Nucleon-Nucleus Collisions at Relativistic Energies

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The Fermi theory of multiple meson production is utilized in the interpretation of high-energy collisions between nucleons and complex nuclei. A relation between the total number of relativistic particles produced, their angular spread, and the number of nucleons involved in the collision is found to be satisfied by all the cases of relativistic showers published thus far.

THE showers of relativistic particles produced by the nucleonic component of the cosmic radiation, as observed in photographic plates, present radical differences in their angular distribution and in their relative proportion of light and heavy tracks.

This has usually been interpreted as due to the fact that one is dealing, in general, not with single nucleon-nucleon collisions, but rather with composite collisions, i.e., events in which the primary nucleon hits several of the nucleons present in the nucleus involved.

Composite collisions are not the ones most suited for the study of the peculiarities of the nucleon-nucleon interaction, so the experimenters have tried to select the cases which could be thought of as those in which only one of the nucleons in the nucleus is hit (single collisions).

Often considered as examples of single collisions have been the cases in which zero or very few slow particles are observed in addition to the relativistic particles, the argument being that, while in composite collisions the nucleus is strongly excited and boils out many charged particles of small energy, in single collisions the nucleon involved supposedly lies at the periphery of the nucleus and the remaining nucleons can be left practically unexcited.

However, the argument can be wrong in many cases, especially at very high energies, since in a composite collision, when the velocity of the center of mass (c.m.) is quite large, all the particles produced in the first nucleon-nucleon encounter are strongly collimated forward, and in going through the nucleus produce a tunnel[1] which can leave the nucleus not strongly excited. In going through a nucleus of $A=100$, no more than 4–5 nucleons can be hit by the primary nucleon and the breaking of this number of bonds can give an excitation of less than 100 Mev; few neutrons can be emitted and no or few charged particles.[2]

In this note an attempt is made to establish on a different basis a criterion for distinguishing composite from single collisions.

Consider a composite collision at very high energy (primary nucleon with $\gamma=E/Mc^2>200$) in which n nucleons are involved. The n nucleons in the nucleus are at distances one from another of the order of the range of nuclear forces, and hence it is not possible to consider the n successive collisions separately.

A more reasonable picture is to assume that all the lump of n nucleons interacts with the incoming nucleon and the number of particles created in the collision is thus a function of γ and n.

The velocity of the center of mass ($c=1$) is

$$\beta_c=(\gamma^2-1)^{\frac{1}{2}}/(\gamma+n),$$

and correspondingly

$$\gamma_c=(\gamma+n)/(2n\gamma+n^2+1)^{\frac{1}{2}} \underset{\gamma\gg n}{\longrightarrow} (\gamma/2n)^{\frac{1}{2}}.$$

The total energy available in the c.m. system ($Mc^2=1$) is:

$$W_c=(2n\gamma+n^2+1)^{\frac{1}{2}} \underset{\gamma\gg n}{\longrightarrow} (2n\gamma)^{\frac{1}{2}}.$$

To obtain the number of particles produced in the collision, it is necessary to assume a model.

It appears that the Fermi model[3] can give the most reasonable results. The multiplicity N of the charged particles produced in the interaction will be given by an expression similar to that given by Fermi (extreme relativistic case)

$$N=K\gamma^{\frac{1}{4}}f(n);$$

$f(n)$ represents the effect of the composite collision and becomes equal to unity for $n=1$.

The form of $f(n)$ depends on the assumptions about the volume Ω inside which the equilibrium of the meson-nucleon gas takes place. It seems reasonable to put

$$\Omega=n\Omega_0/\gamma_c=\Omega_0(2n^3/\gamma)^{\frac{1}{2}},$$

where

$$\Omega_0=(4/3)\pi(\hbar/\mu c)^3.$$

The factor $1/\gamma_c$ represents the relativistic contraction in the direction of motion, and the factor n comes from the fact that n nucleons are involved. Since the n nucleons are not aligned, it is not possible to define an impact parameter, and the correction for the conservation of momentum does not have to be considered (the correction, a factor $\frac{1}{2}$ in the multiplicity, stands when $n=1$, single collisions). With these assumptions, the dependence of N on n is easily found.

The density of energy in the volume Ω is

$$\delta=W_c/\Omega\propto\gamma n^{-1}.$$

[1] M. F. Kaplon and D. M. Ritson, Phys. Rev. **88**, 386 (1952). F. C. Roesler and C. B. A. McCusker, Nuovo cimento **10**, 127 (1953).

[2] This problem has been considered by W. Heitler and C. Terraux in Proc. Phys. Soc. (London) **A66**, 929 (1953).

[3] E. Fermi, Progr. Theoret. Phys. **5**, 570 (1950).

TABLE I. Summary of all the showers reported in the literature, produced either by a single ionizing particle or by a neutral particle, with (a) $\eta<0.10$; (b) $\eta<0.20$. The rows give the numbers of cases with $N\eta^{\frac{1}{2}}$ comprised between 0 and 1 (row 0), between 1 and 2 (row 1), etc. In the columns the cases are subdivided according to the number N_h of heavy tracks accompanying the shower. The subscript "*nb*" refers to unbiased samples, i.e., those cases published by the authors without discarding the showers accompanied by a large number of heavy prongs (references k and q). The subscript "all" refers to all cases reported, most of them biased in favor of events with a few heavy prongs. Column "?" refers to showers originated in thin sheets of brass and detected in nearby plates, for which there is no information about N_h (reference j).*

N_h / $N\eta^{\frac{1}{2}}$	0 *nb*	0 all	1–2 *nb*	1–2 all	3–5 *nb*	3–5 all	6–10 *nb*	6–10 all	>10 *nb*	>10 all	?	Totals *nb*	Totals all
(a) $\eta<0.10$													
0		1		2									3
1	3	4	2	3	2	3	1	1	1	1	6	9	18
2	1	4	1	2				1			4	2	11
3											8		8
4				1	1	1					2	1	4
5	1	1				1		1		1	2	1	6
6			1	1	1	1						2	2
12							1	1	1	1		2	2
(b) $\eta<0.20$													
0		1		2									3
1	3	4	2	3	2	3	1	1	1	1	6	9	18
2	1	4	1	3	1	3		1			4	3	15
3			2	3	1	3					8	3	14
4	1	1		3	1	2		1			2	2	9
5	1	1	1	1	1	2		1		2	2	3	9
6		1	1	1	1	1						2	3
7						1							1
8		1			2	2						2	3
9													
10													
11			1	1	1	1			1	1		3	3
12							1	1	1	1		2	2
13							1	1				1	1
14					1	1						1	1
23									1	1		1	1

* Papers from which the cases were taken are:
[a] Bertolino, Debenedetti, Lovera, and Vigone, Nuovo cimento **10**, 991 (1953).
[b] Bradt, Kaplon, and Peters, Helv. Phys. Acta. **23**, 24 (1950).
[c] Brown, Camerini, Fowler, Heitler, King, and Powell, Phil. Mag. **40**, 862 (1949).
[d] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[e] K. Gottstein and M. Teucher, Z. Naturforsch. **8a**, 120 (1953).
[f] Hopper, Biswas, and Darby, Phys. Rev. **84**, 457 (1951).
[g] S. Biswas and V. D. Hopper, Phys. Rev. **86**, 209 (1952).
[h] Lal, Pal, Peters, and Swami, Proc. Indian Acad. Science **36**, 75 (1952).
[i] Kaplon, Peters, and Ritson, Phys. Rev. **85**, 900 (1952).
[j] M. F. Kaplon and D. M. Ritson, Phys. Rev. **88**, 386 (1952).
[k] Kaplon, Ritson, and Walker, Phys. Rev. **90**, 716 (1953).
[l] Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950).
[m] L. S. Osborne, Phys. Rev. **81**, 239 (1951).
[n] E. Pickup and L. Voyvodic, Phys. Rev. **84**, 1190 (1951).
[o] Demeur, Dilworth, and Schönberg, Nuovo cimento **9**, 92 (1952).
[p] A. Gerosa, and R. Levi Setti, Nuovo cimento **8**, 601 (1951).
[q] Daniel, Davies, Mulvey, and Perkins, Phil. Mag. **43**, 753 (1952).
[r] T. F. Hoang, J. phys. et radium **14**, 395 (1953).

The temperature, proportional to $\delta^{\frac{1}{4}}$, is then:

$$T\propto\gamma^{\frac{1}{4}}n^{-\frac{1}{4}}$$

and the total number of charged particles, proportional to W_c/T, is:

$$N=K\gamma^{\frac{1}{4}}n^{\frac{3}{4}}.$$

$K=2.1$ if only π mesons are produced; $K=2.4$ if π mesons and nucleons are produced.[3] Also for a composite collision it is plausible to admit that the particles are emitted, in the c.m. system, symmetrically with respect to the plane normal to the direction of the incoming particle; the observed shower will then satisfy the condition

$$\gamma_c=1/\tan\eta\approx1/\eta$$

where η is the polar angle, in the laboratory system, which contains $\frac{1}{2}$ of the emitted particles.

Combining this expression of γ_c with that obtained for N, it follows that

$$1.2Kn=N\eta^{\frac{1}{2}}.$$

If the assumptions are reasonable, no relativistic shower observed in emulsions could have

$$N\eta^{\frac{1}{2}}>\sim1.2\times2.2\times5=13.$$

For all the relativistic showers thus far described in the literature (54 cases), the maximum value found for $N\eta^{\frac{1}{2}}$ is 12.

A more detailed analysis of this survey is given in Table I. Table I(a) refers to the cases where $\eta<0.10$, Table I(b) to cases where $\eta<0.20$. Though the relation derived before could be applied only to the cases of Table I(a) the cases of Table I(b), statistically more significant, will also be analyzed in the same manner.

From the columns "all" of both tables it appears that several of the events with number of heavy tracks $N_h=0$ or 1 (i.e., the events previously believed to be due to single collisions) give values of $N\eta^{\frac{1}{2}}$ well above the value expected for a single collision, which is: $1.2\times(2.2/2)\times1=1.3$. These events are probably composite collisions which leave the nucleus not strongly excited and emitting only neutrons.[4] On the other hand, several of the events with $N_h>3$, and actually also one event with $N_h=15$, should be considered as single collisions according to our criterion. Events of this kind could happen when the collision is single, but one or more other nucleons recoil with relatively high energy and are absorbed by the nucleus.

It is worth pointing out that a picture of the composite collision as given in this paper could not be possible assuming the model proposed by Heisenberg[5] instead of the Fermi model. In fact, according to Heisenberg, the production of mesons takes place in nucleons up to a distance

$$r\simeq(\hbar/\mu c)(0.8+\log_{10}\gamma)$$

from the incoming nucleon; if this is the case, at relativistic energies, when the nucleus is hit centrally, nearly all nucleons would be involved in the collision and it would be much less probable for the nucleus to be left with low excitation.

Actually, this large value of the radius of influence at high energies makes it quite difficult to understand how single collisions with no strong excitation of the nucleus can happen at all. Since after all these collisions are observed quite frequently, this can be considered as an argument in favor of the Fermi model, or of other models in which the radius of action does not extend beyond $\sim\hbar/\mu c$.

[4] Arguments in favor of this conclusion are given also by C. B. A. McCusker and F. C. Roesler, Phys. Rev. **91**, 769 (1953).

[5] W. Heisenberg, *Kosmische Strahlung* (Springer, Berlin, 1953), p. 148.