Compton Scattering of 2.62-Mev Gamma Rays by Polarized Electrons*,†

S. B. GUNST‡ AND L. A. PAGE
University of Pittsburgh, Pittsburgh, Pennsylvania
(Received July 20, 1953)

The differential cross section for Compton scattering of a circularly polarized photon by an electron with given initial spin orientation can be written as a sum of the common Klein-Nishina formula for no polarization and a term sensitive to polarization. The total cross section is $\sigma=\sigma_0\pm\sigma_1$. A measurement of the transmission of 2.62-Mev gamma rays through iron magnetized along the transmission direction relative to that through unmagnetized iron gives the absolute value of σ_1 for this energy, if the number ν_s of polarized electrons per iron atom at saturation is known. For $\nu_s=2.06$, $\sigma_1/\pi r_0^2=0.089\pm0.007$. This agrees with the theoretical value 0.093. Alternatively, the theoretical σ_1 and the measurements would yield $\nu_s=1.97\pm0.15$.

The application of the method of this experiment to measurement of gyromagnetic ratios for ferromagnets is suggested, as is its application to the analysis of circularly polarized radiation.

I. INTRODUCTION

THE analysis of circularly polarized photons by Compton scattering from polarized electrons has been mentioned a number of times.[1] The differential cross section for the process (see Appendix A) may be written as the sum of two terms. One term is the common Klein-Nishina formula for unpolarized incident quanta and unpolarized electrons and the other term takes account of polarization. The total cross section is $\sigma=\sigma_0\pm\sigma_1$, where σ_0 is independent of polarization and σ_1 depends on polarization.

The term σ_1 is measured in this experiment by the Compton scattering of gamma rays by polarized electrons. An apparatus that measures σ_1 must act, so to speak, like a "quarter-wave plate for gamma rays."[2]

II. EXPERIMENTAL METHOD

Let gamma rays from a radioactive source pass through a ferromagnet magnetized along the axis defined by the incident gamma rays in one case and unmagnetized in the other. It can be shown (see Appendix B) that the ratio of the difference between the transmissions through the magnetized and unmagnetized ferromagnet to the transmission through the unmagnetized ferromagnet is given by

$$[T(\nu)-T(0)]/T(0)=\tfrac{1}{2}(NL\nu\sigma_1)^2. \quad (1)$$

Here N is the number of atoms per unit volume, L is the axial length of the ferromagnet, ν is the number of axially polarized electrons per atom, and σ_1 is the part of the total Compton cross section dependent on polarization. The experimental determination of the ratio expressed by (1) leads to the evaluation of σ_1 and a test of the theory. The 2.62-Mev gamma ray of ThC″ (see Fig. 3) is used in this experiment.

* Work done in the Sarah Mellon Scaife Radiation Laboratory and assisted by the U. S. Office of Ordnance Research.
† Based on Ph.D. thesis by SBG.
‡ Now at Westinghouse Atomic Power Division, Pittsburgh, Pa.
[1] Biedenharn, Rose, and Arfken, Phys. Rev. **83**, 683 (1951); O. Halpern, Phys. Rev. **82**, 753 (1951); F. P. Clay and F. L. Hereford, Phys. Rev. **85**, 675 (1952).
[2] S. P. Lloyd, Phys. Rev. **88**, 906 (1952).

III. DESCRIPTION OF APPARATUS

A. Magnet Design

An electromagnet 12 inches long (approximately 8 mean free paths at 2.62 Mev) was designed. Tapered ends of the Armco-iron core ($1\frac{1}{2}$-in. diameter) fit snugly in mild-steel end plates which are secured to a length of cold-rolled seamless steel tubing (5-in. o.d. by 4-in. i.d.) by screws at each end. The end plates and tubing, which complete the magnetic circuit, have a reluctance only a few percent of that of the core.

A water-cooled, copper coil form was fabricated to fill the annulus. Primary and secondary windings of copper wire fill the coil form. Current through the series-connected primary windings magnetizes the core; or, by switching connections to "anti-series" (alternate windings bucking), the same coil current induces negligible magnetization in the core. The secondary was wound so as to permit a survey of the state of magnetization of the core in ten steps along the axis.

B. Arrangement of Equipment

Equipment is arranged as shown in Fig. 1. The thorium source is contained in a lead housing, opened adjacent to the core of the magnet. The magnet is enclosed in a loosely fitting magnetic shield. A lead shield with a 1-inch collimation hole at the exit end of the core surrounds a $1\frac{1}{2}$-inch diameter by 1-inch thick NaI(Tl) crystal. Mineral oil couples the crystal to a light pipe of lucite 46-in. long; the other end of the lucite is similarly coupled to a type 5819 photomultiplier tube. Cylindrical magnetic shields of mumetal and steel surround the photomultiplier tube. Signals from the

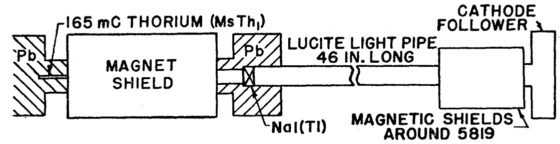

FIG. 1. Arrangement of equipment for final experiments. The cathode follower feeds a linear amplifier and then two discriminator scalers.

photomultiplier tube go through a cathode follower and linear amplifier before being fed to two discriminator scalers.

IV. PRELIMINARY EXPERIMENTS

Extensive preliminary experiments were made in order to obviate certain possible difficulties in the interpretation of final results. These experiments were concerned with the stability of electronic equipment, the linearity of the pulse-height energy response for the NaI(Tl) crystal and its associated electronic equipment, the spectrum of the thorium source used and the portion of the spectrum admitted to the discriminator scalers for any discriminator settings, the distribution of magnetization in the iron core, the coil current required to saturate the core, and the demonstration that the magnet does not perturb the operation of the detector prohibitively. In addition, it was established that the "saturation current" induces negligible magnetization in the core when the primary windings are connected in anti-series.

The investigation of the spectrum of the thorium source employed the Hofstadter and McIntyre method[3] for Compton scattering of collimated gamma rays from a trans-stilbene crystal into a second *trans*-stilbene crystal at approximately 90°. Results show that the thorium source emits gamma rays of the same energies and in approximately the same intensities as found by other investigators[4] for typical thorium sources.

V. FINAL EXPERIMENTS

A. Fraction of Counts Caused by Transmission through Magnet Core

For equipment and source arranged as in Fig. 1, not all the counts obtained in the scalers are caused by gamma rays which pass through the core of the magnet. Some of the counts arise from scattering around the core as well as cosmic-ray and other background activities. To minimize cosmic-ray effects, one of the discriminators was operated at a setting just above the 2.62-Mev photoline. Counts observed on this discriminator scaler were subtracted from those on the other discriminator scaler to give "net counts."

To determine what fraction of the net counts is caused by gamma rays passing through the magnet core, a substitute core was used. This core has the same over-all dimensions as the iron core but consists of a two inch length of iron joined to a ten inch length of lead. Substitution of this composite core for the iron core, with the iron portion adjacent to the source, permits determination of the fraction f of the net counts caused by gamma rays passing through the iron core, as a function of discriminator setting. This fraction is thus a function of net counting rate.

[3] R. Hofstadter and J. A. McIntyre, Phys. Rev. **78**, 619 (1950).

[4] I. E. Dayton (private communication); G. D. Latyshev, Revs. Modern Phys. **19**, 132 (1947).

Expressed in terms of net counting rate, Eq. (1) can now be written

$$(1/f)(\Delta C/C)=\tfrac{1}{2}(NL\nu\sigma_1)^2. \quad (2)$$

Here C is the net counting rate for the core unmagnetized, and ΔC is the difference in net counting rates for the magnetized and unmagnetized core.

B. Experimental Procedure

Since the primary windings can be connected in series or anti-series and can carry current in either of two directions, there are four possible ways in which the "saturation current" may flow through the primary windings. These are designated anti-series left AL, series left SL, anti-series right AR, and series right SR. Net counts/500 sec were obtained for magnet coils switched to AL, SL, AR, and SR in succession. Before the AL and AR "runs" the magnet was carefully degaussed so that AL and AR give an unmagnetized core, SL and SR a magnetized core.

The same current flowed in the magnet primary windings during counting runs for both the magnetized and unmagnetized conditions. This was done in order to minimize thermal expansion effects which might arise if the magnet-core temperature differed between the magnetized and unmagnetized runs. Instruments showed the magnet-core temperature changed negligibly.

The AL, SL, AR, and SR runs were repeated in that order many times and the net counts per 10 runs for each comprises a "sequence."

C. Experimental Result

Two experiments were done as follows.

Twenty-eight sequences (280 runs for each magnet condition) were made with the discriminator set to give a net counting rate of approximately 7 counts/sec; and twenty-two sequences were made with the discriminator set to give a net counting rate of approximately 10 counts/sec. The uncertainties in the total net counts for the AL and AR runs consistently overlapped, as did the uncertainties for the SL and SR runs. The net counts obtained for the core magnetized minus net counts for the core unmagnetized are tabulated in Table I for each sequence of the two experiments. The totals in the table give values of ΔC, and the values of $(1/f)(\Delta C/C)$ for the two experiments are 0.0047 $\pm$0.00135 and 0.0069$\pm$0.00121. (The value of f is in the neighborhood of 0.8 for these runs.) Each uncertainty is based on the observed rms deviation per run and exceeds the standard deviation associated with counting by less than 10 percent. For the two experiments the mean of $(1/f)(\Delta C/C)$ is 0.0059$\pm$0.00090.

This gives σ_1 from (2) if N, L, and ν are known. The number N of atoms/unit volume is obtained from atomic constants and the measured density of the iron core. The effective length L of the core is determined

TABLE I. Net counts with iron magnetized minus net counts with iron unmagnetized for each sequence (20-ksec running time). The net counting rates during the experiments were approximately those shown in the column headings. Totals give values of ΔC.

7 counts/sec		10 counts/sec	
283	53	555	−413
228	324	−134	278
396	−151	443	−23
638	−126	428	734
229	52	1078	1333
134	411	415	1213
267	1227	44	1002
457	222	113	−78
26	83	591	622
204	612	695	1310
−386	567	517	1487
674	−211		
827	256		12 210
229	105		
	7630		

from the preliminary experiment (with uncertainty ~one percent).

The number ν of polarized electrons/atom is determined as follows. Fluxmeter measurements on the secondary windings of the magnet give 2.20 Bohr magnetons/atom at the "saturation current." If the figure for saturated iron is taken as 2.22 Bohr magnetons/atom,[5] ν is the product of 2.20/2.22 and the number ν_s of polarized electrons/atom at saturation. Argyres and Kittel[6] give 2.06 for ν_s.

The result for σ_1 is $(2.22\pm0.18)\times10^{-26}$ cm^2.

D. Comparison with Theory

The theoretical value of $\sigma_1/\pi r_0^2$ at 2.62 Mev is 0.093, while the experimental result is 0.089 ± 0.007.

The experimental σ_1 is actually the effective value for that portion of the thorium spectrum transmitted and counted. For 100 gamma rays at 2.62 Mev, we estimate three at ~2.2 and one at ~1.8 Mev are counted. From the theoretical behavior of σ_1 over these energies (see Fig. 3), the influence of the lower-energy components on the experimental σ_1 should be ~0.2 percent and is neglected.

VI. CONCLUSIONS

The total Compton cross section for scattering of a circularly polarized photon by a polarized electron has been investigated at 2.62 Mev and the polarization sensitive part σ_1 is found to agree with theory to 8 percent standard deviation.

The experiment demonstrates the use of a ferromagnetic absorber as a polarizer or analyzer for circularly-polarized radiation.

If the theoretical value for σ_1 is accepted, the method of this experiment could be applied to the determination of gyromagnetic ratios for ferromagnets. This would serve as an independent check of the results obtained by microwave resonance techniques.

In particular, this experiment would give the number ν_s of polarized electrons per iron atom at saturation as 1.97 ± 0.15. Increased precision might be obtained by using a very strong source of gamma rays. Energies higher than 2.6 Mev are preferable as curve σ_1/σ_0 in Fig. 3 indicates. The source need not be monochromatic; any spectrum from 2 to 7 Mev should be usable since σ_1 is fairly flat over these energies.

VII. ACKNOWLEDGMENT

The authors are indebted to Professor A. J. Allen whose encouragement and support made the investigation possible; to Professor P. Stehle for theoretical results, and for stimulating discussions throughout the experiment; to Professor Frederic Keffer for information concerning recent work in ferromagnetism; to Mr. Milton Heinberg, Mr. R. H. Bassel, and Mr. B. J. Mathewson for assisting with experimental operations; to Mr. Eugene Perkins for supervising the construction of equipment; and to the other members of the Laboratory who gave valuable aid. The loan of a thorium source from the Cornell University Laboratory of Nuclear Studies is gratefully acknowledged. Materials kindly furnished by the Gulf Research and Development Company expedited the investigation.

APPENDIX A: UNDERLYING THEORY

Let a circularly polarized photon $\mathbf{k}_0$, incident along the 3 axis, be Compton scattered into the 2,3 plane (outgoing photon $\mathbf{k}$ at an angle θ with the 3 axis) by a free electron at rest whose spin direction is given by polar angle ψ (with the 3 axis) and azimuthal angle ϕ (with the 1,3 plane). The cross section[7] is

$$\frac{d\sigma}{d\omega}=\frac{1}{2}r_0^2\frac{k^2}{k_0^2}\left\{\left[\frac{k_0}{k}+\frac{k}{k_0}-\sin^2\theta\right]\mp\left[\left(\frac{k_0}{k}-\frac{k}{k_0}\right)\cos\theta\cos\psi+\left(1-\frac{k}{k_0}\right)\sin\theta\sin\phi\sin\psi\right]\right\}, \quad (A.1)$$

where $r_0=e^2/mc^2$, and $d\omega=d\phi d(\cos\theta)$. The upper sign refers to an incident l.c.p. (left circularly polarized) photon.

For $\psi=0$, integration over ϕ gives

$$d\sigma/d\Omega=d\sigma_0/d\Omega\pm d\sigma_1/d\Omega, \quad (A.2)$$

where σ_0 is the common Klein-Nishina expression for unpolarized photons and electrons, σ_1 is the part of the

[5] L. F. Bates, *Modern Magnetism* (Cambridge University Press, Cambridge, 1951), third edition, p. 317; R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., New York, 1951), p. 867.

[6] P. Argyres and C. Kittel, Acta Metallurgica **1**, 241 (1953).

[7] The expression (A.1) was calculated by *P.* Stehle. The simple case, $\sin\psi=0$, follows from W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, Cambridge, 1944), second edition, Eq. (18) f.f., p. 149. It has been pointed out to us by Professor O. Halpern that the above cross section is contained in a paper by W. Franz, Ann. Physik **33**, 698 (1938).

Compton cross section sensitive to circular polarization and spin, and the upper sign refers to an l.c.p. photon as before. The ratio $(d\sigma_1/d\Omega)/(d\sigma_0/d\Omega)$ is sketched in Fig. 2 for several values of k_0/mc^2.

Integration over the sphere yields the total Compton cross section,

$$\sigma=\sigma_0\pm\sigma_1. \tag{A.3}$$

Here,

$$\frac{\sigma_1}{2\pi r_0^2}=\frac{1+4\gamma+5\gamma^2}{\gamma(1+2\gamma)^2}-\frac{1+\gamma}{2\gamma^2}\ln(1+2\gamma),$$

where $\gamma=k_0/mc^2$. Figure 3 shows σ_1 and σ_1/σ_0. For k_0

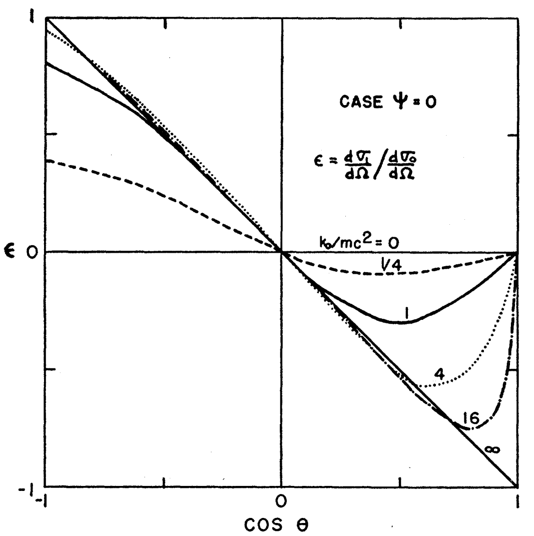

FIG. 2. The fractional change in the differential Compton cross section which obtains because electrons are polarized along the axis of the incident circularly polarized photon plotted as a function of the angle through which the photon is scattered.

greater than $1.25mc^2$, the *forward* "lobe" in $d\sigma_1/d\Omega$ predominates in the total cross section.

APPENDIX B: TRANSMISSION THROUGH A FERROMAGNET

Let a ferromagnetic absorber of N atoms per unit volume and uniform magnetization along the 3 axis have ν polarized electrons per average atom. Circularly polarized photons of a certain sense, propagating along the 3 axis, can undergo Compton scattering with the

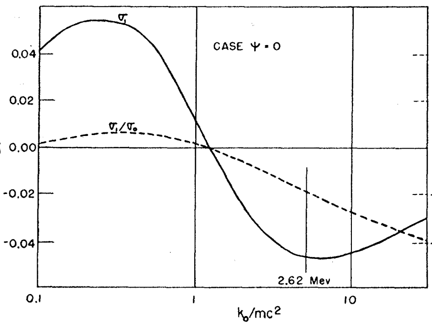

FIG. 3. The polarization-sensitive part σ_1 of the total Compton cross section is in units of $2\pi r_0^2$. The part independent of polarization is σ_0.

$\frac{1}{2}(Z+\nu)Ndx_3$ electrons according to one sign in (A.3) and with the $\frac{1}{2}(Z-\nu)Ndx_3$ electrons according to the opposite sign.

The probability that a photon having no circular polarization will survive (in good geometry) a length L of the ferromagnet is

$$\begin{aligned}T(\nu)=&\tfrac{1}{2}\exp\{-NL[(\sigma_0+\sigma_1)\tfrac{1}{2}(Z+\nu)\\&+(\sigma_0-\sigma_1)\tfrac{1}{2}(Z-\nu)+\sigma_2]\}\\&+\tfrac{1}{2}\exp\{-NL[(\sigma_0-\sigma_1)\tfrac{1}{2}(Z+\nu)\\&+(\sigma_0+\sigma_1)\tfrac{1}{2}(Z-\nu)+\sigma_2]\}\\=&\cosh(NL\nu\sigma_1)\exp[-NL(Z\sigma_0+\sigma_2)],\end{aligned} \tag{B.1}$$

where σ_0 and σ_1 are the terms defined in (A.3) and σ_2 represents all processes in addition to Compton scattering. Spin-polarization effects in σ_2 are taken to be nil; for example, photoeffect involving the polarized electrons has a very small cross section compared to σ_1 at energies of interest in this experiment.

The ratio of magnetized transmission to unmagnetized transmission is

$$T(\nu)/T(0)=\cosh(NL\nu\sigma_1). \tag{B.2}$$

Ideally, $(NL\nu\sigma_1)$ may be measured in the shortest time if $NL(Z\sigma_0+\sigma_2)=4$.

The transmission ratio R of l.c.p. relative to r.c.p. photons is, by (B.1),

$$R=\exp(2NL\nu\sigma_1), \tag{B.3}$$

where the magnetization is in the direction of transmission.