Nuclear Cross Sections at Low Energies

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The note is concerned with estimates of the accuracy of the one-term formula for the energy dependence of the reaction cross section for charged particles. The second term in an expansion of the cross section in powers of the energy is worked out for a one-body model. Comparison with data of Sawyer and Phillips on the bombardment of Li and Be with protons and deuterons shows that the deviations from the one-term formula are of the correct order of magnitude to be accounted for by the second term. This term was compared with the effect of the 107-kev resonance on the cross section of $H^3(d,n)He^4$. Employing a conservative central-field model the correction term to the one-body asymptotic form is found to be smaller than the effect of the resonance expected from the Breit-Wigner formula. The f-function generalization to $L>0$ which has been made by Breit is studied and a table facilitating its use for the calculation of reaction cross sections is given.

I. INTRODUCTION

THE present note is concerned primarily with the influence of the Coulomb field on the dependence of nuclear reaction cross sections σ on energy. It was first pointed out by Gamow[1] that the cross sections should depend critically on the energy if the Coulomb barrier prevents direct contact between the colliding particles. In this somewhat qualitative approach it was not important to specify the mechanism of the nuclear reaction, since the Coulomb barrier effect is to some degree independent of the interaction between the particles at distances smaller than that corresponding to contact. On a tentative picture of the flux of relative motion being the determining factor, Breit[2] worked out the asymptotic form of the cross section at low energies in connection with estimates on the possibility of nuclear transmutations by artificial sources of charged particles. This work has been extended by Ostrofsky, Breit, and Johnson,[3] employing a more detailed view of the reaction mechanism. It was supposed by them that the influence of interactions in the region of configuration space corresponding to distances smaller than the contact radius b may be schematically represented through the introduction of an imaginary part of the potential energy and this quantity was related to the inherent probability of distintegration P by means of the conservation theorem for the number of particles. The quantity P represents the probability per second and per unit volume of the space of relative motion of the colliding particles that a disintegration should take place. This quantity is used also in the present work, although its effect on the damping of the incident wave is not taken into account so as to simplify the form of the answers. In the work of OBJ it was shown that the asymptotic form of σ is independent of the value of the orbital angular momentum L even though the centrifugal barrier depends on this quantity. Their paper also shows that, except for the entrance of a factor of proportionality in σ, the same asymptotic form applies independently of the value of P. Generalizations of these results both regarding generality of derivation and of applicability to a wide variety of situations have been made by Wigner.[4] The asymptotic form is in good agreement with experimental data as has been shown by OBJ and later work.

One of the main objects of the present note is to present estimates of the energy region in which deviations from the asymptotic one term form are negligible and to assign approximate values to its probable accuracy. This purpose is accomplished by deriving the second term in the expansion of the ratio of σ to the value of the one term approximation of OBJ in powers of the energy of relative motion E. Since the principal interest here is to have an approximate estimate, the effect of P on the internal function is neglected. There being no special reason for believing that the main term and the correction term are affected very differently by the imaginary part of the potential energy, it is probable that the estimates are valid also for non-negligible absorptions.

For small absorption inside b the behavior of the phase shift as a function of E can be represented conveniently through the employment of the f function of Breit, Condon, and Present[5] and its generalization to $L\neq0$ made by Breit.[6] In this representation the critical dependence of K_L on E at low E does not interfere with a convenient graphical treatment of experimental material somewhat as in the effective range plots for p-p and p-n scattering. Since the phase shift determines the value of the wave function at the nuclear boundary it is possible to estimate the energy

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[1] G. Gamow, Z. Physik 52, 510 (1928).
[2] G. Breit, Phys. Rev. 34, 817 (1929).
[3] Ostrofsky, Breit, and Johnson, Phys. Rev. 49, 22 (1936). Hereafter this paper will be referred to as OBJ. The same point of view regarding the role of nuclear absorption has been taken by Feshback, Peaslee, and Weisskopf, Phys. Rev. 71, 145 (1947).
[4] E. P. Wigner, Phys. Rev. 73, 1002 (1948).
[5] Breit, Condon, and Present, Phys. Rev. 50, 825 (1936).
[6] G. Breit, Revs. Modern Phys. 23, 238 (1951).

dependence of σ by means of the f_L function. The present note contains, therefore, a table and a few graphs facilitating the calculation of f_L and showing effects of varying L and other parameters on the (f_L,E) curves.

Notation

The notation for the Coulomb functions is the same as that used by Yost, Wheeler, and Breit,[7] and by Bloch, Hull, Broyles, Bouricius, Freeman, and Breit.[8]

F_L,G_L regular and irregular Coulomb functions for angular momentum $L\hbar$ normalized to be asymptotic to the sine and cosine of the same phase at large r with $F_L>0$ at small r. Where no ambiguity arises the subscript L is omitted in this note.

$\nu=2L+1$.

r,v relative distance and velocity.

$k=\mu v/\hbar$, where μ is the reduced mass.

$\rho=kr$. In this note it is in most cases taken at the nuclear boundary, $r=b$.

$\eta=ZZ'e^2/\hbar v=1/ka$.

$x=(8\rho\eta)^{\frac{1}{2}}$.

$C_L=\{2^L/(2L+1)!\}\{[L^2+\eta^2][(L-1)^2+\eta^2]\cdots\times[1+\eta^2]\}^{\frac{1}{2}}C_0$.

$C_0=[2\pi\eta/(e^{2\pi\eta}-1)]^{\frac{1}{2}}$.

u regular solution of the radial equation for $r<b$, normalized so that $u(b)=1$.

$\delta=F'/F-u'/u$; $'=d/d\rho$.

P adjustable parameter representing probability of reaction occurring if the bombarding particle is inside the nuclear boundary. In general different values of P can be chosen for different L.

U depth of square well representing internal potential in Mev. In all applications the centrifugal potential is supposed to extend in to the origin and is superimposed on this square-well potential.

II. EXPANSION OF THE LOW-ENERGY CROSS SECTION

OBJ derived the formula for the partial cross section for distegration on the one-body model without spin in the limiting case of no absorption, obtaining[3]

$$\sigma_L=\frac{4\pi Pb^3}{v}\frac{(\nu)(F_L^2/\rho^2)\langle u_L^2\rangle}{(1-F_LG_L\delta_L)^2+F_L^4\delta_L^2},\tag{1}$$

where

$$\langle u_L^2\rangle=\int_0^b w_Lu_L^2dr;\quad \int_0^b w_L(r)dr=1.\tag{1.1}$$

Here the function w_L is the relative intrinsic probability of inducing the disintegration in dr at r as in Bloch, Hull, Broyles, Bouricius, Freeman, and Breit,[8] and in this note will be assumed to be a constant, $1/b$, for simplicity, as in OBJ.

Yost, Wheeler and Breit[7] gave an expansion for Φ and sufficient relations to obtain Φ^* as a power series in $1/\eta^2$ (proportional to energy) and Breit and Hull[9] obtained a corresponding expansion for Θ where the coefficients are simple expressions involving Bessel functions of imaginary argument as defined by Whittaker and Watson.[10]

These expansions were introduced into Eq. (1) in order to obtain the second term in an asymptotic expansion about zero energy of the cross section. The quantity

$$C_0^2=2\pi\eta/(e^{2\pi\eta}-1),$$

which enters through the F^2 in the numerator of the cross-section formula, contains the main energy dependence and was kept intact. Except for the case where a resonance occurs at a very low energy, the term $F^4\delta^2$ is small enough to be neglected and it has therefore been dropped. The result of the calculation thus performed is

$$\sigma=\frac{2\pi P}{v}\frac{H\beta}{(R+S\alpha)^2}\left\{1+(1/\eta^2)\left[\frac{M+N\alpha+O\beta}{R+S\alpha}+\left(\frac{x}{2}\right)^4\frac{\gamma}{\beta}\right]+\cdots\right\}\frac{2\pi\eta}{e^{2\pi\eta}-1},\tag{2}$$

where

$$H\equiv\nu b^3(x/2)^{-2}I_\nu^2(x)=(\nu/8)a^3(x/2)^4I_\nu^2(x),\tag{2.1}$$

$$R\equiv[\nu+1+2(x/2)^2/(\nu+1)]I_\nu(x)K_\nu(x)-[2(x/2)^2/(\nu+1)]I_\nu(x)K_{\nu+2}(x),\tag{2.2}$$

$$S\equiv-2I_\nu(x)K_\nu(x),\tag{2.3}$$

$$M\equiv\left[\frac{(\nu+2)(x/2)^4}{3(\nu+1)}-\frac{\nu(\nu-1)(x/2)^2}{12}-\frac{\nu(\nu-1)(\nu+1)^2}{24}\right]I_\nu(x)K_\nu(x)+\left[-\frac{(x/2)^4}{3(\nu+1)}+\frac{(\nu-1)(x/2)^2}{12}\right]I_\nu(x)K_{\nu+2}(x),\tag{2.4}$$

$$N\equiv\left[-\frac{(x/2)^4}{3(\nu+1)}+\frac{\nu(\nu-1)(\nu+1)}{12}\right]I_\nu(x)K_\nu(x)+\left[\frac{(x/2)^4}{3(\nu+1)}-\frac{(\nu-1)(x/2)^2}{6}\right]I_\nu(x)K_{\nu+2}(x),\tag{2.5}$$

$$O\equiv(x/2)^4S/4.\tag{2.6}$$

[7] Yost, Wheeler, and Breit, Phys. Rev. **49**, 174 (1936).
[8] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).
[9] G. Breit and M. H. Hull, Jr., Phys. Rev. **80**, 561 (1950).
[10] E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, Cambridge, 1927), Sec. 17.7.

TABLE I. Values of parameters corresponding to typical nuclear interactions.[a]

Reaction	$\eta^2 E$	x ($10^{13}r_0=1.4$)	E/ρ^2 ($10^{13}r_0=1.4$)	x ($10^{13}r_0=1.7$)	E/ρ^2 ($10^{13}r_0=1.7$)	x ($10^{13}r_0=2.1$)	E/ρ^2 ($10^{13}r_0=2.1$)
$\alpha+p$	0.0991	1.027	5.70	1.132	3.87	1.258	2.533
$C^{12}+p$	0.892	2.241	2.264	2.469	1.535	2.744	1.006
$Ne^{20}+p$	2.477	3.183	1.545	3.51	1.049	3.90	0.687
$S^{32}+p$	6.34	4.38	1.102	4.83	0.747	5.37	0.490
Li^7+d	0.446	1.935	2.034	2.133	1.379	2.370	0.904
$O^{16}+d$	3.171	3.79	0.983	4.18	0.666	4.64	0.437
$Al^{27}+d$	8.37	5.35	0.649	5.90	0.442	6.56	0.2898
$Li^7+\alpha$	3.92	3.62	1.464	3.99	0.994	4.43	0.651
$C^{12}+\alpha$	15.70	5.91	0.822	6.52	0.557	7.24	0.365
$Ne^{20}+\alpha$	43.6	8.61	0.508	9.49	0.344	10.55	0.2257

[a] The nuclear boundary is given by $b=r_0A^{\frac{1}{3}}$ where A is the sum of the mass numbers of the incident and the bombarded nuclei and $10^{13}r_0$ is 1.4, 1.7, and 2.1. Lengths are in centimeters and energies in Mev.

H, R, S, M, N, and O are functions only of the external region, and the other parameters in (2) are defined by the expansions

$$\rho u'/u=\alpha+(1/\eta^2)[-\beta(x/2)^4/8]+\cdots, \tag{3}$$

$$2\langle u^2\rangle=\beta+(1/\eta^2)[\gamma(x/2)^4]+\cdots, \tag{3.1}$$

with energy-independent α, β, and γ. It has been assumed that the nuclear potential does not extend beyond b.

The relation of η to the bombarding energy in Mev is[8]

$$E=(0.1574ZZ')^2M_i/\eta^2, \tag{4}$$

where M_i is the mass of the incident particle. The choice of a nuclear radius b determines

$$\rho\eta=ZZ'\mu b/(2.905\times10^{-12}\text{ cm}), \tag{4.1}$$

and thus x, and establishes the relation between E and ρ^2. The phase z of the wave function at b can be obtained from the knowledge of the internal potential. Thus if U is the depth of a square well potential inside b, the ratio of the square of the phase of the wave function at b to $E+U$ is the same as that of ρ^2 to E, and for zero energy $z^2/U=\rho^2/E$. In Table I values of the parameters are given for various reactions.

The functions defined by Eqs. (2.2)–(2.6) are tabulated in Table II for values of x less than 10. The functions α, β, and γ are obtainable from their definitions, Eqs. (3) and (3.1).

This expansion was compared for a simple case with calculations using the Coulomb function tables.[8] The parameters chosen, $x=2.76$ and $z=4$, correspond to $C^{12}+p$ with a square well potential of depth, $U=14.75$ Mev and range, $b=4.99\times10^{-13}$ cm. The proton energy is obtainable as $E=0.892/\eta^2$ Mev.

The solid straight lines in Fig. 1 represent the correction to the one-term low-energy formula due to terms linear in energy in this case. For $L=0$ accurate computations of σ using the Coulomb function tables give the same result as the expansion. The agreement is fortuitous in this case, as an examination of the energy dependence of $1-FG\delta$ indicates that the nonlinearity of $\rho\delta$ tends to cancel that of FG/ρ. For $L=1$ the difference of 10 percent when $1/\eta^2=0.5$ between the result of the computations using the Coulomb function tables and those using the expansion is largely due to the nonlinearity of $1-FG\delta$. The results of obtaining the values of the various components of the cross section, $2\langle u^2\rangle$, $1-FG\delta$, and F/ρ, from their low-energy expansions at $1/\eta^2=0.5$ and then combining these numbers directly indicate that terms of order $1/\eta^4$ are large enough to be seen on an ordinary graph even at this energy.

TABLE II. Values of coefficients $\begin{pmatrix} R & S \\ M & [N] \end{pmatrix}$ for different L.[a]

x \ L	0		1		2		3		4	
0	0	1.000	0.333	0.3333	0.400	0.2000	0.429	0.1429	0.444	0.1111
	0	[0]	0.333	[0.333]	2.000	[1.000]	6.00	[2.000]	13.33	[3.333]
1	0.238	0.680	0.353	0.3149	0.404	0.1960	0.430	0.1414	0.445	0.1104
	0.0312	[−0.0383]	0.354	[0.311]	2.020	[0.979]	6.02	[1.979]	13.35	[3.312]
2	0.362	0.445	0.383	0.2754	0.414	0.1853	0.434	0.1372	0.447	0.1084
	0.0604	[−0.1345]	0.395	[0.251]	2.064	[0.913]	6.07	[1.915]	13.40	[3.249]
3	0.412	0.3175	0.412	0.2345	0.426	0.1711	0.439	0.1312	0.450	0.1054
	0.00567	[−0.2736]	0.396	[0.110]	2.091	[0.799]	6.11	[1.806]	13.46	[3.143]
4	0.436	0.2437	0.432	0.1995	0.437	0.1558	0.446	0.1239	0.454	0.1015
	−0.197	[−0.453]	0.277	[−0.0665]	2.035	[0.635]	6.10	[1.651]	13.47	[2.991]
5	0.449	0.1968	0.446	0.1713	0.447	0.1412	0.452	0.1161	0.457	0.0971
	−0.609	[−0.673]	−0.0360	[−0.288]	1.795	[0.422]	5.96	[1.448]	13.39	[2.795]
6	0.458	0.1649	0.455	0.1491	0.455	0.1279	0.457	0.1083	0.461	0.0924
	−1.292	[−0.934]	−0.612	[−0.553]	1.378	[0.163]	5.62	[1.197]	13.13	[2.551]
7	0.464	0.1417	0.462	0.1314	0.461	0.1161	0.462	0.1010	0.464	0.0877
	−2.308	[−1.237]	−1.516	[−0.860]	0.629	[−0.140]	5.02	[0.902]	12.66	[2.263]
8	0.469	0.1243	0.467	0.1170	0.466	0.1060	0.466	0.0940	0.468	0.0830
	−3.72	[−1.581]	−2.813	[−1.208]	−0.495	[−0.487]	4.07	[0.559]	11.86	[1.925]
9	0.472	0.1106	0.471	0.1055	0.470	0.0971	0.470	0.0875	0.471	0.0784
	−5.60	[−1.969]	−4.57	[−1.600]	−2.062	[−0.880]	2.70	[0.1690]	10.67	[1.540]
10	0.475	0.0996	0.474	0.0958	0.473	0.0894	0.474	0.0817	0.474	0.0741
	−7.98	[−2.394]	−6.84	[−2.027]	−4.14	[−1.308]	0.859	[−0.2557]	8.27	[1.118]

[a] Graphical or parabolic interpolation is recommended.

The expansion has been used to investigate the effect of decreasing the nuclear radius while increasing the well depth so as to keep the same zero-energy value of the logarithmic derivative of the wave function at b. This makes z, and thus α, β, and γ remain unchanged. The correction to the one-term formula due to the second term is given in Fig. 1 by the solid straight lines when $x=2.76$, the dashed lines when $x=2.48$, and the dotted lines when $x=2.25$. The corresponding radii b are 4.99×10^{-13} cm, 4.04×10^{-13} cm and 3.32×10^{-13} cm, and the well depths are 14.75 Mev, 22.5 Mev and 33.2 Mev, respectively. For these cases $H\beta/(R+S\alpha)^2$ is given by $3.25b^3$, $2.18b^3$, and $1.55b^3$ for $L=0$, and $1.80b^3$, $0.409b^3$, and $0.150b^3$ for $L=1$.

Sawyer and Phillips[11] have measured cross sections at 90° for several of the reactions which occur when lithium or beryllium is bombarded by protons or deuterons in the 30- to 250-kev energy range. By assuming an isotropic distribution of the reaction particles they obtained values for the total cross sections. For most of their reactions $\ln(\sigma E)$ was a linear function of $E^{-\frac{1}{2}}$ with a slope not far from that predicted by the one-term asymptotic formula

$$\sigma E = A e^{-2\pi\eta}. \tag{5}$$

The slope corresponding to the two-term formula is

$$d\ln(\sigma E)/dE^{-\frac{1}{2}} = -2\pi\eta E^{\frac{1}{2}}[1+\Xi E/\eta\pi], \tag{5.1}$$

where

$$\Xi = \left[\frac{M+N\alpha+O\beta}{R+S\alpha} + \left(\frac{x}{2}\right)^4 \frac{\gamma}{\beta}\right]/\eta^2 E. \tag{5.2}$$

For reasonable models the ratio of this slope to that of the one-term formula was found to be of the same order of magnitude as the ratio obtained from the experimental curves. In the case of $Li^6(p,\alpha)He^3$ the ratio from experiment was 1.024 while the ratio computed from the two-term formula varied from 1.020 when $E=20$ kev to 1.118 when $E=250$ kev. These values were very insensitive to the nuclear radius chosen, showing little change as the radius was increased from 2.87×10^{-13} cm to 4.02×10^{-13} cm. The order of magnitude is not very sensitive to the well depth in the region of good convergence of Eqs. (3) and (3.1). For the ratio quoted, the well depth had been adjusted for a phase of 1.7 radians at the nuclear radius. The ratio obtained from experiment for the $Li^6(d,\alpha)He^4$ reaction was 0.967. If the well is adjusted to give a phase of about 1.7 radian at a nuclear radius of 4×10^{-13} cm, the calculated ratio varies from 1 to about 0.97 in the energy range considered. The calculated ratio shows considerable sensitivity both to the radius and the depth in this case. These calculations were made for $L=0$. No significance is being attached to the possibility of fitting experimental values exactly.

[11] G. A. Sawyer and J. A. Phillips, Los Alamos Report LA-1578 (unpublished).

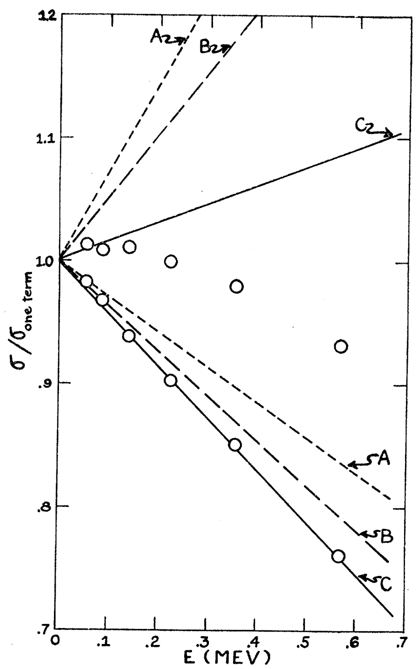

FIG. 1. Correction factor, $\{1+(1/\eta^2)[(M+N\alpha+O\beta)/(R+S\alpha)+(x/2)^4\gamma/\beta]\}$, to the one-term formula for the reaction cross section for $C^{12}+p$ as a function of the bombarding energy E. The dotted lines A correspond to a nuclear well of range $b=3.32\times10^{-13}$ cm and depth $U=33.2$ Mev, the dashed lines B to $b=4.04\times10^{-13}$ cm, $U=22.5$ Mev, and the solid lines C to $b=4.99\times10^{-13}$ cm, $U=14.75$. The points marked by circles represent the results of an exact calculation using the Coulomb function tables for parameters corresponding to lines C. The lower curves are for $L=0$ and the upper for $L=1$.

It appears appropriate to state again that in the correction terms discussed above, the decay of the wave function inside the nucleus is not taken into account.

Behavior of the f Function

Breit[6] has generalized the f function for cases where $L>0$:

$$f_L = [\nu C_L^2 \cot K_L - p_L \ln\eta + q_L]/\eta^\nu. \tag{6}$$

This function was computed for some one-body models for reactions between protons, deuterons, or alpha particles and nuclei smaller than Si^{28}. Values of f and $\nu C^2 \cot K/\eta^\nu$ were compared for several choices of $\rho\eta$, keeping $U=0$. For $\rho\eta=0.1$, f_0 and f_1 are almost constant, f_0 increasing from 79 to 81 as $1/\eta^2$ goes from 0 to 10 and f_1 changing from 5.4×10^4 to 5.5×10^4 in the same range. When $\rho\eta=0.538$, which represents the interaction of a deuteron with Li^7, f_0 changes almost linearly from 1.5 to 3.4 while f_1 goes from 41 to 60 with some curvature as $1/\eta^2$ goes from 0 to 10. These curves are not illustrated but are similar to those for $\rho\eta=0.952$ which corresponds to a proton incident on C^{12} where $b=4.99\times10^{-13}$ cm. The $\rho\eta=0.952$ curves, presented in Fig. 2, illustrate the effect produced by changing L

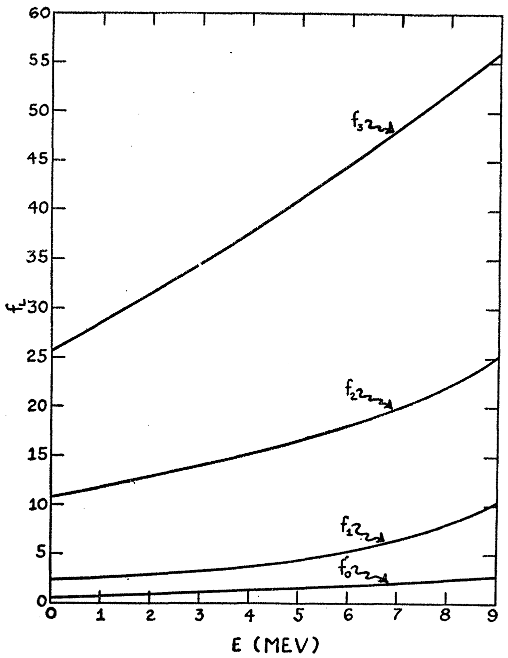

FIG. 2. The f_L function as a function of the bombarding energy, E, for $C^{12}+p$ with $b=4.99\times10^{-13}$ cm($\rho\eta=.952$) and $U=0$.

and are reasonably well represented by the series:

$$f_0=0.50+0.175/\eta^2+0.005/\eta^4; \tag{7}$$

$$f_1=2.45+0.055/\eta^2+0.055/\eta^4; \tag{7.1}$$

$$f_2=10.70+0.713/\eta^2+0.072/\eta^4; \tag{7.2}$$

$$f_3=25.70+2.24/\eta^2+0.084/\eta^4. \tag{7.3}$$

For values of $\rho\eta$ greater than 2, the term $(-p\ln\eta+q)/\eta^\nu$ tends to mask all the effects of the phase shift. In these cases a simple plot of $C_L{}^2 \cot K_L/\eta^\nu$ used in the spirit of Breit, Thaxton, and Eisenbud[12] could still be useful.

In order to facilitate the use of the f function, values of the term $(-p \ln\eta+q)/\eta^\nu$ have been computed for $L\leq4$ for values of $1/\eta^2<40$ and are presented in Table III.

For the case $L=1$, calculations were performed to see the effect of varying the range of the nuclear well. The well depth was adjusted in each case to restore the f function at zero energy to its previous value. The well depth is increased to 5.05 Mev when $\rho\eta$ is decreased to 0.72 which corresponds to $C^{12}+p$ where $b=3.77\times10^{-13}$ cm. Similarly $U=24.16$ Mev when $\rho\eta=0.50$ for which $b=2.62\times10^{-13}$ cm. The results are shown in Fig. 3. The straight lines indicate the f function as found from an expansion in powers of the energy similar to Eq. (2). The slope at zero energy rapidly steepens as the nuclear radius is decreased, illustrating the fact, pointed out by Breit,[6] that for zero range $\partial f/\partial E=-\infty$ for $L>0$.

In order to investigate the effect of changing the Coulomb field, the effect of changing a proton to a neutron in a reaction was considered. As f is proportional to $\eta^{-\nu}$ it was necessary to introduce a modification for zero charge. The function, $(e^2/mc^2a)^\nu f_L$, was used, the dependence of a on the charge Z canceling that of η, and the length, e^2/mc^2, being introduced to reduce the function to a convenient order of magnitude. For zero charge the term, $-p \ln\eta+q$, reduces to zero. In Fig. 4, comparisons are made for $L=1$ between $C^{12}+p$ and $C^{12}+n$, the nuclear radii and the potentials inside the nuclear boundary being the same as those of the lower two curves in Fig. 3. The case corresponding to $\rho\eta=0.952$ is not shown, as the phase shift would vanish since $U=0$ so that the modified f function would be infinite.

In changing from a proton to a neutron reaction some consideration must be made of the effect of the change in the internal potential. To estimate this the potential corresponding to a uniform distribution of charge inside the nucleus was calculated. The average value of this potential was then added to the depth of the internal well for the neutron reaction calculations. The modified f functions are shown by dotted lines in Fig. 4 for the same radii as above but with larger well depths. When $b=3.77\times10^{-13}$ cm the well depth is increased to 8.15 Mev and for $b=2.62\times10^{-13}$ cm, $U=28.61$ Mev.

The cross section of $H^3(d,n)He^4$ has recently been studied at energies below 120 kev by Arnold, Phillips,

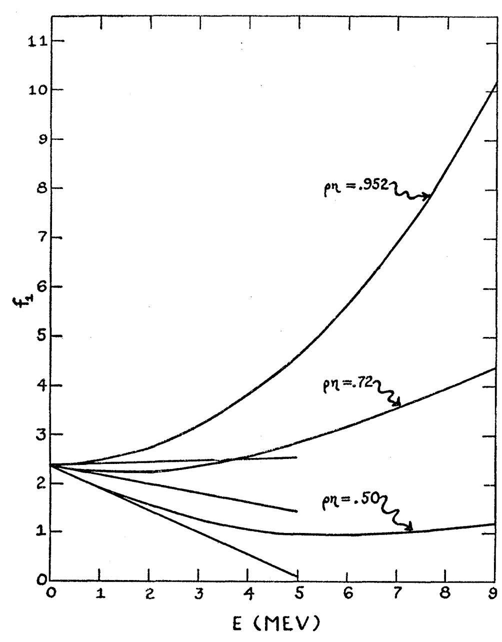

FIG. 3. The effect on the f function for $L=1$ of decreasing b and increasing U to keep the same zero-energy intercept. The curves, representing $C^{12}+p$, have parameters: $b=4.99\times10^{-13}$ cm ($\rho\eta=0.952$), $U=0$; $b=3.77\times10^{-13}$ cm ($\rho\eta=0.72$), $U=5.05$ Mev; and $b=2.62\times10^{-13}$ cm ($\rho\eta=0.50$), $U=24.16$ Mev. The straight lines indicate the zero-energy slopes as obtained from an asymptotic expansion.

[12] Breit, Thaxton, and Eisenbud, Phys. Rev. **55**, 1018 (1939).

Sawyer, Stovall, and Tuck,[13] as well as by Argo, Taschek, Agnew, Hemmendinger, and Leland[14] and by Conner, Bonner, and Smith.[15] At the peak of the 107-kev resonance the total cross section is 4.95 barns. Within experimental error it is isotropic[16] with angle at the resonance as well as at lower energies. There is no special objection therefore to assuming the reaction to be caused by incident deuterons in s states. Since in the comparison of resonance and potential well effects one is primarily interested in relative values the exact interpretation of the cross section in terms of contributions from different values of total angular momentum J is probably not too important. For this reason only the two extremes were tried in assignments of partial cross sections to $J=\frac{1}{2}$ and $\frac{3}{2}$, it being assumed that the whole cross section at resonance is associated with one or the other of these values of J. The experimental value of 4.95 barns is large enough to exclude the assignment of the reaction to $J=\frac{1}{2}$.[14,15] For $J=\frac{3}{2}$, the resonance denominator has the approximate form

$$(44\ \text{kev})^2+(E-E_0)^2,$$

giving a variation in σ of a factor $\frac{5}{2}$ as one changes E from about 60 kev to 20 kev. The denominator of the Breit-Wigner formula is readily seen to give a sufficiently strong dependence on E to make the direct application of Eq. (2) difficult. No attempt is being made here to find the best one-level formula fit since it is desired to illustrate the lack of constancy of the denominator. Even if one were to use 88 kev in place of 44 kev with constant Γ and no energy-dependent level shift, the factor $\frac{5}{2}$ would change to about 1.6 which is still larger than the one-body effects unless the latter are made to have distinct resonance features.

TABLE III. Values[a] of $(-p_L \ln\eta + q_L)/\eta^\nu$.

$1/\eta^2$	$L=0$	$L=1$	$L=2$	$L=3$	$L=4$
0	0.3089	−0.453	−0.013		
1	0.4982	−0.612	−0.091	−0.004	
2[b]	0.697	−0.632	−0.200	−0.012	
3[b]	0.875	−0.540	−0.303	−0.040	−0.003
4[b]	1.037	−0.383	−0.409	−0.074	−0.007
6[b]	1.300	0.138	−0.552	−0.170	−0.020
8[b]	1.510	0.851	−0.531	−0.271	−0.049
10	1.679	1.711	−0.364	−0.399	−0.099
12	1.828	2.689	0.050	−0.477	−0.162
14	1.956	3.777	0.712	−0.480	−0.235
16[b]	2.071	4.94	1.66	−0.370	−0.310
18	2.172	6.211	2.914	−0.082	−0.363
20	2.265	7.517	4.463	0.410	−0.381
22	2.350	8.915	6.425	1.199	−0.325
24[b]	2.429	10.36	8.85	2.30	−0.140
26[b]	2.503	11.84	11.50	3.03	0.180
28	2.569	13.39	14.34	5.873	0.695
30	2.633	14.98	17.78	8.406	1.502
32	2.693	16.63	21.71	11.54	2.617
34	2.750	18.30	25.94	15.33	4.159
36	2.802	20.00	30.60	19.82	6.172
38	2.855	21.80	35.86	25.29	8.861
40[b]	2.900	23.40	41.37	30.65	11.67

[a] Linear interpolation is accurate to ±0.03.
[b] Values were not calculated directly but were obtained graphically from neighboring calculated values.

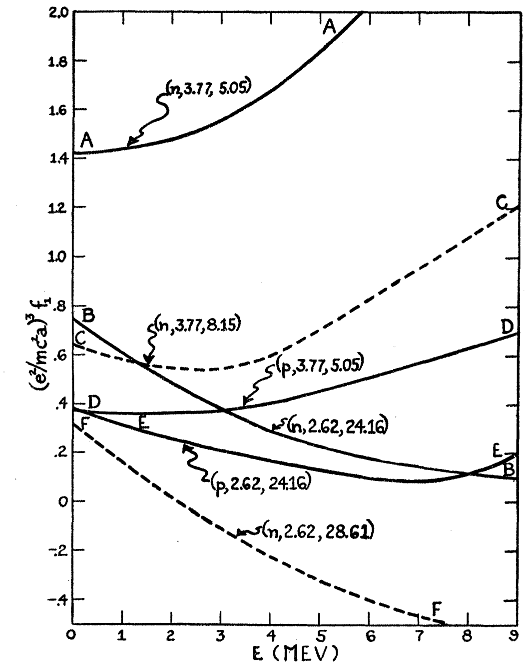

FIG. 4. Comparison of $(e^2/mc^2a)^\nu f_L$ for $C^{12}+p$ and $C^{12}+n$ reactions. For the solid curves the parameters representing the nuclear well are the same as for the two bottom curves in Fig. 3, *viz.*, curves D and E represent a proton reaction where $b=3.77\times10^{-13}$ cm ($\rho\eta=0.72$), $U=5.05$ Mev, and $b=2.62\times10^{-13}$ cm ($\rho\eta=0.50$), $U=24.16$ Mev, and the curves A and B represent the corresponding neutron reactions ($\rho\eta=0$). For the dotted curves the well depths have been increased so as to take approximate account of the Coulomb potential inside the nucleus. Thus for curve C the well depth has been increased to 8.15 Mev, the radius being kept at $b=3.77\times10^{-13}$ cm, and for curve F where $b=2.62\times10^{-13}$ cm the well depth is 28.61 Mev. In order to ensure positive identification of the graphs the triplet of symbols consisting of the designation of the reaction particle (p or n), the radius in 10^{-13} cm and the well depth in Mev are used as an additional label for each graph. Thus graph A has the additional identification (n, 3.77, 5.05).

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[13] Stovall, Arnold, Phillips, Sawyer, and Tuck, Phys. Rev. **88**, 159 (1952).
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