We find that for (1), $Q=(103_{-15}^{+20})$ Mev and the calculated mass is $2315m_e$; for (2), $Q'=(101_{-15}^{+20})$ Mev and the mass is $2650m_e$.

Support for scheme (2) is provided by recent cloud-chamber observations of cascade V-particle events in which a charged primary disintegrates into a V_1^0 and a light meson,[6,7]

$$V^- \rightarrow V_1^0 + (\pi^- \text{ or } \mu^-) + Q. \tag{3}$$

Evidence thus exists for the emission of a V_1^0 as a neutral decay product of a charged heavy particle, whereas there is no corresponding evidence, so far as we know, for a secondary neutron. It seems reasonable, therefore, as a working hypothesis, to identify our scheme (2) with (3) of the Pasadena group.[7] The respective Q values, 101_{-15}^{+20} Mev (NRL) and 60 ± 15 (Pasadena) are not inconsistent.

As for the *charged* secondary, the cloud-chamber observations do not permit a choice between π and μ. However, under the foregoing hypothesis, we conclude that the charged secondary in the V-particle cascades is more likely to be a pion than a muon.

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Intermediate Coupling and Nuclear Reactions

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PREVIOUS discussion of the shell model has centered around such "static" experimental quantities as magnetic moments, spins, etc. It is the purpose of this letter to point out that this discussion can be extended to include the "dynamical" quantities of the type found in nuclear reactions. These quantities include the reduced widths of energy levels for nucleon emission and the matrix elements of electromagnetic transitions. There is a considerable accumulation of such data[1] in light nuclei, most of which has never been seriously used to investigate nuclear structure.

If an extreme mode of coupling is assumed in nuclei (i.e., $L-S$ or $j-j$), then theoretical expressions for these dynamical quantities can be found very simply using the theory of fractional parentage.[2] For instance, consider the reduced width in $L-S$ coupling for a nucleon transition of the type:

$$\text{compound nucleus } \lambda \rightarrow \text{residual nucleus } p + \text{nucleon};$$

i.e.,

$$l^n(\alpha TSL, J) \rightarrow l^{n-1}(\alpha_p T_p S_p L_p, J_p) + l; \tag{1}$$

i.e., the emission of one nucleon from a state λ of n equivalent nucleons to leave a residual nucleus of $(n-1)$ equivalent nucleons. The symbols α, T, S, L, J have their usual meaning. The reduced width[3] for this process can be shown to be

$$\gamma_{\lambda s l}{}^2 = \gamma^2(l) n \langle \alpha]\!] \alpha_p \rangle^2 U^2(L_p S_p s \tfrac{1}{2}, J_p S) U^2(S L_p J l, s L), \tag{2}$$

where the U functions are Recah functions[2] and where $\langle \alpha]\!] \alpha_p \rangle$ is an abbreviated notation for the total fractional parentage coefficient[2] appropriate to the two states of (1). α and α_p, as usual, denote symmetry characters. The symbol s denotes the values of the coupled spins of the nucleon and residual nucleus,[3] and $\gamma^2(l)$ is the reduced width for a single l nucleon in the potential well of the shell model (the value will be $\sim\hbar^2/Ma$, where M is the reduced mass of the nucleon and a is the radius of the well). In $j-j$ coupling, the reduced width for the transition

$$j^n(\alpha TJ) \rightarrow j^{n-1}(\alpha_p T_p J_p) + j \tag{3}$$

emerges as

$$\gamma_{\lambda s l}{}^2 = \gamma^2(l) n \langle \alpha]\!] \alpha_p \rangle^2 U^2(J_p \tfrac{1}{2} J l, s j). \tag{4}$$

Formulas can be derived similarly for the values of the matrix elements of electromagnetic transitions in $L-S$ and $j-j$ coupling.

The formulas both for reduced widths and electromagnetic matrix elements have been extensively compared with experimental values in light nuclei. It is found that neither extreme coupling mode can fit the data, but there is strong suggestion that intermediate coupling can do so.

In order to test this conclusion, the mirror nuclei C^{13} and N^{13} have been examined in detail. The properties of these nuclei are well known[1] and altogether there are eleven useful independent data on the first four levels of these nuclei:

(1) the binding energies of the first-excited states in the two nuclei,

(2) the slow-neutron scattering length for C^{12} and the "effective range" of slow-neutron scattering,[4]

(3) the reduced widths of the first- and third-excited states in N^{13} (these are essentially "single-particle" levels, the first $(\frac{1}{2}+)$ being simply a $2s$ nucleon added to the C^{12} "core," and the third $(5/2+)$ being a $1d$ nucleon added similarly),

(4) the $E1$ and $M1$ transition widths from the first- and second-excited states of N^{13} to the ground state,

(5) the reduced widths of the ground- and second-excited states [the latter is observed in N^{13} from proton scattering in C^{12}, and the former is obtained for C^{13} from the yield of the (d, p) stripping reaction on C^{12}],

(6) the magnetic moment of C^{13}.

Of these data, the predicted values of (1), (2), and (3) are not sensitive to the mode of coupling in the shell model, but only to the potential well of the model. These six independent data can all be fitted with a certain choice of square well (depth 35 Mev, radius 4.05×10^{-13} cm). The predictions for the other data (4), (5), and (6) are all sensitive to the mode of coupling in the shell model, some of them being extremely so. Analysis shows that all the five data of (4), (5), and (6) are fitted extremely well by an intermediate coupling model with a value of the usual parameter a/K of about 4.5 (a being the strength of the spin-orbit coupling and K the exchange integral).[5]

Consequently every observable feature of the first four states of C^{13} and N^{13} is consistent with an intermediate coupling model of the nucleus. This appears to give by far the most consistent support for the model found hitherto. A complete account of this work will appear in a series of articles in the *Proceedings of the Physical Society* (*London*),[6] and as part of a review article on nuclear reactions.[7]

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Periodic Ellipse of the Strong-Focusing Equation[*]

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THE equation of free betatron oscillations in the strong-focusing synchrotron is a Hill's equation of the form[1]

$$d^2x/d\theta^2 + n(\theta)x = 0, \tag{1}$$

where $n(\theta)$ can be taken (by including a scale factor in θ) to alternate between 1 in converging sectors and -1 in diverging