Hyperfine Structure in the Spectra of Sb, Sm, Hg, and Cd⁺

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Using two samples that were enriched in Sb¹²¹ and Sb¹²³, respectively, the hyperfine structure (hfs) of the spectrum of Sb II was studied, and the quadrupole moments were determined to be $O(Sb^{121}) = (-0.52 \pm 0.10)$ $\times 10^{-24}$ cm², $Q(Sb^{123}) = (-0.67 \pm 0.10) \times 10^{-24}$ cm². Study of the hfs of the spectra of Sm I and Sm II, using two samples enriched in Sm^{147} and Sm^{149} , respectively, yielded the result that the nuclear spins of Sm^{147} and Sm¹⁴⁹ are 7/2 and their nuclear magnetic moments are $\mu(\text{Sm}^{147}) = -0.76 \pm 0.08$ nm and $\mu(\text{Sm}^{149}) = -0.64$ ± 0.06 nm with μ (Sm¹⁴⁷)/ μ (Sm¹⁴⁹) = 1.198 \pm 0.015. Using a sample containing 1.5 percent of Hg¹⁶⁶, the hfs of the line Hg 11 M6150 was studied, and the existence of the Hg¹⁹⁸ component was just observed. Shifts of the
even isotopes in the lines Hg 1 N5461, Hg 1 N6123, and Hg 111 N4797 were measured, and it was found that they follow the regularity mentioned earlier. Using natural cadmium, the hfs of the line Cd II λ 4415 was studied, and the shift $Cd^{114}-Cd^{116}$ was found to be anomalously small compared with the shift $Cd^{112}-Cd^{114}$.

I. HFS OF THE SPECTRUM OF Sb II

TACH of the antimony isotopes contains only one & proton outside closed proton shells, so that their quadrupole moments would be especially suited for comparison with theory. In a previous work by Suwa and the author,¹ the quadrupole moment Q of Sb¹²¹ was deduced from the hyperfine structure (hfs) of the line Sb II λ 5640 (5*p*6s³ P_2 –5*p*6*p*³S₁)² and Q(Sb¹²³) was deduced from the hfs of the line Sb II λ 5895 (5 p 6s ³ P_1 – 5 p 6 p ³ P_0) measured by Tomboulian and Bacher.³ A simple calculation shows that the quadrupole effect is far larger in the term $5p6s^{3}P_{2}$ than in the term 5γ 6s 3P_1 , so that it is desirable to measure the hfs of $5p6s \,^3P_2$ for Sb¹²³, but this was almost impossible owing to the complexity of the hfs of the line λ 5640, if natural antimony only was used.

The separated isotopes Sb^{121} and Sb^{123} were available to the author in 1951.Using a hollow cathode discharge tube⁴ and a Fabry-Perot etalon the hfs of λ 5640 was examined. All the important components could be measured, but some weak components could not owing to the ghosts of the strong helium line X5876. The hfs diagram of X5640 for the natural antimony could now be completely constructed and is shown in Fig. 1. A number in parentheses was calculated, because the

The spins of Sb are known to be $I^{121} = 5/2$ and $I^{123} = 7/2$ from
the works of Badami (reference 2) and of M. F. Crawford and
Bateson, Can. J. Research 10, 693 (1934).
³ D. H. Tomboulian and R. F. Bacher, Phys. Rev.

O. H. Arroe and J. E. Mack, J. Opt. Soc. Am. 40, ³⁸⁶ (1950).

expected component was disturbed by the ghost and could not be measured accurately. The components b, c, e, \overline{b} , \overline{c} , \overline{e} , \overline{g} , and \overline{i} were measured on plates that were taken using separated isotopes. The other components were measured on plates that were taken using natural antimony.

From Fig. 1 we get

$$
A^{121} = 73.8 \times 10^{-3} \text{ cm}^{-1}, \quad B^{121} = -0.159 \times 10^{-3} \text{ cm}^{-1};
$$

$$
A^{123} = 40.0 \times 10^{-3} \text{ cm}^{-1}, \quad B^{123} = -0.098 \times 10^{-3} \text{ cm}^{-1}
$$

for the term $5p6s^{3}P_{2}$. From the ratio of B^{121} and B^{123} . we get the ratio of the quadrupole moments of Sb^{121} and Sb^{123} :

$$
Q^{123}/Q^{121} = 21B^{123}/10B^{121} = 1.29 \pm 0.10.
$$

This value is in agreement with that published by Loomis and Strandberg' (1.263) and that published by Dehmelt and Krüger⁶ (1.2689). Putting the values

FIG. 1. Hfs of the line Sb II λ 5640.

⁵ C. C. Loomis and M. W. P. Strandberg, Phys. Rev. 81, 798 (1951) ^s H. G. Dehmelt and H. Kriiger, Z. Physik 130, 385' (1951).

⁾The work with enriched isotopes was performed at the University of Wisconsin in 1950—1951 and was supported by the U. S. Office of Naval Research. The work with natural samples was performed in Tokyo. The enriched isotopes were produced by the Y-12 plant, Carbide and Chemicals Division, Oak Ridge, and were obtained by allocation from the U. S. Atomic Energy Commission.

K. Murakawa and S. Suwa, Phys. Rev. 76, 433 (1949).

² The Sb II spectrum was classified by J. S. Badami, Z. Physik
79, 224 (1932) and extensively by R. J. Lang and E. H. Vestine Phys. Rev. 42, 233 (1932). In the course of the study of the hfs of Sb II it was found that the classifications of a considerable number of lines given by Lang and Vestine should be modified, and the modified and extended classification was published by K. Murakawa and S. Suwa, Rept. Inst. Sci. Tech. Univ. Tokyo 1, 90 (1947) in Japanese.

of B in Casimir's formula (quoted in reference 1) we get

$$
Q^{121} = (-0.52 \pm 0.10) \times 10^{-24} \text{ cm}^2,
$$

$$
Q^{123} = (-0.67 \pm 0.10) \times 10^{-24} \text{ cm}^2.
$$

These values are more accurate than those given in the previous work.¹ Loomis and Strandberg set the upper limit of Q^{121} and Q^{123} at -0.50 and -0.62 (in unit of limit of Q^{121} and Q^{123} at -0.50 and -0.62 (in unit of Q^{121} and 10^{-24} cm²), respectively and the lower limit of Q^{121} and Q^{123} at -0.35 and -0.45 , respectively. Their estimates are in essential agreement with the values deduced here. Quite recently Sprague and Tomboulian⁷ have deduced Quite recently Sprague and Tomboulian⁷ have deduced
the values $Q^{121} = -1.3 \times 10^{-24}$ cm² and $Q^{123} = -1.7 \times 10^{-24}$ cm'. From their short description it is not possible to find source of the discrepancy.

II. HFS OF THE SPECTRA OF Sm I AND Sm II

In a previous work by Ross and the author⁸ the hfs of the spectra of Sm I and Sm II was examined, using two samples that were enriched in Sm¹⁴⁷ and Sm¹⁴⁹ (see Table I and reference 9), respectively; and it was concluded $Sm¹⁴⁷$ and Sm¹⁴⁹ have the same spins equal to 5/2, that because the maximum number of hyperfine components of any Sm line with a fIag pattern was observed to be six. This conclusion was based unfortunately on the

TABLE I. Isotopic constitution (percent) of the samples of samarium.

Sample	Isotope									
label	144	147	148	149	150	152	154			
149	0.55	5.09	11.88	71.53	3.98	4.85	2.13			
147	1.10	81.63	6.96	3.94	1.20	3.41	.77			
Natural	3.16	15.07	11.27	13.84	7.47	26.63	22.53			

assumption that the contribution of the complex $4f^6$ to the hfs splitting of the configuration $4f^6$ 6s or $4f^6$ 6 ϕ was negligible, so that the hfs of the transition $4f^66s - 4f^66p$ represented directly the approximate hfs of $4f⁶6s$. The contribution of $4f^6$ is small only when the J value of the term is small, for example, equal to 1 or $\frac{1}{2}$. However, in order to determine the nuclear spin, just those lines¹⁰ that involve sufficiently high J values should be chosen. In this case the aforementioned finding leads to the conclusion that the spin is probably equal to $5/2$ or larger, because the hfs of any line attributed to the transition $4f^66s - 4f^66p$ does not represent the splitting of the term $4f⁶$ 6s directly, it being possible that both the upper and the lower term have the same order of magnitude of hfs splitting. It is, therefore, necessary to determine the nuclear spins and magnetic moments, keeping these points in mind.

- ⁷ G. Sprague and D. H. Tomboulian, Phys. Rev. 91, 476 (1953).
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^s K. Murakawa and J. S. Ross, Phys. Rev. 82, ⁹⁶⁷ (1951). 'The isotropic constitution of natural samarium was taken

from Inghram, Hayden, and Hess, Phys. Rev. 73, 180 (1948).
¹⁰ The classification of the spectra of Sm I and Sm II was published by W. Albertson, Phys. Rev. 47, 370 (1935); 52, 644 (1937)
and Astrophys. J. 84, 26 (1936), respectively. That of Sm I was
extended by P. Brix, Z. Physik 126, 431 (1949). The literature
concerning the shift of even

FIG. 2. Hfs of the lines Sm II λ 4467 and Sm I λ 5626.

After measurement of the hfs splittings of numerous terms of Sm II, it was found that the term $4f^6$ 6s $^6F_{11/2}$ has a small splitting, and finally the line Sm π λ 4467 $(4f⁶6s⁶F_{11/2} - 84^o_{13/2})$ was chosen for determining the spin. This line is so weak that it was overlooked in the previous work.⁸ The measured hfs for the sample 147 is given in Fig. 2. The two tail components were not so well resolved, but they could still be recognized as two. From Fig. 2 we can conclude that the term $84^{\circ}_{13/2}$ of Sm¹⁴⁷ is inverted (hfs level with the largest F lies deepest) and splits into eight hfs levels, so that the spin of Sm¹⁴⁷ is $7/2$. The hfs of λ 4467 of the sample 149 is quite similar to that of the sample 147; however, the scale is somewhat smaller. The spin of $Sm¹⁴⁹$ is, therefore, also 7/2. The hfs of the line Sm 1 λ 5626 (4f⁶6s² $7F_0$ – 28₁) shown in Fig. 2 also supports qualitatively this spin value. This conclusion is in agreement with that of Bogle and Scovil, $¹¹$ who found the spins from para-</sup>

FIG. 3. Hfs of the line Sm II λ 4515.

¹¹ G. S. Bogle and H. E. D. Scovil, Proc. Phys. Soc. (London) 65, 368 (1952).

magnetic resonance measurements of samarium ethyl sulfate.

The ratio of the nuclear magnetic moments (μ^{147}/μ^{149}) can be determined from the line Sm $I₁$ λ 4515, which was classified in the previous work⁸ as due to a transition $J=\frac{1}{2} \rightarrow J'=\frac{1}{2}$. In the mean time this line has been classified as $4f^66s^6F_4-4f^55d^6s X_4(X_4=23660.03)$. The hfs of this line is given in Fig. 3, from which we get

$$
\mu^{147}/\mu^{149} = (\bar{d}-\bar{a})/(d-a) = 1.198 \pm 0.015.
$$

This value is more accurate than that of the previous work,⁸ because the hfs was now measured more carefully and on more plates. Bogle and Scovil¹¹ got the value 1.222 ± 0.008 for the ratio. This is probably in essential agreement with the value obtained here.

From Fig. 3 we get $A^{147}(4f^66s^6F_*) = -0.0175$ cm⁻¹. In the previous work⁸ it was found that the splitting of the term $4f^{6}6s^{8}F_{\frac{1}{3}}$ of Sm¹⁴⁷ is -0.120 cm⁻¹. We, therefore, get $A^{147}(4f^66s^8F_*) = -0.0300$ cm⁻¹. $A(^8F_*)$ $+A(^{6}F_{4})$ should be independent of coupling, and is equal to $\frac{2}{3}a(6s)$, where the contribution of the complex $4f⁶$ is neglected. We get, therefore, $a¹⁴⁷(6s) = -0.0712$ cm⁻¹. Putting this value and $n^{*3}/(dn^{*}/dn) = 9.85$ in the Fermi-Segrè-Goudsmit formula, we get finally

$$
\mu^{147} = -0.76 \pm 0.08
$$
 nm.

Using the above-mentioned value of μ^{147}/μ^{149} , we get

$$
\mu^{149} = -0.64 \pm 0.06
$$
 nm.

Elliott and Stevens¹² obtained $|\mu^{147}| = 0.68 \pm 0.1$ nm and $|\mu^{149}| = 0.55 \pm 0.1$ nm from the data of Bogle and Scovil.¹¹

From the hfs of the line Sm I λ 5626 (4f⁶6s² r_{0} – 28₁) shown in Fig. 2, the following constants for the term 281 are obtained:

$$
A^{147} = -13.33 \times 10^{-3} \text{ cm}^{-1},
$$

\n
$$
B^{147} = (-0.021 \pm 0.08) \times 10^{-3} \text{ cm}^{-1}.
$$

This A^{147} , when combined with the above-mentioned value of μ^{147}/μ^{149} , gives the value of A^{149} , and then we get from the distance ac the value of B^{149} :

$$
A^{149} = -13.20 \times 10^{-3} \text{ cm}^{-1},
$$

$$
B^{149} = (-0.00 \pm 0.08) \times 10^{-3} \text{ cm}^{-1},
$$

where the uncertainty in the values of B comes mainly from the disturbance due to the existence of minor quantities of Sm¹⁵⁰, Sm¹⁵², and Sm¹⁵⁴ in the enriched isotope samples. In the same way the constants B in some terms with $J=1$ were measured, but no definite quadrupole effect could be detected. On the other hand it is possible that the upper terms with $J=1$ perturb each other, and it is dificult to calculate accurate wave functions for these terms. The limit for O was roughly estimated, and it is possible at the present time

to give the estimates only:

$$
|Q^{147}|, |Q^{149}| < 1 \times 10^{-24} \text{ cm}^2.
$$

Elliott and Stevens¹² got the limit $|Q^{147}|$, $|Q^{149}| \le 0.72$ $\times 10^{-24}$ cm².

In order to get more accurate values of Q , it would be necessary to get the sample in an isotopically pure state. Especially Sm¹⁴⁷ would deserve further study in view of the discussion by Hill and Wheeler¹³ about the nuclear deformation in connection with the alpha activity¹⁴ of Sm¹⁴⁷.

III. ISOTOPE SHIFT IN THE SPECTRA OF Hg I, Hg II, AND Hg III

In order to study the isotope shift of Hg^{196} , the enriched 196 sample (see Table II) was placed in an iron hollow cathode discharge tube and, using helium as the carrier gas, the line Hg II λ 6150 (7s²S_i-7p²P_i) was obtained with strong intensity. The hfs of λ 6150 is shown in Fig. 4, in which the calculated relative intensity (in percent) is given under the notation of each component. If the shift 198—196 were of the same order of magnitude as 204—202, the 196 component should lie about midway between 198 and a , but this is not the case. The 198 component is somewhat diffuse to the smaller-frequency side, and this was interpreted as due to the unresolved 196 component, whose position was estimated to be about -0.110 cm^{-1} . The shift 198–196 is then about 0.019 cm⁻¹.

Since the 196 sample contains larger percentage of $Hg²⁰⁴$ than natural mercury, shift of even isotopes in

FIG. 4. Hfs of the line Hg $\scriptstyle\rm II$ λ 6150.

¹³ D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953). ¹² R. J. Elliott and K. W. H. Stevens, Proc. Phys. Soc. (London) ¹⁴ Rasmussen, Reynolds, Thompson, and Ghiorso, Phys. Rev. 65, 370 (1952). 65, 370 (1952). 80, 475 (1950).

the line Hg r λ 5461 (6s6p³ P_2 –6s7s³S₁) was also measured. The shift $204 - 202$ was measured to be 0.0306 cm⁻¹. Since the 202 component (intensity 25.2) is expected to have superimposed on it a partial component (intensity 5.4) of Hg^{199} lying 0.0299 cm⁻¹ to the smaller-frequency side of the 204 component, the corrected shift $204-202$ is 0.0309 cm⁻¹. The shift of the even isotopes is given in Table III, together with the measurement of Sterner¹⁵ for the same line. The position of Hg¹⁹⁸ given by Sterner deviates somewhat from that of the present author, but the source of the discrepancy is difficult to find at present.

In the measurement using natural mercury, a highfrequency electrodeless discharge tube was used. This light source was found to give sharper lines than a hollow cathode tube. Hg I λ 6123 (7s³S₁-5d⁹6s²6p¹D₂) and Hg III λ 4797 (5d⁸6s² 12₄ – 5d⁹6p 2₃⁰)¹⁶ were especially strong, and the result of the measurement is given in Table III. The present measurement for λ 6123 is

TABLE III. Isotopic shift of the even isotopes of mercury^a $(\Delta \nu \text{ in cm}^{-1})$.

		Isotope					
	Line	198	200	202		204	
					0.0298		
					0.0309		
	Hg I λ 5461 $\begin{cases} \Delta \nu (J.S.)^b & 0.0276 & \\ \Delta \nu (K.M.)^c & 0.0276 & 0.0311 & 0.089 \\ \text{Ratio}(K.M.)^c & 0.89 : 1.01 : 1 \\ \pm 0.03 & \pm 0.03 \end{cases}$						
		0.1886		0.2131	0.2090		
Hg 1 λ6123 $\left\{\n\begin{array}{c}\n\Delta \nu \\ \text{Ratio}\n\end{array}\n\right\}$		± 0.012 ± 0.012		0.902 : 1.020 : 1			
		0.552		0.604	0.599		
Hg 111 λ 4797 $\begin{cases} \Delta \nu \\ \text{Ratio} \end{cases}$				$0.922 : 1.009 : 1$ ± 0.006 ± 0.006			

a In all three lines listed the 204 component lies on the highest-frequency s1de. ^b J. Sterner (see reference 15). & Our results.

out results.
more accurate than the previous one.^{17,18} From Table III we see that the ratio $\Delta \nu (198-200)$: $\Delta \nu (200-202)$: $\Delta \nu (202 - 204)$ is constant at least as a first approximation, as was inferred in references 17 and 18. If the

FIG. 5. Hfs of the line Cd π λ 4415 (c.g. means center of gravity).

result of Sterner¹⁵ were accepted, $\Delta \nu (198-200)$ $\Delta\nu(202-204)$ for λ 5461 would have the anomalous value 0.805. Such an anomalous value for λ 5461 only is very improbable, however.

IV. ISOTOPE SHIFT IN THE SPECTRUM OF Cd II

Koch and Rasmussen¹⁹ and Ross and the author²⁰ found an anomalous shift of the even isotopes in the spectra of Xe I and of Te II, respectively. The neutron numbers of the Cd isotopes are fairly near to those of the Te isotopes, and the isotope shift in the spectrum of Cd deserves investigation. This was studied by of Cd deserves investigation. This was studied by
previous workers,²¹ but they detected components due to Cd¹¹⁰, Cd¹¹², and Cd¹¹⁴ only.

In the present work the hfs of the line Cd π λ 4415 $(4d^{10}5p^2P_4-4d^95s^2D_4)^{22}$ was studied, using natural cadmium and a liquid-air-cooled hollow-cathode discharge tube. The result of measurement is shown in Fig. 5. The hfs splittings of the odd isotopes were calculated, using the magnetic moments given in the calculated, using the magnetic moments given in the
literature.²³ The distance 114—116 is anomalously small, and

$$
\Delta\nu(114-116)/\Delta\nu(112-114)=0.70\pm0.02.
$$

This kind of anomaly would be probably interpreted according to the idea recently proposed by Wilets, Hill, and Ford.²⁴

The author would like to express his appreciation for the kind cooperation of Dr. J. S. Ross in the work with enriched isotopes. It was a great pleasure to be able to talk about interesting related subjects with Professor J. E. Mack, Dr. J. G. Hirschberg, and Dr. J. S. Ross during my stay in Madison, Wisconsin.

¹⁵ J. Sterner, Phys. Rev. 86, 139 (1952). All the data of previous workers concerning λ 5461 are listed in his article. See also K. Burns and K. B. Adams, J. Opt. Soc. Am. 42, 56, 716 (1952). ¹⁶ The hfs of the line Hg III λ 4797 was interpreted by S. Mrozow

ski, Phys. Rev. 57, 207 (1940), and later the classification was
published by E. W. Foster, Proc. Roy. Soc. (London) A200, 429
(1950). E. W. Foster [Proc. Roy. Soc. (London) A208, 367 (1951)]
also measured the hfs of λ 4 resolving power, he could resolve the structure only incompletely. In the present work the hfs of '44797 could be resolved completely, and the result of the measurement is: —1.¹⁵⁶⁰ (198), —1.⁰⁷⁴⁶ and the result of the measurement is: $-1.1500 (198)$, $-1.0/40$
(199), $-0.6043 (200)$, $-0.4293 (201)$, 0 (202), 0.5994 (204) cm⁻¹,
the accuracy being ± 0.0012 cm⁻¹. Contrary to the calculation of
Foster, the 199 an has a splitting smaller than about 0.010 cm⁻¹.

¹⁷ K. Murakawa, Phys. Rev. 78, 480 (1950).

¹⁸ K. Murakawa and S. Suwa, J. Phys. Soc. Japan 5, 429 (1950).

¹⁹ J. Koch and E. Rasmussen, Phys. Rev. 77, 722 (1950).
²⁰ J. S. Ross and K. Murakawa, Phys. Rev. 85, 559 (1952).
²¹ H. Schüler and H. Westmeyer, Z. Physik 82, 685 (1933);
P. Brix and A. Steudel, Z. Physik 128, 260

²² This and other lines of Cd II were classified by Y. Takahashi,

Ann. Physik 3, 27 (1929).

²³ J. E. Mack, Revs. Modern Phys. 22, 64 (1950); P. F. A.

Klinkenberg, Revs. Modern Phys. 24, 63 (1952).

²⁴ Wilets, Hill, and Ford, Phys. Rev. 91, 1488 (1953).