The Angular Correlation of the Cascade Gamma Rays from the Decay of Au^{198}†

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The angular correlation of the 0.411-Mev and 0.64-Mev cascade gamma rays resulting from the beta decay of Au^{198} to Hg^{198} has been measured with scintillation counters. If only pure multipole radiations are assumed, the experimental data do not agree with theoretical calculations for any reasonable values of the spins of the relevant states in Hg^{198}. They do agree well, however, with spins of 0, 2, and 2 for the ground state, 0.411-Mev state, and 1.09-Mev state of Hg^{198}, respectively, if the first transition (0.68-Mev gamma) is assumed to be a mixture of 60 percent dipole and 40 percent quadrupole radiation and the second transition (0.41-Mev gamma) is assumed to be pure quadrupole radiation. The parities of all three states are the same, presumably even. These results are in general agreement with the internal conversion measurements of other investigators.

I. INTRODUCTION

It has recently been discovered[1–3] that about two percent of the decay of Au^{198} occurs through the emission of a 0.295-Mev beta to a 1.09-Mev level in Hg^{198}. This level decays both by a direct 1.09-Mev gamma to the ground state and by cascade emission of gammas of energies 0.68 and 0.411 Mev. Cavanagh[4] measured the intensities of the 0.68 and 1.09-Mev gammas relative to the 0.411-Mev gamma and found values of 1.4 percent and 0.4 percent, respectively. Elliott and Wolfson[5] have measured the internal conversion coefficients of all three gammas and in addition have reported a very low-intensity 1.38-Mev beta to the ground state. A determination of the angular correlation of the two cascade gamma rays should serve to verify the spin and parity assignments made on the basis of the internal conversion measurements, and if sufficiently precise should permit the determination of the multipole mixing ratio of the radiations.

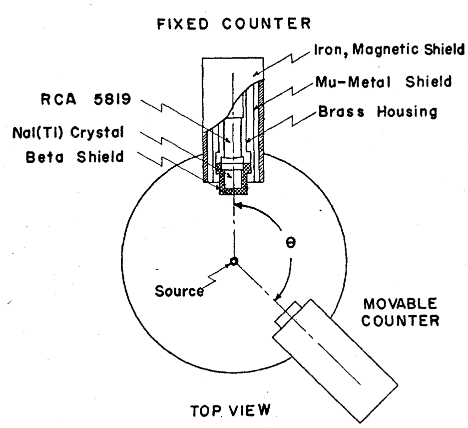

FIG. 1. Top view of scintillation spectrometer table. The details of the magnetic shielding of the photomultiplier tube and the β shields of the crystals are shown.

II. APPARATUS, PROCEDURE, AND EXPERIMENTAL RESULTS

The Au^{198} source used for the angular correlation was in the form of a metallic colloidal suspension. The source holder was made of aluminum and was constructed in the form of a hollow needle with a wall thickness of 0.020″. The scattering of the gamma rays in the walls of the source holder was estimated to have a negligible effect on the angular correlation. The scintillation counters were NaI(Tl) crystals on RCA 5819 photomultiplier tubes. The crystal and phototube were sealed in a light tight housing as shown in Fig. 1. The Au^{198} differential pulse-height curve measured in one of the counters is shown in Fig. 2. The shoulder on the curve at 0.82 Mev is due to the coincidence of random 0.411-Mev gammas within the single counter. The peak near 0.20 Mev is the Compton peak of the 0.411-Mev gammas. The peak near 0.069 Mev is thought to be due to the K x-ray of gold.[3]

The electronic components of the two counting channels and coincidence circuit are shown schematically in Fig. 3. The discriminator of the amplifier for counter No. 1 was set at a point which corresponded approximately to point A of Fig. 2. This point was high enough to avoid pile-up from low-energy pulses and noise, low enough for the counter to have very good efficiency for the 0.411-Mev gamma, and at this setting the effect of amplifier drift was also minimized. The discriminator of the amplifier for counter No. 2 was set at a point corresponding to B of Fig. 2. This setting again minimized the effect of amplifier drift while permitting good efficiency for the counting of 0.68-Mev gammas without allowing a tremendously high accidental coincidence rate from the intense 0.411-Mev gammas.

The discriminator settings were calculated to be high enough so that it was energetically improbable

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* Now at the University of California Radiation Laboratory, Berkeley, California.
[1] R. W. Pringle and S. Standil, Phys. Rev. 80, 762 (1950).
[2] Brosi, Ketelle, Zeldes, and Fairstein, Phys. Rev. 84, 586 (1951).
[3] Cavanagh, Turner, Booker, and Dunster, Proc. Phys. Soc. (London) A64, 13 (1951).
[4] P. E. Cavanagh, Phys. Rev. 82, 791 (1951).
[5] L. G. Elliott and J. L. Wolfson, Phys. Rev. 82, 333 (1951).

for any gammas from the decay of Au^{198} to be counted in one counter, scatter, and be counted in the other counter. The same settings obviated any contribution from coincidences between possible 0.97-Mev beta bremsstrahlung and a subsequent 0.411-Mev gamma (see Fig. 4).

The problem of accidental coincidences was particularly acute in this experiment since the gammas following the direct beta transition to the 0.411-Mev level are approximately seventy times more intense than the cascade gammas. The accidental coincidence rate was determined by delaying the pulses in channel two by 1.3 microseconds, approximately three times the resolving time of the coincidence circuit. This eliminated all true coincidences but left the accidental coincidence rate unchanged. The accidental coincidence rate was approximately 20 percent of the total coincidence rate. Also, because the true cascade gamma coincidence rate from Au^{198} was so small, the cosmic-ray background coincidence rate became an important correction to the observed coincidence rate. This background rate was approximately 10 percent of the total coincidence rate.

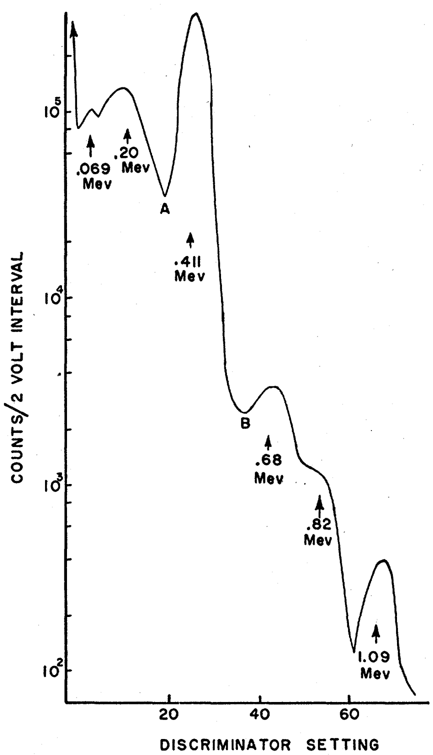

FIG. 2. Differential pulse-height distribution of pulses produced in NaI crystal by Au^{198} γ rays.

The final experimental quantity actually obtained was $W_{exp}(\theta)$. This is the ratio of the true coincidence rate (observed rate corrected for background and accidental rate) to the true rate in counter No. 1. The division by the true rate in counter No. 1 automatically corrected for decay of the source and any slight decentering of the source.

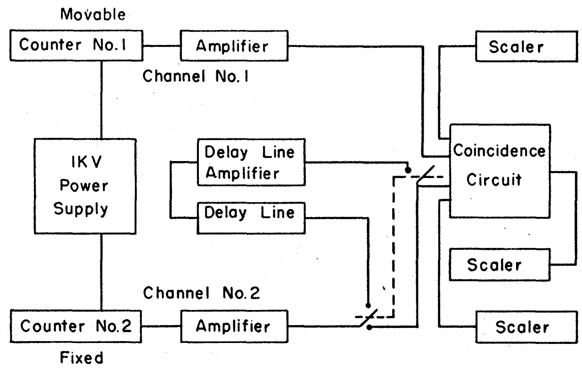

FIG. 3. Block diagram of electronic equipment.

Since theory shows that angular correlation functions are expressible in terms of Legendre polynomials, $W_{exp}(\theta)$ was fitted to such an expansion by the method of weighted least squares. The normalized result was

$$W_{exp}(\theta)=1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta),$$

where

$$A_2=-0.261\pm0.023,\quad A_4=0.137\pm0.012.$$

Although the interpretation of the data will be made in terms of the coefficients A_2 and A_4 of the Legendre polynomials, a conventional plot of $W_{exp}(\theta)$ against θ is shown in Fig. 5. The vertical bars through the experimental points represent the standard deviations based on counting statistics only.

III. DISCUSSION AND INTERPRETATION OF RESULTS

Since the lifetime of the intermediate (0.411-Mev) level has been measured[6,7] and is known to be about 10^{-11} second, the observed correlation should not have been perturbed by extra-nuclear fields. Thus a direct comparison with theoretical calculations can be made. It will be shown that such a comparison leads to a unique assignment of the spin for the second excited state of Hg^{198} and that this result agrees with all other information concerning the decay of Au^{198}.

Since $_{80}Hg^{198}$ is an even-even nucleus, its ground

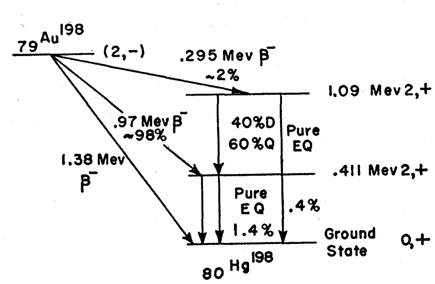

FIG. 4. Decay scheme of Au^{198}.

[6] P. B. Moon, Proc. Phys. Soc. (London) **A64**, 76 (1951).
[7] Bell, Graham, and Petch, Can. J. Phys. **30**, 35 (1952).

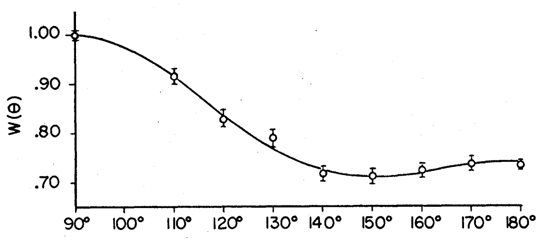

FIG. 5. Coincidence rate as function of angle between the counters. The vertical bars through the experimental points indicate the standard deviation due to counting statistics. The theoretical curve shown is calculated for a 2(60 percent D, 40 percent Q)2(Q)0 cascade.

state is assumed to have 0 spin and + parity (see also Burger and Van Cittert[8]). Moon[9] and Simons[10] are among the latest of several workers to measure the internal conversion coefficient for the 0.411-Mev gamma. All results show that this gamma is pure electric quadrupole radiation. This leads to the assignment of spin 2 and + parity for the 0.411-Mev level.

The spin of the 1.09-Mev level in Hg^{198} could be either 1, 2, or 3; a spin greater than 3 would mean a spin change of more than 3 in the crossover gamma-ray transition which is very highly improbable since the crossover gamma competes favorably with the first cascade gamma. Table I shows the theoretical Legendre polynomial coefficients for possible pure multipole transitions, and also shows the same coefficients corrected for the finite resolution of the counters used in this experiment. (The correction was made according to the method of Church and Kraushaar.)[11] Since there is no theoretical set of pure 1, 2, or 3 (D or Q) 2(Q)0 angular correlation coefficients which agree with the experimental coefficients, it becomes necessary to calculate the possible multipole mixtures which might occur in the first transition.

Theoretical expressions for angular correlations

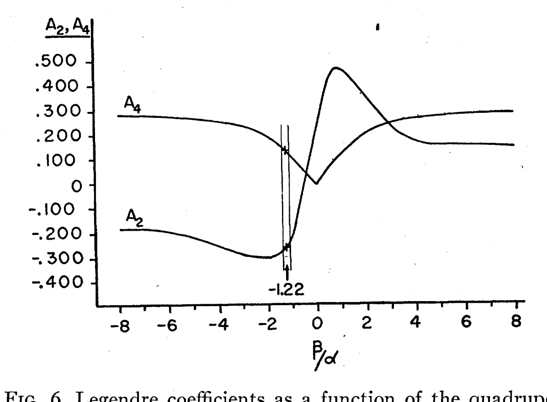

FIG. 6. Legendre coefficients as a function of the quadrupole-dipole amplitude ratio, β/α. The mean value of β/α which is compatible with the experimentally possible values of A_2 and A_4 (enclosed by the vertical lines) is -1.22.

involving mixed radiations have been calculated by several authors, including Coester and Jauch.[12] The particular equations used were taken from Sec. III of Rose and Biedenharn.[13] The general angular correlation function for the cascade emission of a mixed dipole-quadrupole radiation and a pure quadrupole radiation may be written as

$$W(\theta)=\alpha^2W_1(\theta)+\beta^2W_2(\theta)+2\alpha\beta W_3(\theta),$$

where W_1 is the correlation function for a pure D-Q cascade, W_2 is the function for a pure Q-Q cascade, and W_3 is the interference term, and where the α's and the β's are the reduced matrix elements of the respective transitions. The W's involve sums of products of Clebsch-Gordon coefficients, which may be calculated from equations given in Condon and Shortley,[14] and Racah coefficients, which can be found in tables prepared under the direction of Biedenharn.[15] The relative intensity of the quadrupole component to the dipole component of the mixed radiation is given by β^2/α^2. Lloyd[16] (see also Coester[17]) has pointed out that α and β are real with their signs undetermined. Thus

TABLE I. Coefficients in the expansion $1+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$.

Transition	Theoretical coefficients for pure multipole transitions A_2	A_4	Coefficients corrected for counter resolution A_2	A_4
1(D)2(Q)0	−0.200	0	−0.192	0
1(Q)2(Q)0	1.895	−0.0764	1.821	−0.0666
2(D)2(Q)0	0.250	0	0.240	0
2(Q)2(Q)0	−0.0775	0.327	−0.0745	0.285
3(D)2(Q)0	−0.0714	0	−0.0686	0
3(Q)2(Q)0	−0.204	−0.0817	−0.196	−0.0713
(Experimental)	···	···	−0.261	0.137

the sign and the magnitude of the ratio β/α are arbitrary and can be adjusted to the data.

In the present case a consideration of the possible dipole-quadrupole mixtures for a spin of either 1 or 3 shows that for all values of β/α the coefficient of the $P_4(\cos\theta)$ term is either zero or negative. Such mixtures are thus ruled out because of the observed positive coefficient of the $P_4(\cos\theta)$ term. Therefore, it is most probable that a theoretical mixture of spin 2-D, Q will agree with the experimental data.

Figure 6 shows the values of the angular correlation coefficients possible for a dipole-quadrupole, spin-2 mixture as a function of β/α. A mixing ratio of 60Q/40D with $\beta/\alpha=-1.225$ gives theoretical values of $A_2=-0.261$ and $A_4=0.136$ which most nearly coincide

[8] H. C. Burger and P. H. Van Cittert, Physica 5, 177 (1938).
[9] M. Moon, Thesis, State University of Iowa, 1951 (unpublished).
[10] L. Simons, Phys. Rev. 86, 570 (1952).
[11] E. L. Church and J. Kraushaar, Phys. Rev. 88, 419 (1952).
[12] F. Coester and J. M. Jauch, Helv. Phys. Acta 26, No. 1 (1953).
[13] M. E. Rose and L. C. Biedenharn, Revs. Modern Phys. (to be published).
[14] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (The Macmillan Company, Inc., New York, 1952), pp. 76–77.
[15] L. C. Biedenharn, "Table of Racah Coefficients," Oak Ridge National Laboratory Report ORNL-1098, 1952 (unpublished).
[16] S. P. Lloyd, Phys. Rev. 81, 161 (1951).
[17] F. Coester, Phys. Rev. 89, 619 (1953).

with the observed coefficients, although any values of β^2/α^2 between about 55/45 and 65/35 (β/α between -1.11 and -1.30) would be within the statistical accuracy of the experiment. The theoretical coefficients have been corrected for finite resolution of the counters.

Because of the excellent agreement of the observed correlation with the theoretical 2(40 percent D, 60 percent Q)2(Q)0 correlation, it is concluded that the spin of the 1.09-Mev level in Hg^{198} is 2 and that the 0.68-Mev radiation consists of a mixture of about 40 percent dipole and 60 percent quadrupole radiation.

Recently Elliott and Wolfson[5,18] have measured the internal converson of all three gamma rays and have found the coefficients to be consistent with the theoretical values of pure $E2$ for the 1.09- and 0.411-Mev gammas and 60 percent $M1$ plus 40 percent $E2$ for the 0.68-Mev gamma. They also measured the intensities of the 0.68- and 1.09-Mev gammas relative to the 0.411-Mev gamma, finding values of 0.48 percent and 0.13 percent, respectively. The pure $E2$ coefficient for the crossover transition allows immediate assignment of 2+ to the 1.09-Mev level. This agrees with the present experiment in the assignment of spin. Also, since the first announcement[19] of the present work, Schiff and Metzger[20] have published an independent account of essentially the same work with essentially the same results.

[18] Dr. L. G. Elliott (private communication, December, 1952).

IV. ACKNOWLEDGMENTS

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[19] Schrader, Nelson, and Jacobs, Phys. Rev. **90**, 159 (1953).
[20] D. Schiff and F. R. Metzger, Phys. Rev. **90**, 849 (1953).

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Method for the Determination of the End-Point Energy of Beta Emitters*

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It was found that the absorption of beta spectra in materials of high atomic number is exponential. The absorption coefficients in gold are inversely proportional to the $\frac{3}{2}$ power of the end-point energy of the spectrum. This experimental law holds with 2 percent accuracy in the energy interval from 0.3 Mev to 2.2 Mev and in the intensity domain from $\frac{1}{2}$ to 1/100 of the original intensity, if a detector with less than 2 percent relative opening is used. The end-point energy of beta emitters can, therefore, be determined by measuring only two points on a gold absorption curve and is expressed by the equation: $E=9\mu^{-\frac{2}{3}}+0.015$ with 2 percent accuracy from 0.17 Mev to 2.2 Mev; where E is the end-point energy in Mev and μ the absorption coefficient in cm^2/g.

The most characteristic feature of a beta spectrum is its end-point energy. Spectroscopic and absorption methods so far used require strong sources and tedious measurements.[1] They cannot be used in cases when the source is weak, has a short lifetime, or when for other reasons a quick determination is required.

The method presented here is based on two experimental observations made by the author.

(1) It was found that the absorption curve of beta emitters in materials of high atomic number, e.g., gold, is a fairly straight line in a semi-logarithmic plotting, if the detector used has a relative opening of less than 2 percent and the absorbers are placed close to the source (see Fig. 1). Consequently the slope of such an absorption curve is determined by measuring only two points on the curve.

(2) It was found that the slope of the absorption curve in gold, i.e., the absorption coefficient, and the maximum energy of the beta spectrum are connected by the equation:

$$E=9\mu^{-\frac{2}{3}}+0.017, \tag{1}$$

where

$$\mu=\frac{\ln(J_1/J_2)}{a_2-a_1},$$

E denoting the end-point energy in Mev; a_1 and a_2 the thicknesses of the gold absorbers in g/cm^2; J_1 and J_2 the measured intensities at the two absorber thicknesses.

All in all, seven substances have so far been investi-

* This research has been supported by Frederick Gardner Cottrell grant of Research Corporation.
[1] See the comprehensive paper discussing absorption methods by L. Katz and A. S. Penfold, Revs. Modern Phys. **24**, 31 (1952).