

Electron Capture in the Decay of Na²²†

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The ratio of electron capture to positron emission in the decay of Na²² to the 1.28-Mev level in Ne²² has been determined by a comparison of the intensities of positron emission and 1.28-Mev radiation. The measurements were made with a coincidence arrangement employing a 4π beta counter to detect the positrons and a scintillation counter for the γ rays. By varying the bias of a discriminator responding to the γ spectrum, one can determine the efficiency of the 4π beta counter, and the product of this efficiency and the fraction of decays which undergo electron capture. These measurements lead to a value of 0.110 ± 0.006 for the ratio of electron capture to positron emission. If one assumes that the Na²² decay is allowed (Δ*J* = 1, no) the theoretical value is 0.1135. By a comparison of these values, we can make an estimate of the magnitude of the Fierz interference term. We find, subject to the above assumption, that the ratio of the axial vector and tensor coupling constants *C*_A/*C*_T = (−1 ± 2) percent.

INTRODUCTION

SEVERAL measurements on electron capture in the decay of Na²² have been reported in the last ten years. Good, Peaslee, and Deutsch¹ obtained essentially negative results (0 ± 5 percent). Bothe² also failed to detect *K* capture. On the other hand Bouchez³ reports 0.10 ± 0.05 for the ratio of *K* capture to positron emission, while Major³ found 0.04 ± 0.03 for the same quantity. The extensive use of Na²² for calibrating the efficiency of scintillation counters has led us to attempt a more precise measurement.

Na²² decays by positron emission to an excited state of Ne²² at 1.28 Mev. [There is a very weak transition (0.06 percent) to the ground state of Ne²²; this is too feeble to have any effect on our measurements and we shall ignore it hereafter.] Electron capture to the 1.28-Mev level will also give rise to the 1.28-Mev gamma ray. Our method is essentially the comparison of the relative numbers of positrons and 1.28-Mev quanta.

Suppose a Na²² source is placed in the vicinity of a β counter (which we shall assume counts β particles only) and a γ counter shielded in such a way as to count γ's only. The β counter registers the positrons with an efficiency ε_β (the efficiency is the number of events detected per event). The γ counter will detect nuclear (1.28-Mev) radiation with an efficiency ε_n and annihilation (0.511-Mev) radiation with an efficiency ε_a. Since the efficiency depends on geometry as well as on energy, ε_a will depend on the place at which the positrons annihilate. We differentiate between that

fraction *f* of the positrons which annihilate after passage through the β counter and those which annihilate elsewhere by designating the γ efficiencies (averaged over geometries) as ε_a' and ε_a'', respectively. Let *P* be the average number of positrons per disintegration and *K* be the average number of electron captures per disintegration (*P* + *K* = 1).

The rate for coincidences between β and γ counts is

$$C = PN_0\epsilon_\beta(2\epsilon_a' + \epsilon_n), \quad (1)$$

where *N*₀ is the disintegration rate. The counting rate of the γ counter is

$$N = 2PN_0f\epsilon_a' + 2PN_0(1-f)\epsilon_a'' + N_0\epsilon_n. \quad (2)$$

Since the counter cannot distinguish between the two categories of annihilation quanta, the first two terms on the right are lumped together to give *N*_a, the counting rate for annihilation quanta and the third term is *N*_n, the counting rate for the nuclear γ ray.

Let α and β be the fractional γ-counting rates:

$$\begin{aligned} \alpha &= N_n/N = N_0\epsilon_n/N, \\ \beta &= N_a/N = 2PN_0/N\{f\epsilon_a' + (1-f)\epsilon_a''\}, \\ \alpha + \beta &= 1. \end{aligned} \quad (3)$$

Dividing (1) by *N* and using (3), we obtain

$$\frac{C}{N} = \epsilon_\beta \left\{ \alpha P + \beta \frac{\epsilon_a'}{\epsilon_a'' + f(\epsilon_a' - \epsilon_a'')} \right\}. \quad (4)$$

The physical interpretation of (4) is simply this: if the γ counter can be made to respond only to the nuclear γ ray (α = 1 and β = 0), then *C*/*N* = ε_β*P*, which is the probability that the positron will be counted times the probability that a positron was emitted. Similarly if β = 1 and α = 0, *C*/*N* will be the product of the probability that a positron was counted times the probability that the annihilation quantum detected was associated with that positron.

Now in general the γ counter will respond to both γ radiations and in order to obtain *P* from this type of

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¹ Good, Peaslee, and Deutsch, Phys. Rev. **69**, 313 (1946).

² W. Bothe, Z. Physik **123**, No. 1E (1944).

³ Unpublished work referred to by R. Bouchez, Physica **18**, 1171 (1952). This paper is an extensive review of the experimental methods and theoretical interpretation of *K* capture measurements.

measurement, the other parameters (α , β , ϵ_β , ϵ_a' , ϵ_a'' , and f) must be determined. Two simplifying conditions are apparent, however. If $f=1$ or $\epsilon_a'=\epsilon_a''$, Eq. (4) reduces to

$$C/N = \epsilon_\beta(\alpha P + \beta). \quad (5)$$

The second condition can be obtained if the β counter is sufficiently small, the source placed close by and all positrons annihilated in the immediate vicinity; if then the γ counter is sufficiently remote, $\epsilon_a'=\epsilon_a''$ since the difference here depends primarily on solid angle. To obtain $f=1$, we can use a 4π beta counter which contains the source within its volume. Actually f cannot be made exactly equal to unity, since the source must be supported and some positrons may enter the support and annihilate there without entering the gas of the counter; however, with care one can achieve $f=0.95$ or better. This is the arrangement which we have used; since the γ counter is performed exterior to the β counter, ϵ_a' (for positrons annihilated after passage out into the counter) and ϵ_a'' (for positrons annihilating in the source and support) are practically equal.

The condition $f=1$ is desirable for additional reasons. Since f is one of the ingredients of the β counter efficiency ϵ_β , the greater its value, the higher will be the coincidence rate. Also, if f is small because the source is located some distance from an end window counter, γ - γ coincidences may introduce additional corrections. The latter are trivial with the 4π counter.

Once conditions are arranged so that Eq. (5) is applicable, the experimental variables ϵ_β and α (or β) must be determined. If the γ counter is a Geiger counter, α may be measured indirectly, leaving ϵ_β to be separately found. However, by using a scintillation counter one can, by varying the pulse-height selection, change the value of α over a considerable range. It is possible to determine α through an analysis of the pulse spectrum. If one rewrites Eq. (5) using the condition that $\alpha+\beta=1$, one obtains

$$C/N = \epsilon_\beta(1 + \alpha K), \quad (6)$$

so that a plot of C/N vs α yields a straight line whose intercept on the C/N axis yields ϵ_β , while at $\alpha=1$, $C/N = \epsilon_\beta(1+K)$. It is therefore possible to determine the desired parameters in one experiment.

EXPERIMENTAL ARRANGEMENT

We have used a 4π counter operating in the high proportional region to detect the positrons. The counter is identical with that described by Borkowski.⁴ It consists of two brass pillboxes ($2\frac{3}{4}$ in. i.d. and $1\frac{5}{16}$ in. deep) with the anodes across a diameter. The two counting regions are separated by a thin (1 to 10 mil) aluminum or copper plate. The source is mounted on a zapon film (40 micrograms/cm²) stretched across a $\frac{1}{4}$ -in. hole in the middle of this plate. The anodes (1-mil

W wire) of the two halves are connected in parallel externally through isolating condensers to the input of a preamplifier. Additional amplification was obtained with a nonoverloading amplifier designed by Chase and Higginbotham.⁵ The gain was set high so that virtually all pulses saturated. By varying the high voltage applied to the counter we found that the counting rate was flat to better than $\frac{1}{2}$ percent from 2.4 to 2.8 kilovolts with a counter filling of methane at a pressure of 1 atmosphere. An integral bias curve of the amplified pulses was also flat from 10 to 64 volts (maximum pulse amplitude was 70 volts). These tests suggest that the counter efficiency must be very nearly 100 percent for those positrons which entered the counter. Actually the overall efficiency is expected to be smaller than 100 percent, since the plate on which the source is mounted can intercept a few percent of the positrons; the magnitude of this effect depends on the thickness and flatness of this plate.

The sources of Na^{22} were made by evaporation of carrier-free NaCl solution which had been separated in an ion exchange column by Dr. T. A. Pond of this laboratory. We used sources ranging from 33 000 to 137 000 counts/min. Attempts to determine the source thickness indicated that on the average it was less than the 40 $\mu\text{g}/\text{cm}^2$ of the supporting foil.

The γ counter was a $1\frac{1}{2}$ -in. diam \times 1 in. high NaI scintillation crystal. It was placed directly under the proportional counter and a house of lead bricks was built around the counters to reduce the background counts. The γ pulses were analyzed with an integral bias discriminator whose output fed a scaler (for N) and a coincidence circuit. Similar apparatus was used in the proportional counter branch. The coincidence circuit resolving time was varied to determine the proper operating region; finally a resolving time of 1 μsec was selected. (With the weak sources used, the random coincidence rate was only a fraction of one percent of the true coincidence rate.)

A typical run consisted in measuring C and N as a function of the bias of the γ -counter discriminator. As expected, the ratio C/N decreases with increasing bias until the bias exceeds the pulse height for annihilation radiation; beyond this it is fairly constant. The integral bias curve for N is then analyzed into two components, one for the 1.28-Mev radiation and one for the 511-keV annihilation radiation. In order to extrapolate the 1.28-Mev spectrum to low amplitudes, a Co^{60} spectrum was taken with a Co^{60} source replacing the Na^{22} source in the 4π counter. The average energy of the two Co^{60} γ rays (1.17 and 1.33 Mev) is sufficiently close to 1.28 Mev so that the shape of Compton spectra below 600 keV should be practically identical. By matching the 1.28-Mev Na^{22} spectrum to the Co^{60} spectrum, the required extrapolation can be carried out.

⁴C. J. Borkowski, Oak Ridge National Laboratory Report ORNL-1056, 1951 (unpublished).

⁵R. L. Chase and W. A. Higginbotham, Rev. Sci. Instr. 23, 34 (1952).

Before describing the experimental results in detail there are two sources of error which must be considered: namely the possibility that the β counter is detecting the K x-rays of Ne^{22} which follow K capture, and secondly, the question of detection of coincident 511-keV and 1.28-MeV quanta in the γ counter.

FLUORESCENT YIELD OF Ne^{22}

Following K capture in Na^{22} , the daughter atom Ne^{22} has an excess energy of about 860 volts corresponding to a vacancy in the K shell. This energy is subsequently liberated in the form of quanta and Auger electrons. If both of these were detected by the β counter the present experiment would lead to the conclusion that there is no K capture. However, the Auger electrons are softer than 840 volts and would be very hard to detect. (According to Ference and Stephenson⁶ the range, for example, of a 2-keV electron is about $17 \mu\text{g}/\text{cm}^2$.) It would require a great deal of care to make a source sufficiently thin to permit these electrons to escape. This was verified by measurements with $40\text{-}\mu\text{g}/\text{cm}^2$ zapon films covering both sides of the Na^{22} source. The β -counting rate decreased by approximately 0.3 percent when 4 films were added to each side of the source. This amount of absorber also completely eliminates L radiation of Ne.

On the other hand, K x-rays of Ne would not be appreciably absorbed by $40 \mu\text{g}/\text{cm}^2$ of Zapon, while the absorption in the counter gas should be almost com-

plete. We do not know whether our counter would register these x-rays, but it seems highly probable that it would. Counts taken covering the source with $\sim 150 \mu\text{g}/\text{cm}^2$ of Zapon and $\sim 150 \mu\text{g}/\text{cm}^2$ of aluminum showed a 0.5 percent decrease. These absorbers should reduce the K x-rays by approximately 50 percent. Since some soft positrons were certainly absorbed, this measurement can only set an upper limit of 1 percent for the β -counting rate (without these foils) ascribable to K x-rays. However, consideration of the fluorescent yield indicates that we should expect at most 0.1 percent effect due to the x-rays.

While numerous measurements of fluorescent yields have been made throughout the periodic table, very few measurements are available for low Z .^{7,8} Locher⁹ has measured the fluorescent yield of Ne to be 8.3 percent. (Crone¹⁰ gives a value of 0.81 percent; however, this was not a direct measurement but was obtained by normalizing his data to higher- Z results on the basis of theoretical expectations.) Locher's measurement is considerably in excess of the expected value. The theory of the Auger process is well understood and Burhop⁷ gives a calculated value of 1.1 percent. Since theory and experiment are in good accord for higher Z , it seems reasonable to accept Burhop's calculations for Ne. (Locher's experiment is a difficult one involving identification of 800-volt Auger electron blobs at the beginning of a photoelectron track in a cloud chamber.) Since the amount of K capture in Na^{22} is approximately 10 percent, the maximum number of K x-rays which can be detected in the β counter is 0.1 percent of the β^+ count, if we take Burhop's calculated value for the fluorescent yield, and 0.8 percent with Locher's result. Our absorption measurement is consistent with either. If Locher's results are correct, our measurement of the electron capture (K) might require a correction of +8 percent, whereas Burhop's value lead to a maximum correction of +1 percent.

CORRECTION FOR SIMULTANEOUS QUANTA

In writing down Eqs. (1) and (2) we ignored the possibility that an annihilation quantum and a 1.28-MeV quantum might enter (and be absorbed by) the γ crystal simultaneously. Because of the finite solid angle this effect is not trivial, especially near the upper end of the 1.28-MeV spectrum and beyond. Pulses of this type will give coincidences and since they are associated with positrons should be included in β in Eq. (5). Since these pulses can be larger than the 1.28-MeV quanta, α in Eqs. (5) and (7) cannot reach unity. As a function of bias, α increases from a low value (~ 0.3) to about 0.95 near the end of the 1.28 MeV spectrum and then decreases rapidly to zero beyond this point. (See Fig. 2.)

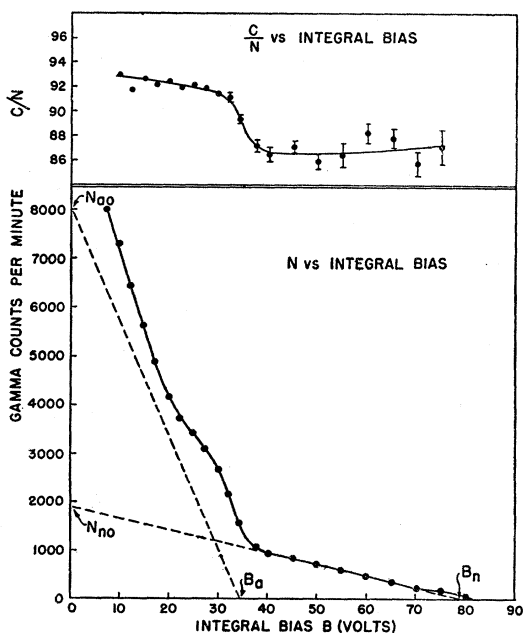


FIG. 1. Lower graph: Integral bias curve for $\text{Na}^{22}\gamma$ rays. The dotted lines show the approximations to the 1.28-MeV and 0.511-MeV spectra used in calculating the spectrum of simultaneous pulses (N_{an}). Upper graph: Integral bias curve for C/N .

⁶ M. Ference and R. J. Stephenson, *Rev. Sci. Instr.* **9**, 246 (1938).

⁷ E. H. S. Burhop, *The Auger Effect* (Cambridge University Press, Cambridge, 1952).

⁸ Broyles, Thomas, and Haynes, *Phys. Rev.* **89**, 715 (1953).

⁹ G. L. Locher, *Phys. Rev.* **40**, 484 (1932).

¹⁰ W. Crone, *Ann. Physik* **27**, 405 (1936).

Correspondingly the ratio C/N decreases, reaches a plateau, and finally increases to a value ϵ_β . (If it were not for the poor statistics obtained in the last region, this would be a good way to find ϵ_β .)

In order to take these simultaneous quanta into account, we have made the following approximate calculation. Figure 1 shows N vs bias for a particular run (after correction for background). N has now three components N_a , N_n , and N_{an} , where the first two correspond to counts resulting from crystal responses to the annihilation quanta and the nuclear γ rays, and N_{an} is the counting rate due to simultaneous responses to these two radiations. Then (in the $f=1$ approximation)

$$\frac{C}{N} = \epsilon_\beta \frac{PN_n + N_a + N_{an}}{N}. \quad (7)$$

If we assume that N_{an} is small, we can decompose N into N_a and N_n as a first approximation. If we approximate these spectra by the straight lines, as shown in Fig. 1, and use these to calculate N_{an} , we find

$$N_{an} = \frac{N_{a0}N_{n0}}{N_0} A_{an}, \quad (8)$$

where N_{a0} and N_{n0} are the extrapolated counting rates at zero bias, and N_0 is the source strength (given with sufficient accuracy by the β count). The quantity A_{an} is given by

$$A_{an} = \begin{cases} 1 - (B/2B_a B_n) & \text{for } B < B_a \\ (B_a + 2B_n - 2B)/2B_n & \text{for } B_a < B < B_n \\ (B_o + B_n - B)^2/2B_a B_n & \text{for } B_n < B < B_a + B_n, \end{cases} \quad (9)$$

where B is the bias setting and B_a and B_n are the upper ends of the straight line approximations to the 0.511- and 1.28-Mev spectra.

This formula was compared with an experimentally determined curve in the region just beyond B_n using a strong source. It was found that our formula gave results about 40 percent high; hence for analysis of the coincidence runs Eq. (8) was multiplied by 0.7. Since this correction is significant only near the end point of the 1.28-Mev spectrum, we feel that this procedure is sufficiently accurate.

The analysis of N to determine α for each bias is carried out as follows. N_{an} is computed as outlined above; its magnitude ($\times 10$) is shown in Fig. 2. These numbers are subtracted from N for biases in excess of 40 volts. This difference is then N_n in this region. To extrapolate to lower bias, we took a Co^{60} spectrum, corrected it for simultaneous pulses, and normalized it to the Na^{22} 1.28-Mev spectrum as shown in Fig. 2. The ratio of the complete 1.28-Mev spectrum to N of Fig. 1 gives α . The latter is also plotted in Fig. 2.

RESULTS

A series of runs were made with several sources to investigate the general features of this experiment.

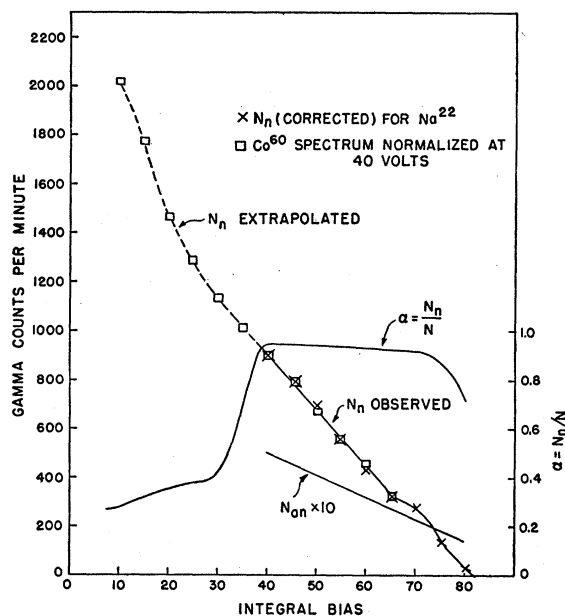


FIG. 2. The 1.28-Mev spectrum (N_n) obtained after correction for simultaneous pulses (N_{an}) and extrapolated to low bias through the use of the Co^{60} spectrum. The scale on the right and the curve labelled α give the fractional counting rate for the 1.28-Mev γ ray.

Analysis of this data gave $\epsilon_\beta = 0.995 \pm 0.007$ and $K = 0.101 \pm 0.011$. The efficiency seemed a bit high (for the source support intercepts some of the positrons) and it was felt that there might have been a systematic error at low γ bias. The electronic equipment was checked carefully and a more extensive set of points were taken. It is these data which we shall discuss. The observed counts (N_{ob} and C_{ob}) had to be corrected for N_b , the background in the γ counter, and C_r and C_b , the random and real (extraneous) coincidences. The last effect is observed with no source in the counter and is probably entirely due to cosmic radiation (7 counts/min) since the rate was insensitive to the bias of the γ counter. The random rate was small (< 0.7 percent). The γ background was not negligible amounting to 30 percent at a bias of 80 volts. In calculating the statistical errors, if we ignore these corrections for the moment, we note that while C and N are large numbers and $N-C$ is small, the error in C/N is determined by $N-C$. If there were no K capture and $\epsilon_\beta = 1$, $N-C$ would be zero and C/N would be exactly unity, whatever the actual magnitude of C . N_{ob} and C_{ob} are not independent since many of the γ counts are the same counts recorded by the coincidence circuit. Therefore we can break up N_{ob} into two parts, a part which is independent of C_{ob} which we shall call X , and a part directly correlated with C_{ob} , which is just C_{ob} . The same discussion applies to N_b which we write as $C_b + N_b'$. Then the corrected ratio of C to N is:

$$\frac{C}{N} = \frac{C_{ob} - C_b - C_r}{N_{ob} - N_b} = \frac{C_{ob} - C_b - C_r}{C_{ob} + X - N_b' - C_b}. \quad (11)$$

Applying the asymptotic formula for the variance,

$$\sigma^2\{f(X, Y \dots)\} = \left(\frac{\partial f}{\partial X}\right)^2 \sigma^2(X) + \left(\frac{\partial f}{\partial Y}\right)^2 \sigma^2(Y) + \dots,$$

we obtain, keeping only important terms:

$$\sigma^2\left(\frac{C}{N}\right) = \frac{C^2}{N^4} \left[N_{ob} - C_{ob} + \left(\frac{t(C_{ob})}{t(N_b)}\right) N_b + \left(\frac{t(C_{ob})}{t(C_b)}\right) C_b \right], \quad (12)$$

where $t(C_{ob})$, $t(C_b)$, and $t(N_b)$ are the intervals of times for determining C_{ob} , C_b , and N_b , respectively. (In the above equations, the actual counts in the run are used and not rates; the correction counts are adjusted to the same interval as the runs.) The vertical bars in Figs. 1 and 3 represent $\pm\sigma$ as determined by Eq. (12).

The final data is presented in Fig. 1, showing C/N as a function of bias, for qualitative comparison with (N vs bias). In Fig. 3 we have plotted C/N vs α , which according to Eq. (7) should give a straight line whose intercept on the (C/N) axis gives ϵ_β , and whose slope is $-K\epsilon_\beta$. The line drawn is a least squares fit to the data and yields $\epsilon_\beta = 0.956 \pm 0.004$ and $K = 0.099 \pm 0.006$.

The efficiency of the counter with this sample was lower than with previous samples. However, it is not unreasonably low for the thin plate which supported the foil was warped and could have intercepted and absorbed a few percent of the positrons. It is precisely effects of this kind which the present method takes care of. On the other hand, the discussion of the theory of this method indicates that if ϵ_a' and ϵ_a'' are not equal a correction arises when f is not unity. We recall that ϵ_a' and ϵ_a'' are, respectively, the γ -counter efficiencies (including solid angle, etc.) for those positrons (the fraction f) which pass into the active volume of the counter, and for those $(1-f)$ which are, in the present case, absorbed in the source and source support. Bias curves were taken with a source uncovered and

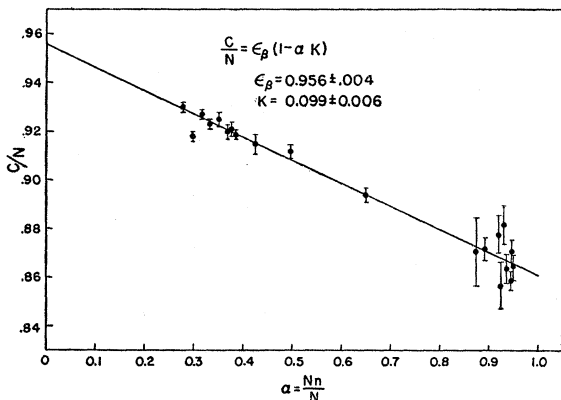


FIG. 3. Plot of C/N vs α . The line drawn through the experimental points and the values of ϵ_β and K correspond to a least-squares analysis of the data.

with the source covered with $\frac{1}{32}$ -in. aluminum. These measurements indicate that $\epsilon_a'/\epsilon_a'' \lesssim 1.08$. If we take the experimental value of ϵ_β , 95.6 percent, and ascribe it entirely to the factor f , and use $\epsilon_a'/\epsilon_a'' = 1.08$, the above value of K should be decreased by 3 percent. However, ϵ_β will be reduced by other factors, such as counting losses in the electronics and in the counter, and this correction will be smaller than 3 percent. Accepting Burhop's estimate of the fluorescent yield, there may be an additional correction of +1 percent for the K x-rays of Ne^{22} if our counter responded to them. These two corrections partially cancel each other and since they are small compared with statistical errors we shall ignore them. To summarize, our conclusion is that the fraction of electron capture ($K+L$) in the decay of Na^{22} is 0.099 ± 0.006 .

We also performed some experiments setting the β counter in anticoincidence with the γ counter. In this case, the ratio of anticoincidence counts to γ counts is $1 - (C/N)$. This gave essentially the same result. Photographs taken of γ pulses in anticoincidence with positrons, showed the 1.28-Mev spectrum with an intensity of ~ 10 percent of the normal spectrum and an annihilation spectrum with an intensity of ~ 2 percent of the normal spectrum. No other lines were visible; there was no evidence that levels other than the 1.28 Mev level are involved in the electron capture in the Na^{22} decay.

We feel that the present method can be extended to higher Z decays by using a " 4π " scintillation counter for the β counter, for one could bias out Auger and x-ray pulses (providing the β^+ energy is higher than these). The use of a single channel discriminator for the γ pulses would allow a wider range of α to be studied, and would reduce the effect of simultaneous pulses.

DISCUSSION OF RESULTS

To compare our experimental value of the electron capture and positron transition rates with theory, we shall assume that the transitions are allowed. The spin of Na^{22} is known to be 3,¹¹ and it seems fairly certain that the spin of the Ne first excited state is 2.¹² The parities of both states are most probably even and both the spectrum shape¹³ and absence of β - γ correlation¹⁴ are consistent with an allowed transition. On the other hand, $\log(ft) = 7.4$ for the Na^{22} decay to Ne^{22*} , a value considerably larger than that usually accepted for a normal allowed transition ($\log ft = 4$ to 5.8).^{15,16} Therefore this transition,

¹¹ Davis, Nagle, and Zacharias, Phys. Rev. **76**, 1068 (1949).

¹² G. Scharff-Goldhaber, Phys. Rev. **90**, 587 (1953); Hinman, Brower, and Leamer, Phys. Rev. **90**, 370 (1953).

¹³ Macklin, Lidofsky, and Wu, Phys. Rev. **78**, 318 (1950); B. T. Wright, Phys. Rev. **90**, 159 (1953).

¹⁴ D. T. Stevenson and M. Deutsch, Phys. Rev. **83**, 1202 (1951).

¹⁵ E. Feenberg and G. L. Trigg, Revs. Modern Phys. **22**, 399 (1950).

¹⁶ E. J. Konopinski and L. M. Langer, Ann. Rev. Nuc. Sci. **2**, 261 (1953).

allowed by Gamow-Teller selection rules may be slowed down by an accidentally poor overlap of the initial and final wave functions for the nucleons, or it may be l forbidden.¹⁷ In the former case, the electron capture to positron ratio is the same as for normally allowed decays. In the latter case, this ratio may be quite different. Bouchez^{3,18} has calculated a value of ~ 0.05 for Na²², assuming that the decay is $\Delta l = 2$ forbidden; this value is clearly in disagreement with our measurement. He calculates ~ 0.09 under the assumption that it is first forbidden ($\Delta J = 0, 1$ yes); however, it is unlikely that the decay involves a change of parity.

Assuming the decay is allowed, ($\Delta J = 1$, no) the transition probabilities P_+ (for positron decay) and P_K (for K capture) are:¹⁹

$$P_+ = \frac{G^2 M^2 C_T^2}{2\pi^3} \int_1^{E_0} p E (E_0 - E)^2 F(Z, E) \times \left\{ 1 - \frac{2\gamma C_A}{E C_T} \right\} dE = \frac{G^2 M^2 C_T^2 f_+}{2\pi^3},$$

$$P_K = \frac{G^2 M^2 C_T^2}{4\pi^2} (E_0 + E_K)^2 g_K^2 \left\{ 1 + 2 \frac{C_A}{C_T} \right\} = \frac{G^2 M^2 C_T^2}{4\pi^2} f_K.$$

The various quantities have their usual meanings: G is the Fermi constant which is a measure of the strength of the β -decay interaction and M is the nuclear matrix element; E and p are the energy and momentum of the positron; $F(Z, E)$ and g_K^2 are determined by the wave functions of the emitted positron and of the K electron respectively; E_K is the total energy of the K electron and is given by $E_K \cong \gamma = (1 - \alpha^2 Z^2)^{1/2}$. The terms involving C_A/C_T are the Fierz interference terms, where $C_A G$ and $C_T G$ are the coupling constants for the axial vector and tensor interactions. We have assumed that the latter is much larger than the former.¹⁶

Let R_0 be the ratio of electron capture to positron emission if $C_A/C_T = 0$. We have used the *Tables for the Analysis of Beta Spectra*, National Bureau of Standards Applied Mathematics Series No. 13 (U. S. Government Printing Office, Washington, D. C., 1952) to evaluate f_+ . The calculation of g_K^2 is straightforward; the effect of screening is included through the use¹⁹ of $Z_{\text{eff}} = Z - 0.3$. The nuclear matrix element disappears in the ratio. With $E_0 = 2.061$ (corresponding to a maximum kinetic energy of 542 keV¹³ for the positron decay), we find $f_+ = 0.2722$ and $f_K = 0.01865$, giving for R_0 the value 0.1076.

There are a few corrections which should be made;³ the only important one for the low Z of the Na²² decay

is that for L capture. According to the calculations of Rose and Jackson²⁰ this correction is 6.5 percent for $Z = 11$, leading to a final value $R_0 = 0.1135$.

From our measurements we obtain (for 9.9 ± 0.6 percent electron-capture) the value $R_{\text{exp}} = 0.110 \pm 0.006$, in good agreement with the above theoretical value. It is now tempting to see the limits which this agreement can set in the ratio C_A/C_T . Letting R be the ratio P_K/P_{β^+} given by the complete expressions above, we find

$$\frac{R}{R_0} = \frac{1 + 2(C_A/C_T)}{1 - 2\langle E^{-1} \rangle_{\text{av}} (C_A/C_T)},$$

where $\langle E^{-1} \rangle_{\text{av}}$ is the average of E^{-1} over the β^+ spectrum. For the Na²² decay it has the value of 0.7. Solving for C_A/C_T , we obtain

$$\frac{C_A}{C_T} \cong \frac{R - R_0}{3.4 R_0}.$$

We take for R our experimental value and for R_0 the value calculated above. However, there is an uncertainty in R_0 due to approximations in the calculations and due to inaccuracy in the measurement of E_0 . The uncertainty in E_0 is of the order of 0.5 percent, leading to an uncertainty in R_0 of 1.5 percent. Thus, it seems reasonable to take $R_0 = 0.1135 \pm 0.002$. Therefore, $R - R_0 = -0.0035 \pm 0.007$ and $C_A/C_T \cong (-1 \pm 2)$ percent.

The usual method of determining C_A/C_T is by analysis of the experimental shapes of allowed spectra for those decays which follow Gamow-Teller selection rules. The Fierz terms modify the shape through the factor $[1 \pm (2\gamma/E)(C_A/C_T)]$. This will introduce a curvature in the Kurie plot. C_A/C_T is then estimated from the maximum curvature consistent with the Kurie plot which is a straight line for $C_A/C_T = 0$. Opinions as to the sensitivity of this type of analysis vary considerably. Konopinski and Langer¹⁶ and Davidson and Peaslee²¹ have concluded that $C_A/C_T < 2$ percent and 4 percent, respectively, while Winther and Kofoed-Hansen²² feel that a ratio as large as 20 percent cannot be excluded by an analysis of the published β spectra. It is therefore helpful to have the additional evidence supplied by the present measurement, which indicates that $C_A/C_T < \text{several percent}$. However, it must be remarked that the significance of our calculation is somewhat weakened by the lack of knowledge of the precise character of the Na²² decay.

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