of levels and the correspondingly poorer resolution. Unfortunately, the close spacing prevented full advantage to be taken of the refinements of the "area method." Results for these resonances are summarized in Table I.

Several additional parameters of interest may be computed and are listed in Table I. The product, $\sigma_0\Gamma$, can be used to obtain $g\Gamma_n$ for each resonance. In several cases only $\sigma_0\Gamma^2$ was measured from the area analysis; so in order to compute $g\Gamma_n$, Γ must be estimated. The value $\Gamma=0.09$ ev appeared to be a resonable estimate for Eu, and was arbitrarily assumed when necessary. The statistical weight factor g cannot be uniquely determined from total cross section measurements; however, it should be noted that the spin[13] of both Eu^{151} and Eu^{153} is 5/2 permitting g to be either 5/12 or 7/12. Thus the error is small if g is assumed to be $\frac{1}{2}$ in all cases. A more useful quantity for comparing the strength of resonances is the "reduced neutron width", defined as $\Gamma_n^0\equiv\Gamma_n/E^{\frac{1}{2}}$. This parameter permits the strength of all resonances to be compared at a reference energy of 1 ev with all energy-dependent factors in the parameters eliminated.

IV. DISCUSSION OF RESULTS

Comparison of the values of Γ_n^0 in Table I shows a range of variation of more than a factor of ten. Large and apparently random fluctuations of Γ_n^0 in an individual isotope are not uncommon, and on the contrary appear to be the usual case.[14,15] At the present time there is no theory sufficiently detailed to account for this behavior.

The ratio of the average "reduced neutron width" to the average level spacing $\overline{\Gamma}_n^0/D$ is of importance to the choice of nuclear models in theories relating to the formation of the compound nucleus.[16] In the "extreme compound nucleus" model of Feshbach, Peaslee, and Weisskopf,[17] $\overline{\Gamma}_n^0/D$ is expected to be independent of the atomic weight and approximately equal to 1.45×10^{-4}. The more recent "cloudy crystal-ball" model of Feshbach, Porter, and Weisskopf predicts variations in $\overline{\Gamma}_n^0/D$ as a function of A. With their present choice of the well depth $V_0=19$ Mev and nuclear absorption coefficient $\xi=0.05$, a large increase in $\overline{\Gamma}_n^0/D$ is expected in the vicinity of an atomic weight of 160.

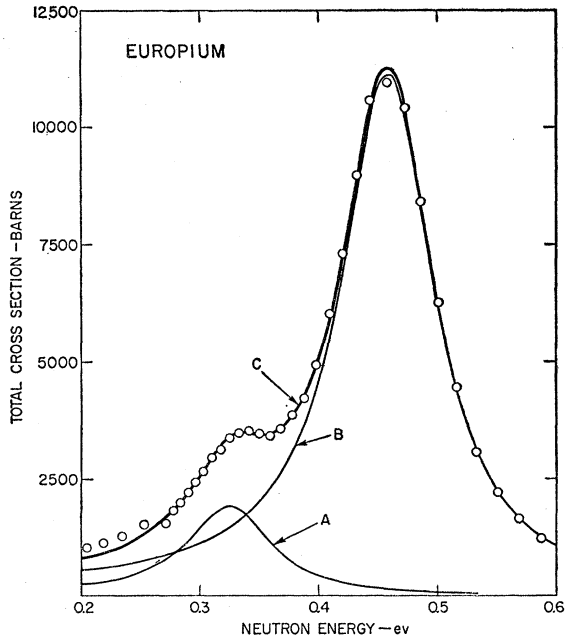

FIG. 2. Theoretical fit to the experimental points for the 0.327- and 0.461-ev resonances. Curves A and B are the best Breit-Wigner curves for the two resonances, and curve C is the sum of A and B.

Only rough averages of the ratio can be obtained from the present data partly because of the small sampling of levels and partly because of the lack of knowledge of the proper value of g for each level. The averaging was carried out for the element as well as for the individual isotopes to serve as a comparison. The averaging of Γ_n^0 for the element is not sensitive to isotopic assignment in Eu because the isotopes are almost equally abundant. The quantity D is defined as the average spacing between resonances of the same g in a given

TABLE III. Average reduced neutron widths and level spacings for europium. These averages are based on the twelve resonances below 10 ev. The averages for the element serve merely as a comparison.

	$\overline{\Gamma}_n^0$, ev	D, ev	$\overline{\Gamma}_n^0/D$
Eu (element)	1.05×10^{-3}	3.3	3.2×10^{-4}
Eu^{151}	1.02×10^{-3}	2.9	3.5×10^{-4}
Eu^{153}	1.08×10^{-3}	3.3	3.3×10^{-4}

[13] H. Schuler and T. Schmidt, Z. Physik **94**, 457 (1935).
[14] V. L. Sailor and L. B. Borst, Phys. Rev. **87**, 161 (1952).
[15] R. L. Christensen, Phys. Rev. **92**, 1510 (1953).
[16] C. Porter (private communication). See also J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 462 ff. Similar importance is attached to this ratio in the theory of the compound nucleus developed by R. G. Thomas (private communication).
[17] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

isotope. Since the value of g for each resonance is unknown, it is assumed that half the resonances have one g and half the other. Only the 12 resonances below 10 ev are used for the averaging, including the one at "zero" energy found by Sturm.[5] These averages are given in Table III. It is difficult to appraise the error in these ratios; however, it is believed to be less than $\pm 0.5\times 10^{-4}$. It is seen that the values of the ratio are about the same for the two isotopes and are about $2\frac{1}{2}$ times larger than the estimated 1.45×10^{-4} derived from the "extreme compound nucleus" model. Specific information for comparison with the "cloudy crystal-ball" model is not available at this time.

V. ACKNOWLEDGMENTS

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A Mixture of Central, Tensor, and Two-Particle Spin-Orbit Interactions for N^{14} and D^2

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The calculations of Elliott on the p-shell nuclei Li^6, Li^7, and B^{10} have been extended by the author to the N^{14} nucleus. Only the s^4p^{10} configuration was considered. The central and tensor potentials were taken to be of the Yukawa shape while spin-orbit potentials of both the Yukawa and Case-Pais forms were considered, the ranges being assumed equal. The conclusions are similar to those of Elliott.

The object of the deuteron work was to discover whether the conclusions drawn from the p-shell calculations were consistent with the deuteron data. Here the potentials were taken to be of the Yukawa form throughout with almost equal ranges. Only one calculation has been carried out, but it indicates that the data required by the p-shell nuclei are not inconsistent for the deuteron.

INTRODUCTION

IT has been shown by Elliott[1] that a mixed central, tensor, and spin-orbit interaction of the form

$$V(\mathbf{r})=\tfrac{1}{3}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)[V_cC_{12}f_0(r/r_0)+V_tS_{12}f(r/r_t)+V_sM_{12}f_s(r/r_0)],$$

with

$$C_{12}=1+\tfrac{1}{2}g(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-1);\quad S_{12}=[3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2]-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2);$$

$$M_{12}=3\hbar^{-1}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)(\mathbf{r}\times\mathbf{p}_{12});$$

$$f_0(z)=\frac{e^{-z}}{z};\qquad f_s(z)=-\frac{1}{z}\frac{d}{dz}\left(\frac{e^{-z}}{z}\right);$$

$$r_0=r_t=1.18\times 10^{-13}\text{ cm},$$

may be used with a suitable value of g and of central, tensor, and spin-orbit depths V_c, V_t, and V_s to fit the ground-state total angular momentum, magnetic moment, and quadrupole moment of the p-shell nuclei Li^6, Li^7, and B^{10}, subject to the various assumptions which he makes.

TABLE I. Values of interaction parameters in N^{14}.

Case (1)	$r_0=1.18\times 10^{-13}$ cm	$2.28>g>0.63$	$x=-0.46+1.24\,g$ $y=-0.034-0.053g$
	$r_0=1.35\times 10^{-13}$ cm	$2.25>g>0.5$	$x=-0.32+1.40\,g$ $y=-0.024-0.043g$
Case (2)	$r_0=1.18\times 10^{-13}$ cm	$g>0$	$x=0.21+2.30\,g$ $y=-0.098-0.153g$
	$r_0=1.35\times 10^{-13}$ cm	$g>-0.12$	$x=0.55+2.90\,g$ $y=-0.112-0.190g$

N^{14} CALCULATION

Subject to these same assumptions, and restricted to the configuration s^4p^{10}, the author has shown that the total angular momentum, magnetic moment, and quadrupole moment of N^{14} may be achieved under the conditions set out in Table I, where x and y are defined by the relationships

$$x=V_t/V_c,\quad y=V_s/V_c.$$

In Case (1),

$$f_0(z)=\frac{e^{-z}}{z},\qquad f_s(z)=-\frac{1}{z}\frac{d}{dz}\left(\frac{e^{-z}}{z}\right),\qquad r_0=r_t.$$

In Case (2),

$$f_0(z)=f_s(z)=\frac{e^{-z}}{z},\qquad r_0=r_t.$$

It may be noted with regard to these calculations that, although the matrix elements of the tensor force are identical for the p^2 and p^{10} configurations, the matrix elements of the spin-orbit interaction for the two configurations are entirely different. These matrix elements were evaluated using the $\langle p^{10}|p^8,p^2\rangle$ fractional per-

[1] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).