

FIG. 6. Temperature dependence of the conductivity slope.

in series with conducting grains. In NiO the grain boundaries are conducting and therefore act in parallel with the grains. For this situation it can be shown that Eq. (7) becomes

$$\varphi = \bar{\varphi} - S^2 / 2kT. \tag{8}$$

Grain boundary conductivity was introduced into samples 7 and 8 by cooling them from sintering temperature in an oxidizing atmosphere. Conductivity is shown in Fig. 1. The low-temperature conductivity behavior is in accord with Eq. (8) if the slope of the Fig. 1 curves is taken to be effective barrier height. The temperature dependence of the conductivity slope is shown in Fig. 6 as plotted points. Straight lines are drawn through the points to determine the intercept at 1/T=0 which is  $\bar{\varphi}$  and the slope which is  $S^2/2k$ . Points deviate from straight line behavior at 200°K probably because the normal conductivity process becomes comparable to grain boundary conductivity. From these

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# Drift Mobilities in Semiconductors. II. Silicon

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The drift mobility of holes in *n*-type silicon and electrons in *p*-type silicon has been measured as functions of impurity concentration and temperature. In single crystals of resistivity greater than 10 ohm-centimeter, the mobility at 300°K of holes is  $\mu_P = 500 \pm 50 \text{ cm}^2/\text{volt-sec}$  and of electrons is  $\mu_N = 1200 \pm 100 \text{ cm}^2/\text{volt-sec}$ . For this high-resistivity material, the temperature dependence of mobility in the same units is  $\mu_N = 5.5 \times 10^6 T^{-1.5}$  and  $\mu_P = 2.4 \times 10^8 T^{-2.3}$ .

#### INTRODUCTION

THIS paper constitutes a companion paper to one dealing with the drift mobilities in germanium.<sup>1</sup> The drift mobility of holes and electrons has been measured in silicon single crystals grown from starting material of relatively high purity. Drift measurements were made on samples that were cut from crystals whose resistivity values ranged from 0.3 to 30 ohmcentimeter at 300°K. The variation of mobility with

 $^1\,M.$  B. Prince, Phys. Rev. 92, 681 (1953) hereinafter referred to as I.

temperature was measured over a temperature range of  $150^{\circ}$ K to  $300^{\circ}$ K in two of these samples. Calculated from the experimental data are curves giving the relationship between the number of impurity centers and the resistivity, and the ratio of the electron mobility to the hole mobility in *n*-type and *p*-type silicon as a function of the number of impurity centers.

A discussion of the experimental techniques, preparation of samples, sources of error, and methods for combining the results of various scattering mechanisms is given in I.

results  $\bar{\varphi} = 0.30$  ev and S = 0.059 ev for sample 7, and  $\bar{\varphi} = 0.55$  ev and S = 0.108 ev for sample 8.

#### 10. SUMMARY

Conductivity, Seebeck effect, and optical transmission of NiO have been analyzed using two alternative models. The first model assumes the following: conduction is in the d levels of nickel; the Seebeck effect depends only upon the location of the Fermi level; the level density involved in conduction equals the cation density. The model yields a quantitative fit to the data. Impurity concentration determined electrically agrees with that determined chemically. Electrical and optical results are consistent. Carrier mobility is found to be below 0.1 cm<sup>2</sup>/volt sec. It has an activation energy dependent upon impurity concentration and temperature which is explained on the basis of wavy bands. At high temperature and impurity concentration mobility activation energy tends towards a minimum value of 0.1 ev which may be the energy involved in the transfer of charge from one cation to another.

The second model assumes d-level conduction and, in addition, conduction in the filled sp band of oxygen by holes having normal mobility behavior. A quantitative investigation has been made of this model with unsatisfactory results. The idea of sp-band plus d-level conduction is shown to be reasonable.

Two instances of grain boundary conduction are presented and analyzed in terms of a modified theory of Henisch.

## MOBILITY VERSUS IMPURITY CENTER DENSITY

# **A.** Experimental Results

The experimental results showing the mobility of electrons in p-type silicon and of holes in n-type silicon are given in Fig. 1 as a function of the resistivity of the sample. All the data presented in this section have been corrected to 300°K by using the temperature variation of the mobility which is discussed in a later section. Each experimental point represents the average of several individual measurements. In the case of electron mobility these results agree with the value



FIG. 1. Drift mobility versus resistivity for electrons in p-type silicon and holes in *n*-type silicon.

obtained by Haynes and Westphal<sup>2</sup> over the range of their resistivity values.

## **B.** Interpretation

Less information can be extracted from the experimental data in the case of silicon than in the case of germanium.<sup>1</sup> The reason for this is twofold. Firstly, the data for silicon are fewer and extend over a smaller range than for germanium. Secondly, the art of purifying silicon has not at present reached the high degree attained in the case of germanium. There is some



FIG. 2. Resistivity versus density of uncompensated impurities for n- and p-type silicon.





FIG. 3. Electron mobility versus density of scattering centers in silicon.

evidence that there is compensation of one type of impurity (donor or acceptor) by the other type in the silicon crystals used in this experiment near the 10<sup>15</sup> cm<sup>-3</sup> level.<sup>3</sup> For the present we shall consider this evidence to be correct and calculate the number of impurities in a specimen  $N_I$  by assuming that the minority type of impurity is present at a density level of  $5 \times 10^{14}$  cm<sup>-3</sup>. While it is known that this assumption may be a poor one, it will give a first order approximation to the parameters we wish to determine. Using the relations  $N_A - N_D = 1/\rho_P q\mu_P$  and  $N_D - N_A = 1/\rho_N q\mu_N$ together with estimates of  $b = \mu_N / \mu_P$ , as explained in



scattering centers in silicon.

I, one can then obtain the density of majority impurities  $(|N_A - N_D| + 5 \times 10^{14})$  and the total density of impurities  $N_I (= N_D + N_A = |N_D - N_A| + 10^{15})$ .  $N_D$  and  $N_A$  represent the density of donors and acceptors,  $\rho_N$ and  $\rho_P$  the resistivity of *n*- and *p*-type material,  $\mu_N$  and  $\mu_P$  the mobility of electrons and holes, and q the charge on an electron. With this information a plot of  $\rho$  versus  $|N_D - N_A|$  can be obtained for p- and n-type silicon (Fig. 2), and plots of  $\mu_N$  and  $\mu_P$  versus  $N_I$  can be obtained (Figs. 3 and 4).

The solid curves in Figs. 1, 3, and 4 are empirical and have the same general shape as theoretical curves that include impurity scattering of the Conwell-Weisskopf<sup>4</sup> type. In drawing these curves, the higher values of mobility in the various parts of the plots are weighted more heavily due to the compensation effect

<sup>&</sup>lt;sup>3</sup> Fuller, Theuerer, and Morin (unpublished). <sup>4</sup> E. Conwell and V. F. Weisskopf, Phys. Rev. **69**, 258 (1946); 77, 388 (1950).



FIG. 5. Electron and hole mobilities in n-type silicon.



FIG. 6. Electron and hole mobilities in p-type silicon.

of impurities on the mobility (see I). Extrapolation of these curves to near the intrinsic range yields a latticescattering mobility of 1500 and 500 cm<sup>2</sup>/volt-second, respectively, for electrons and holes. Using these curves, one can obtain curves of drift mobility of the majority



FIG. 7. Drift mobility versus temperature in silicon.

carrier and b for *n*-type and *p*-type silicon.<sup>5</sup> These are given as Figs. 5 and 6.

#### MOBILITY VERSUS TEMPERATURE

The results of measuring the temperature dependence of the drift mobility of electrons in *p*-type and of holes in *n*-type silicon are shown in Fig. 7. Down to  $150^{\circ}$ K the results are consistent with the lines  $\mu_N = 5.5 \times 10^6 T^{-1.5}$  cm<sup>2</sup>/volt-sec and  $\mu_P = 2.4 \times 10^8 T^{-2.33}$ cm<sup>2</sup>/volt-sec. Since there was appreciable impurity scattering in the *p*-type sample, the exponent for pure lattice scattering of electron mobility may be larger than the given -1.5. In the case of the *n*-type sample, the impurity scattering was negligible and therefore the exponent -2.3 is probably the correct exponent for pure lattice scattering. These results indicate that silicon, just as in the case of germanium, probably does not have a simple picture in the band theory of solids.

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 $<sup>^{5}</sup>$  See I for a discussion of the difference between the drift mobility of a majority carrier and the drift mobility of a minority carrier.