We find that for (1), $Q=(103_{-15}^{+20})$ Mev and the calculated mass is $2315m_e$; for (2), $Q'=(101_{-15}^{+20})$ Mev and the mass is $2650m_e$.

Support for scheme (2) is provided by recent cloud-chamber observations of cascade V-particle events in which a charged primary disintegrates into a V_1^0 and a light meson,[6,7]

$$V^- \rightarrow V_1^0 + (\pi^- \text{ or } \mu^-) + Q. \tag{3}$$

Evidence thus exists for the emission of a V_1^0 as a neutral decay product of a charged heavy particle, whereas there is no corresponding evidence, so far as we know, for a secondary neutron. It seems reasonable, therefore, as a working hypothesis, to identify our scheme (2) with (3) of the Pasadena group.[7] The respective Q values, 101_{-15}^{+20} Mev (NRL) and 60 ± 15 (Pasadena) are not inconsistent.

As for the *charged* secondary, the cloud-chamber observations do not permit a choice between π and μ. However, under the foregoing hypothesis, we conclude that the charged secondary in the V-particle cascades is more likely to be a pion than a muon.

We are grateful to Mr. B. Stiller and Mr. F. W. O'Dell for their assistance, and to Mrs. N. T. Redfearn for much of the scanning.

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Intermediate Coupling and Nuclear Reactions

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PREVIOUS discussion of the shell model has centered around such "static" experimental quantities as magnetic moments, spins, etc. It is the purpose of this letter to point out that this discussion can be extended to include the "dynamical" quantities of the type found in nuclear reactions. These quantities include the reduced widths of energy levels for nucleon emission and the matrix elements of electromagnetic transitions. There is a considerable accumulation of such data[1] in light nuclei, most of which has never been seriously used to investigate nuclear structure.

If an extreme mode of coupling is assumed in nuclei (i.e., $L-S$ or $j-j$), then theoretical expressions for these dynamical quantities can be found very simply using the theory of fractional parentage.[2] For instance, consider the reduced width in $L-S$ coupling for a nucleon transition of the type:

$$\text{compound nucleus } \lambda \rightarrow \text{residual nucleus } p + \text{nucleon};$$

i.e.,

$$l^n(\alpha TSL, J) \rightarrow l^{n-1}(\alpha_p T_p S_p L_p, J_p) + l; \tag{1}$$

i.e., the emission of one nucleon from a state λ of n equivalent nucleons to leave a residual nucleus of $(n-1)$ equivalent nucleons. The symbols α, T, S, L, J have their usual meaning. The reduced width[3] for this process can be shown to be

$$\gamma_{\lambda s l}^2 = \gamma^2(l) n \langle \alpha]\!\} \alpha_p \rangle^2 U^2(L_p S_p s \tfrac{1}{2}, J_p S) U^2(S L_p J l, s L), \tag{2}$$

where the U functions are Recah functions[2] and where $\langle \alpha]\!\} \alpha_p \rangle$ is an abbreviated notation for the total fractional parentage coefficient[2] appropriate to the two states of (1). α and α_p, as usual, denote symmetry characters. The symbol s denotes the values of the coupled spins of the nucleon and residual nucleus,[3] and $\gamma^2(l)$ is the reduced width for a single l nucleon in the potential well of the shell model (the value will be $\sim\hbar^2/Ma$, where M is the reduced mass of the nucleon and a is the radius of the well). In $j-j$ coupling, the reduced width for the transition

$$j^n(\alpha TJ) \rightarrow j^{n-1}(\alpha_p T_p J_p) + j \tag{3}$$

emerges as

$$\gamma_{\lambda s l}^2 = \gamma^2(l) n \langle \alpha]\!\} \alpha_p \rangle^2 U^2(J_p \tfrac{1}{2} J l, s j). \tag{4}$$

Formulas can be derived similarly for the values of the matrix elements of electromagnetic transitions in $L-S$ and $j-j$ coupling.

The formulas both for reduced widths and electromagnetic matrix elements have been extensively compared with experimental values in light nuclei. It is found that neither extreme coupling mode can fit the data, but there is strong suggestion that intermediate coupling can do so.

In order to test this conclusion, the mirror nuclei C^{13} and N^{13} have been examined in detail. The properties of these nuclei are well known[1] and altogether there are eleven useful independent data on the first four levels of these nuclei:

(1) the binding energies of the first-excited states in the two nuclei,

(2) the slow-neutron scattering length for C^{12} and the "effective range" of slow-neutron scattering,[4]

(3) the reduced widths of the first- and third-excited states in N^{13} (these are essentially "single-particle" levels, the first $(\frac{1}{2}+)$ being simply a $2s$ nucleon added to the C^{12} "core," and the third $(5/2+)$ being a $1d$ nucleon added similarly),

(4) the $E1$ and $M1$ transition widths from the first- and second-excited states of N^{13} to the ground state,

(5) the reduced widths of the ground- and second-excited states [the latter is observed in N^{13} from proton scattering in C^{12}, and the former is obtained for C^{13} from the yield of the (d, p) stripping reaction on C^{12}],

(6) the magnetic moment of C^{13}.

Of these data, the predicted values of (1), (2), and (3) are not sensitive to the mode of coupling in the shell model, but only to the potential well of the model. These six independent data can all be fitted with a certain choice of square well (depth 35 Mev, radius 4.05×10^{-13} cm). The predictions for the other data (4), (5), and (6) are all sensitive to the mode of coupling in the shell model, some of them being extremely so. Analysis shows that all the five data of (4), (5), and (6) are fitted extremely well by an intermediate coupling model with a value of the usual parameter a/K of about 4.5 (a being the strength of the spin-orbit coupling and K the exchange integral).[5]

Consequently every observable feature of the first four states of C^{13} and N^{13} is consistent with an intermediate coupling model of the nucleus. This appears to give by far the most consistent support for the model found hitherto. A complete account of this work will appear in a series of articles in the *Proceedings of the Physical Society (London)*,[6] and as part of a review article on nuclear reactions.[7]

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Periodic Ellipse of the Strong-Focusing Equation*

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THE equation of free betatron oscillations in the strong-focusing synchrotron is a Hill's equation of the form[1]

$$d^2x/d\theta^2 + n(\theta)x = 0, \tag{1}$$

where $n(\theta)$ can be taken (by including a scale factor in θ) to alternate between 1 in converging sectors and -1 in diverging sectors. If each sector is of length $\Delta\theta=\nu$, then $n(\theta)$ is periodic with period 2ν. The amplitude x and slope $x'=dx/d\theta$ at $\theta+2\nu$ are linear functions of the amplitude and slope at θ,

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\theta+2\nu} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\theta}. \tag{2}$$

The translation matrix $A=||A_{ij}(\theta)||$ has unit determinant, thus preserving density in phase space. The motion is stable if $|\frac{1}{2}\,\mathrm{Tr}A|=|\frac{1}{2}(A_{11}+A_{22})|<1$, in accordance with Floquet's theory of differential equations with periodic coefficients.

In the stable case, A can be written as the equivalence transform $A=S^{-1}(\theta)R(\beta)S(\theta)$ of an orthogonal matrix $R(\beta)$ representing a rotation through angle β, where $\cos\beta=\frac{1}{2}\,\mathrm{Tr}A$ is independent of θ. Multiplication of Eq. (2) on the left by S shows that A leaves invariant a quadratic form in x and x', corresponding for any given set of initial conditions to an ellipse (a hyperbola in the unstable case) with center at the origin of phase space. As θ increases by successive increments of 2ν, the point (x, x') advances clockwise around the perimeter. The radius vectors to two adjacent positions enclose a fraction $\beta/2\pi$ of the total area of the ellipse.

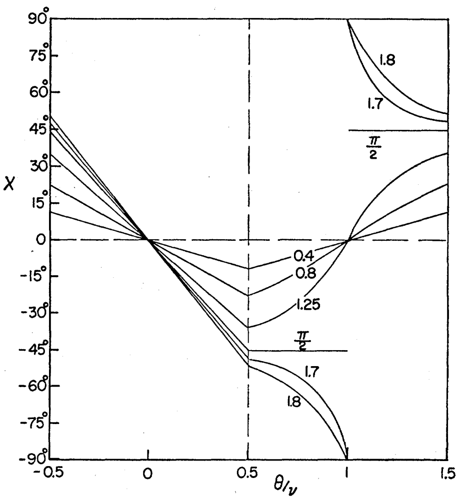

FIG. 1. The angle χ (defined modulo π) between the major axis of the ellipse and the x axis in phase space. The region $-0.5<\theta/\nu<0.5$ is a converging sector and $0.5<\theta/\nu<1.5$ is a diverging sector. The curves are labeled by values of ν, where 2ν is the period of $n(\theta)$.

Since the matrix elements A_{ij} are periodic functions of θ with period 2ν, the eccentricity and orientation of the ellipse will also have this period. The continuous history of a point in phase space, as distinguished from the preceding account of its positions at equal intervals of θ, therefore, can be described as motion on an ellipse whose shape and orientation change periodically with the period of $n(\theta)$. Its area, which is determined by the initial conditions, remains constant since Eq. (1) can be written in Hamiltonian form and satisfies Liouville's theorem.

The following parametrization of the unimodular group leads to a simple description of the periodic motion of the ellipse:

$$\begin{aligned} A_{11}&=\cos\beta-\sin\beta\,\sinh\psi\,\sin2\chi,\\ A_{22}&=\cos\beta+\sin\beta\,\sinh\psi\,\sin2\chi,\\ A_{12}&=\sin\beta(\sinh\psi\,\cos2\chi+\cosh\psi),\\ A_{21}&=\sin\beta(\sinh\psi\,\cos2\chi-\cosh\psi). \end{aligned} \tag{3}$$

Equations (3) correspond to writing the transformation (2) as a rotation of phase space viewed from rectangular axes that have been rotated and stretched. The ellipse left invariant by the transformation (2) is

$$e^{-\psi}(x\cos\chi+x'\sin\chi)^2+e^{\psi}(-x\sin\chi+x'\cos\chi)^2=l^2=\text{constant}. \tag{4}$$

If by definition $\psi\geq0$, then the major axis makes an angle χ with the x axis; the semimajor and semiminor axes are of length $le^{\psi/2}$ and $le^{-\psi/2}$, respectively.

The dependence of ψ and χ on θ is easily obtained by writing down the translation matrix (a product of simple factors, one for each interval of θ in which $n(\theta)$ is constant) and solving Eqs. (3) for the parameters. One finds that $\cos\beta=\cos\nu\,\cosh\nu$ and that in a converging sector with center at the origin of θ (i.e., for $-\nu/2\leq\theta\leq\nu/2$),

$$\begin{aligned} \coth\psi&=\sin\nu\,\coth\nu,\\ \chi&=-\theta. \end{aligned} \tag{5}$$

Thus the ellipse rotates without change of shape in a converging sector. In a diverging sector ($\nu/2\leq\theta\leq3\nu/2$),

$$\begin{aligned} \cosh\psi&=\sin\nu(\sin\beta)^{-1}\cosh2(\theta-\nu),\\ \tan2\chi&=\tan\nu(\sinh\nu)^{-1}\sinh2(\theta-\nu). \end{aligned} \tag{6}$$

Equations (5) and (6) are plotted in Figs. 1 and 2 for various values of ν in the first stability region of Eq. (1). A transition

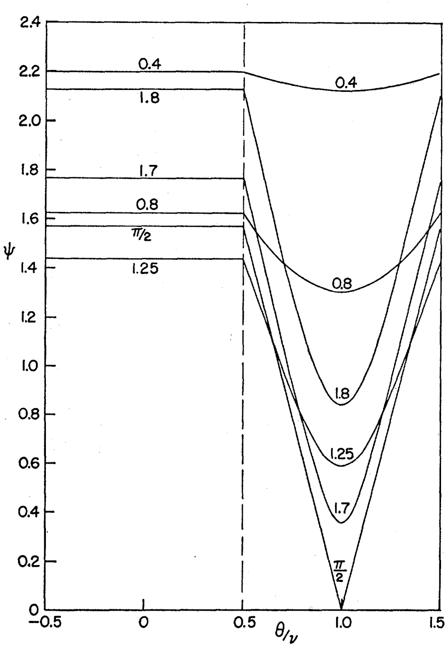

FIG. 2. ψ is the natural logarithm of the ratio of major to minor axis of the ellipse. See caption of Fig. 1 for the abscissa. The curves are labeled by values of ν.

occurs at $\nu=\pi/2$ between libration of the ellipse and end-over-end rotation. At the transition point the ellipse does not rotate at all in a diverging sector but simply degenerates at the center of the sector to a circle, for which χ is indeterminate. At the end of a diverging sector the ellipse returns, of course, to its shape and orientation at the beginning of a converging sector. The maximum excursion in x of any point on the ellipse occurs at the center of a converging sector, where χ vanishes and ψ is a maximum.

If $n(\theta)$ is an arbitrary periodic function instead of being piecewise constant, one cannot in general write down the explicit form of the translation matrix, but it is still possible to find the differential equations of motion of the ellipse. Differentiation with respect to θ of $Fx^2+2Hxx'+Gx'^2=\text{constant}$, and substitution of $-nx$ for x'' leads to a quadratic form in x and x' whose coefficients must vanish identically, since x and x' can be varied independently by varying the initial conditions. The resulting equations are $F'=2nH$, $G'=-2H$, and $H'=nG-F$, and they satisfy $FG-H^2$

=constant. In terms of ψ and χ, these equations become

$$(\chi'+1)2\sin 2\chi=\psi'(\coth\psi\cos 2\chi+1),$$
$$\psi'=(1-n)\sin 2\chi. \quad (7)$$

It can be verified readily that Eqs. (5) and (6) are a periodic solution of Eqs. (7) when n alternates between 1 and -1.

This work resulted from a conversation with Hartland S. Snyder, who suggested investigating the periodic motion of the invariant ellipse.

* This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.

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PHYSICAL REVIEW VOLUME 92, NUMBER 3 NOVEMBER 1, 1953

Proceedings of the American Physical Society

Minutes of the 1953 Spring Meeting of the New York State Section at Schenectady, April 10 and 11, 1953

THE twenty-ninth meeting of the New York State Section convened at the General Electric Research Laboratory, The Knolls, Schenectady, on April 10 and 11, 1953. About 130 members and friends of the section registered for the two-day session.

Friday morning was devoted to tours of the laboratories, and to a relatively short session of two invited papers. Five invited papers were presented at the afternoon session. At the dinner meeting on Friday evening, Mr. James Stokley of the G. E. Research staff discussed "Calendars—Past, Present, and Future."

The closing session, again consisting of invited papers, followed the business meeting of the section on Saturday morning. The local committee, Messrs. Harold Way, Roger Powell, and Frank Studer, under the chairmanship of G. C. Baldwin, deserve commendation.

The principal item of business at the Saturday session was the election of officers. Resignation of H. Hoerlin from the executive committee produced an unanticipated third vacancy; it was ruled that this vacancy be filled by the candidate receiving the third highest number of votes in the balloting for executive committee membership. Officers and members of the executive committee for the next two years are as follows:

Chairman: L. P. Smith, *Cornell University*
Vice-Chairman: Harold E. Way, *Union College*
Secretary: L. W. Phillips, *University of Buffalo*
Treasurer: William I. Caldwell, *Taylor Instrument Company*
Executive Committee:
G. P. Smith, *Corning Glass Works*
G. C. Baldwin, *General Electric Research Laboratories*
R. J. Gladieux, *Kenmore High School*
G. W. Hazzard, *St. Lawrence University*
T. E. Renzema, *Clarkson College*

Titles of the invited papers presented at the various sessions are listed below.

L. W. Phillips, *Secretary*
New York State Section
The University of Buffalo
Buffalo 14, New York

Invited Papers

Electrical Conductivity of Metals at Low Temperatures. Milan Fiske, *General Electric Research Laboratory.*

Crystal Growth and Spirals, Etch-Figures and Holes. F. H. Horn, *General Electric Research Laboratory.*

Studies of the Biological Effects of High-Voltage X-Radiation. H. M. Rozendaal, *General Electric Research Laboratory*

Apparatus for Studies with Cathode Rays. E. J. Lawton, *General Electric Research Laboratory*

Biological Studies with High-Energy Cathode Rays. M. D. Bellamy, *General Electric Research Laboratory.*

Experimental Evidence for Inheritance of Radiation Effects. G. Heidenthal, *Russell Sage College*

Effects of High-Energy Radiation on Glass. S. D. Stookey, *Corning Glass Works*

Electroluminescence. W. W. Piper, *General Electric Research Laboratory*

Double-Crystal Neutron Spectrometer. V. C. Wilson, *General Electric Research Laboratory*

Neutron Velocity Spectrometer with Betatron Source. E. R. Gaerttner, *Knolls Atomic Power Laboratory.*

Čerenkov Radiation. J. Rich, *General Electric Research Laboratory*

Minutes of the Meeting Held at Rochester, New York, June 18–20, 1953

(Corresponding to the *Bulletin of the American Physical Society*, Volume 28, No. 4).

THE American Physical Society held its 1953 Summer Meeting in the East at Rochester, New York, on Thursday, Friday, and Saturday, June 18, 19, and 20. The host was the University of Rochester. The Secretary was unable to attend this meeting, but avails himself of a report made by Professor J. B. Platt. To quote: "The statistics are: a total of 363 persons registered, 281 from