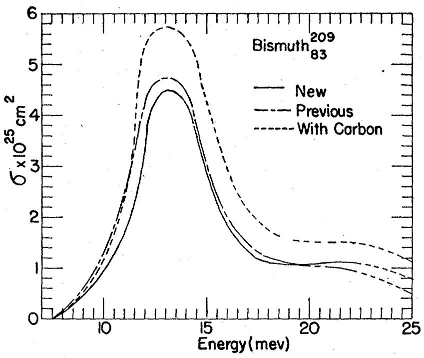

FIG. 2. Bismuth (γ,n) excitation functions constructed from yield curves taken with and without carbon absorber.

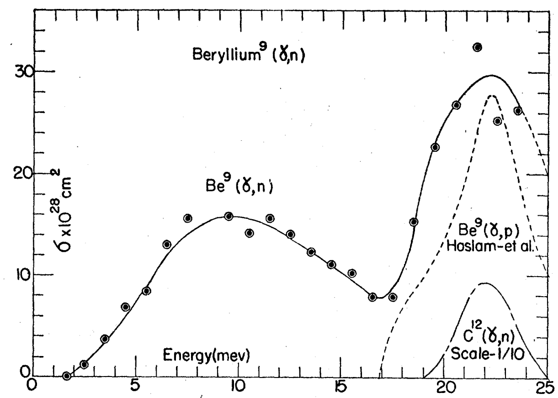

FIG. 3. Beryllium (γ,n) excitation function.

the (γ,n) excitation function of carbon. The method of constructing cross section from yield curves does not permit detailed check with Guth and Mullin near threshold, but the general behavior of the first portion of the beryllium excitation function is in agreement with their model for the $P\rightarrow D$ transitions. In fact, our measured maximum value of 15.8×10^{28} cm^2 for the cross section is in remarkable agreement with the calculations. The coincidence of the second peak with the (γ,p) curve and the (γ,n) excitation in carbon suggests the excitation of the Be^8 core as the photon energy increases and indicates the onset of the large dipole resonance for the core.

The integrated (γ,n) cross section in beryllium up to 25 Mev. is measured to be 0.037 Mev-barns. The (γ,p) integrated cross section of 0.013 Mev-barns[3] is not sufficient to bring the total to 0.187 Mev-barns as calculated[6] from the dipole sum rules. Thus, in beryllium as in other light elements, other modes of disintegration are important or there are large contributions to these reactions at energies above 25 Mev.[7]

[6] J. S. Levinger and H. A. Bethe, Phys. Rev. **85**, 577 (1952).

[7] Jones and Terwilliger [Phys. Rev. 91, 699 (1953)] report a large high-energy tail on the Be$^9(\gamma,n)$ excitation function above 25 Mev.

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Resonant Effects and Spin Dependence in Potential Scattering of Slow Neutrons*

W. SELOVE
Harvard University, Cambridge, Massachusetts

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The potential scattering of a nucleus can be considerably different from $4\pi R^2$, where R is the size of the nucleus, if the potential well parameters happen to give an appreciable resonance effect. Strong resonance effects are expected to be accompanied by strong spin dependence. The potential scattering of beryllium shows an appreciable resonant effect but very small spin dependence in the scattering cross section. From an examination of the numerical parameters involved it is concluded that the small spin dependence in the cross section does not preclude a moderate amount of spin dependence in the potential wells for the two spin states—the close equality of the two scattering lengths involved may be the result of a coincidental combination of well depths and ranges.

THE "potential" scattering cross section of most nuclei for slow neutrons, especially of heavier nuclei, is of the order of $4\pi R^2$, where the nominal nuclear radius R is given by $r_0A^{\frac{1}{3}}$ with $r_0\sim1.5\times10^{-13}$ cm. Some nuclei, however, show "potential" cross sections considerably different from this value.

These differences can be accounted for by "potential-well resonances." These are purely scattering resonances, associated with resonant values of the scattering phase shift on the potential-well model, and are to be distinguished from resonances of the compound nucleus. These potential-well resonances will have a "width" given roughly by $ka\sim1$, where k is the neutron wave number and a is the magnitude of the scattering amplitude $(\sigma_{\text{scat}}/4\pi)^{\frac{1}{2}}$ for $k\rightarrow0$.

In the case of nuclei with nonzero spin, strong potential-well resonant effects are likely to be accompanied

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by relatively strong spin dependence of potential scattering, because near a resonance the cross section is especially sensitive to the strength of the interaction and so a relatively small change in the potential well can produce a relatively large effect on the cross section. Na is an example of such effects and has been discussed previously.[1] Resonant effects and spin dependence may be detectable by measurement of the total scattering and coherent scattering cross sections, although care must be exercised in the analysis of such data, to separate the potential scattering from compound-nucleus effects.[2]

For a rectangular well of depth V_0, in which a neutron wave number is K_0, and of radius R, the scattering cross section at $k\rightarrow 0$ is $4\pi[\tan(K_0R)/K_0-R]^2$. Potential-well resonances thus occur for $K_0R=(2n+1)\pi/2$, $n=0, 1, 2, \cdots$. Taking $V_0\sim 20$ Mev,[3] resonances occur for $R\approx 1.6, 4.8, 8.0, \cdots \times 10^{-13}$ cm. If we take $R=1.5\times 10^{-13}A^{+\frac{1}{3}}$ cm, this gives $A\approx 1$, 33, 152. The resonance at $A=1$ corresponds to the large cross section for $n-p$ scattering (especially in the singlet state). A number of the light elements ($A<16$) also show resonant effects in the potential scattering. These are probably to be accounted for by variations in the effective values of V_0 and the nuclear radius, from the simple values given above. As R becomes larger, i.e., for heavier nuclei, resonant effects are less likely to be observed, since K_0R must be closer to a resonant value in order to make the cross section appreciably different from $4\pi R^2$.

It is one purpose of this note to discuss the scattering of slow neutrons by Be^9. The thermal scattering cross section of Be^9 is 6.04 barns, as compared to a value of $4\pi(1.5A^{\frac{1}{3}}\times 10^{-13})^2=1.3$ barns. This indicates a near-resonance in the potential scattering. Such a near-resonance would be expected to be accompanied by an appreciable spin dependence and, consequently, appreciable incoherent scattering. Yet the incoherent scattering of Be is less than 0.01 barn,[4] which means the scattering lengths for the two spin states are within 6 percent of each other, the scattering cross sections within 12 percent. The average scattering length is 6.9×10^{-13} cm. If we take a potential well of range equal to the nominal radius $R=1.5\times 10^{-13}A^{\frac{1}{3}}=3.24\times 10^{-13}$, the depth of the well is found to be 8.7 Mev. This is rather smaller than the average value of $\sim$20 Mev mentioned above[3] but may be reasonable in view of the alpha-particle nature of the "core" of Be; in fact, a similar calculation shows that the effective potential for interaction of a slow neutron with C^{12} appears to be similarly weak.

If both spin states have this same range of 3.24×10^{-13}, then the difference in depths of the potential wells is restricted to be <0.35 Mev, from the smallness of the incoherent scattering. This would represent a rather rigid requirement as to accidental equality of the potential wells for the two spin states. However, the scattering amplitude is determined essentially by V_0 times the square of the range, and so much more leeway can be allowed in the well depths for the two spin states if the two states have different ranges. For example, for a range of 3×10^{-13} cm, the proper scattering length would be obtained with a well of depth 9.7 Mev. A rough attempt has been made to see if the effective ranges for the two spin states could be determined from the cross section of Be as a function of energy.[5] Matching was attempted up to 0.35 Mev. (The calculations become considerably more involved, and more shape-dependent, at higher energies. Moreover, there is some uncertainty as to the width of the energy region over which the effect of possible negative-energy compound-nucleus resonances can safely be neglected.) Within this energy range the experimental uncertainties in the data make it impossible to detect a difference in effective ranges for the two spin states. For a rectangular-well model the data are fitted by a range of $(3\pm 0.5)\times 10^{-13}$ cm.[6]

One can thus conclude that the small incoherent scattering of Be does not preclude a moderate amount of spin dependence in the scattering. It may be that the close equality of the scattering lengths for the two spin states is at least partially due to a coincidental combination of well depths and ranges.

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[1] W. Selove, Phys. Rev. **80**, 290 (1950).

[2] K. W. Ford and D. Bohm, Phys. Rev. **79**, 745 (1950), have made an analysis of resonant effects but did not make a proper separation of potential-well and compound-nucleus effects.

[3] This value seems to give good agreement over a large region; see Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953).

[4] D. J. Hughes (private communication).

[5] C. K. Bockelman, Phys. Rev. **80**, 1011 (1950).

[6] It should be noted that this analysis has been carried out in the spirit of a potential-well effective range approach, with the energy region so restricted as to make the estimated effects of compound-nucleus resonances negligible. For a discussion of the comparison between this formulation and a resonance level formulation, see T. Teichmann, Phys. Rev. **83**, 141 (1951), and R. G. Thomas, Phys. Rev. **88**, 1109 (1952), especially p. 1115 and p. 1125.