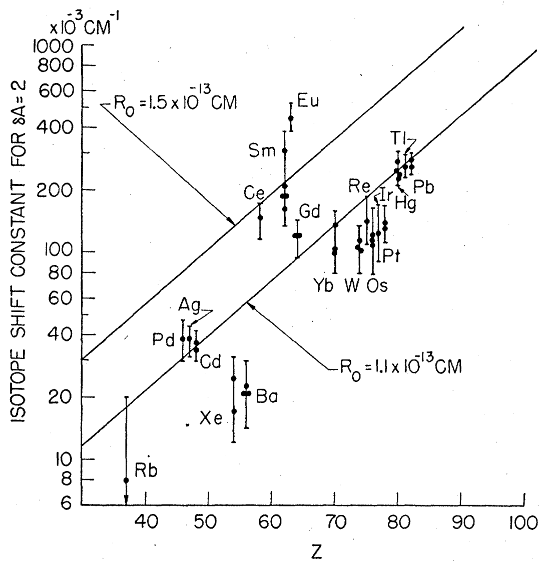

FIG. 1. Isotope shift constant as a function of atomic number.

between mirror nuclei is near the nuclear surface and will have a smaller than average Coulomb interaction. Cooper and Henley have tested these suggestions by a model and have shown it to be plausible that both of these effects are sufficient to reconcile the values of R_s and R_N. Finally, we have the evidence from nuclear reactions. It is perhaps not too surprising that these radii are large because the strong absorption properties of nuclear matter would tend to weight the surface regions more heavily. Actually, this may be seen by defining a radius in terms of an effective range theory.[11] Of course it no longer is possible to take the nucleus as having a uniform distribution, but rather a tail of some extension must be assumed.

* This work was assisted in part by the U. S. Army Signal Corps, the Air Materiel Command, and the U. S. Office of Naval Research; and in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

1 L. N. Cooper and E. M. Henley, Phys. Rev. **91**, 480 (1953).
2 V. L. Fitch and J. Rainwater (private communication).
3 Lyman, Hanson, and Scott, Phys. Rev. **84**, 626 (1951).
4 Hammer, Raka, and Pidd, Phys. Rev. **90**, 341 (1953).
5 Hofstadter, Fechter, and McIntyre, Phys. Rev. **91**, 439 (1953).
6 H. Feshbach, Phys. Rev. **84**, 1206 (1951).
7 L. K. Acheson, Phys. Rev. **82**, 488 (1951).
8 P. Brix and H. Kopfermann, Festschr. Akad. Wiss., Göttingen 17–49 (1951).
9 A. E. S. Green and N. A. Engler, Phys. Rev. **91**, 40 (1953).
10 L. R. B. Elton (private communication).
11 J. Schwinger (private communication).

Charged Particles of Mass Intermediate between Proton and Deuteron

D. T. KING, NATHAN SEEMAN, AND MAURICE M. SHAPIRO
Nucleonics Division, Naval Research Laboratory, Washington, D. C.
(Received September 2, 1953)

OBSERVATIONS of singly charged unstable particles with mass intermediate between those of proton and deuteron have been described by Leighton[1] and by Levi-Setti.[2] Two examples of such heavy particles have been found in this laboratory. One was observed as a moderately fast product of a fundamental nucleon-nucleon collision.[3] The other came to rest, and apparently decayed with the emission of light meson, probably a pion. The mass estimates are in fair agreement, although different methods of mass determinations were appropriate for the two cases: measurements of ionization and multiple scattering for the fast particle, scattering and residual range for the slow one. Both events occurred in Ilford G.5 emulsions exposed in combination with C and Pb absorbers at an atmospheric depth of ~11 g/cm² and geomagnetic latitude 56°N. Table I summarizes the track data.

The track of our first example originates in a "fundamental" collision; i.e., there are no tracks attributable to evaporation particles or to a recoil nucleus. The star comprises a total of 5 tracks, all of grain density less than 2.5 times the "plateau" value, and one of these tracks is very probably due to a charged primary particle. The nature of the generating interaction suggests that the incident particle had at least several Bev of energy. With respect to the forward direction of the assumed primary, the particle of interest was emitted at an angle of ~120° in the laboratory system, and it left the emulsion at a distance of 18.6 mm from its origin. This considerable length of track, and the fact that its grain density lies well above the insensitive region of the ionization minimum, permit the mass determination, $2560 \pm 500 m_e$. For velocity calibration, 34 tracks of protons and pions in the same emulsions, having comparable lengths and grain densities, were measured.

The primary track of our second example exhibits the increase in scattering and ionization characteristic of a charged particle coming to rest. The "constant saggita" method of multiple scattering measurement, which utilizes the range-energy relation for known particles, was applied to this track and to those of 18 calibration tracks of stopped protons. It can be shown that when the scattering cell size s is varied with residual range R according to the relation $s \sim R^{0.385}$, then the mean saggitas (second differences $\bar{D}$ and $\bar{D}_p$, respectively) for a singly charged particle of mass m and for the proton mass m_p are related by $m/m_p = (\bar{D}_p/\bar{D})^{2.37}$. Thus, using the $\bar{D}$ values in Table I, the mass $(2860 \pm 850) m_e$ is obtained. The sensitivity of mass to $\bar{D}$ leads to considerable error; nevertheless this method seems the best available for particles arrested in emulsion. Application of the alternative constant-cell method of scattering yields a mass value $2940 m_e$ which agrees with the one above within experimental error. In arriving at the $\bar{D}$ values, cutoff was applied for large single scatters and correction was made for spurious scattering noise. Omitting the data for the last 800 microns and the last 200 microns of range, respectively, led to values $m = 1.42 m_p$ and $m = 1.70 m_p$. We have provisionally adopted the mean value $(1.56 \pm 0.45) m_p$.

As seen from Table I, the secondary of the slow heavy particle is very probably a pion. It is possible definitely to rule out a proton secondary. A similar example, involving a $\pi^{\pm}$ secondary, has been reported by Peters.[4]

Using the velocity of our meson secondary, we have computed the Q value and primary mass for each of two assumed decay schemes,[5]

$$Y^{\pm} \rightarrow \text{neutron} + \pi^{\pm} + Q, \tag{1}$$

$$Y^{\pm} \rightarrow V_1^0 + \pi^{\pm} + Q'. \tag{2}$$

TABLE I. Summary of track data.

Fast particle	
Track length	18.6 mm
Multiple scattering (600μ cells)	$(0.068° \pm 0.012°)/(100\mu)^{\frac{1}{2}}$
Grain density/plateau density	2.25 ± 0.07
$p\beta$	390 ± 70 Mev/c
Mass (in m_e)	2560 ± 500
Stopped particle	
(a) Primary	
Track length	3.7 mm
Scattering: mean saggita	0.34μ
Mean saggita, protons	0.41μ
Mass (in m_e)	2860 ± 850
(b) Secondary	
Track length	2.2 mm
Multiple scattering	$(0.17° \pm 0.04°)/(100\mu)^{\frac{1}{2}}$
Grain density/plateau density	1.15 ± 0.07
$p\beta$	150 ± 35 Mev/c
Mass (in m_e)	330 ± 90

We find that for (1), $Q=(103_{-15}^{+20})$ Mev and the calculated mass is $2315m_e$; for (2), $Q'=(101_{-15}^{+20})$ Mev and the mass is $2650m_e$.

Support for scheme (2) is provided by recent cloud-chamber observations of cascade V-particle events in which a charged primary disintegrates into a V_1^0 and a light meson,[6,7]

$$V^- \rightarrow V_1^0 + (\pi^- \text{ or } \mu^-) + Q. \tag{3}$$

Evidence thus exists for the emission of a V_1^0 as a neutral decay product of a charged heavy particle, whereas there is no corresponding evidence, so far as we know, for a secondary neutron. It seems reasonable, therefore, as a working hypothesis, to identify our scheme (2) with (3) of the Pasadena group.[7] The respective Q values, 101_{-15}^{+20} Mev (NRL) and 60 ± 15 (Pasadena) are not inconsistent.

As for the *charged* secondary, the cloud-chamber observations do not permit a choice between π and μ. However, under the foregoing hypothesis, we conclude that the charged secondary in the V-particle cascades is more likely to be a pion than a muon.

We are grateful to Mr. B. Stiller and Mr. F. W. O'Dell for their assistance, and to Mrs. N. T. Redfearn for much of the scanning.

[1] R. B. Leighton, Rochester Conference on High Energy Physics, December, 1952 (Interscience Publishers, New York, 1953); York, Leighton, and Bjornerud, Phys. Rev. **90**, 167 (1953).
[2] R. Levi-Setti, Royal Society Conference on Fundamental Particles, London, January, 1953 (unpublished); Bonetti, Levi-Setti, Panetti, and Tomasini, Nuovo cimento **10**, 345 (1953); and International Congress on Cosmic Radiation, Bagnères de Bigorre, July, 1953 (unpublished).
[3] King, Seeman, and Shapiro, International Congress on Cosmic Radiation, Bagnères de Bigarre, July, 1953 (unpublished).
[4] Peters, Lal, and Pal, International Congress on Cosmic Radiation, Bagnères de Bigarre, July, 1953 (unpublished).
[5] We have used the symbol Y to denote the heavy particle.
[6] Armenteros, Barker, Butler, Cachon, and York, Phil. Mag. **43**, 597 (1952).
[7] Anderson, Cowan, Leighton, and van Lint, Bull. Am. Phys. Soc. **28**, No. 5, 17 (1953).

Intermediate Coupling and Nuclear Reactions

A. M. LANE
Department of Mathematical Physics, Birmingham University, Birmingham, England
(Received September 14, 1953)

PREVIOUS discussion of the shell model has centered around such "static" experimental quantities as magnetic moments, spins, etc. It is the purpose of this letter to point out that this discussion can be extended to include the "dynamical" quantities of the type found in nuclear reactions. These quantities include the reduced widths of energy levels for nucleon emission and the matrix elements of electromagnetic transitions. There is a considerable accumulation of such data[1] in light nuclei, most of which has never been seriously used to investigate nuclear structure.

If an extreme mode of coupling is assumed in nuclei (i.e., $L-S$ or $j-j$), then theoretical expressions for these dynamical quantities can be found very simply using the theory of fractional parentage.[2] For instance, consider the reduced width in $L-S$ coupling for a nucleon transition of the type:

compound nucleus λ →residual nucleus p +nucleon;

i.e.,

$$l^n(\alpha TSL, J) \rightarrow l^{n-1}(\alpha_p T_p S_p L_p, J_p) + l; \tag{1}$$

i.e., the emission of one nucleon from a state λ of n equivalent nucleons to leave a residual nucleus of $(n-1)$ equivalent nucleons. The symbols α, T, S, L, J have their usual meaning. The reduced width[3] for this process can be shown to be

$$\gamma_{\lambda s l}^2 = \gamma^2(l) n \langle \alpha]\!] \alpha_p \rangle^2 U^2(L_p S_p s \tfrac{1}{2}, J_p S) U^2(S L_p J l, sL), \tag{2}$$

where the U functions are Recah functions[2] and where $\langle \alpha]\!] \alpha_p \rangle$ is an abbreviated notation for the total fractional parentage coefficient[2] appropriate to the two states of (1). α and α_p, as usual, denote symmetry characters. The symbol s denotes the values of the coupled spins of the nucleon and residual nucleus,[3] and $\gamma^2(l)$ is the reduced width for a single l nucleon in the potential well of the shell model (the value will be $\sim\hbar^2/Ma$, where M is the reduced mass of the nucleon and a is the radius of the well). In $j-j$ coupling, the reduced width for the transition

$$j^n(\alpha TJ) \rightarrow j^{n-1}(\alpha_p T_p J_p) + j \tag{3}$$

emerges as

$$\gamma_{\lambda s l}^2 = \gamma^2(l) n \langle \alpha]\!] \alpha_p \rangle^2 U^2(J_p \tfrac{1}{2} J l, sj). \tag{4}$$

Formulas can be derived similarly for the values of the matrix elements of electromagnetic transitions in $L-S$ and $j-j$ coupling.

The formulas both for reduced widths and electromagnetic matrix elements have been extensively compared with experimental values in light nuclei. It is found that neither extreme coupling mode can fit the data, but there is strong suggestion that intermediate coupling can do so.

In order to test this conclusion, the mirror nuclei C^{13} and N^{13} have been examined in detail. The properties of these nuclei are well known[1] and altogether there are eleven useful independent data on the first four levels of these nuclei:

(1) the binding energies of the first-excited states in the two nuclei,

(2) the slow-neutron scattering length for C^{12} and the "effective range" of slow-neutron scattering,[4]

(3) the reduced widths of the first- and third-excited states in N^{13} (these are essentially "single-particle" levels, the first $(\frac{1}{2}+)$ being simply a $2s$ nucleon added to the C^{12} "core," and the third $(5/2+)$ being a $1d$ nucleon added similarly),

(4) the $E1$ and $M1$ transition widths from the first- and second-excited states of N^{13} to the ground state,

(5) the reduced widths of the ground- and second-excited states [the latter is observed in N^{13} from proton scattering in C^{12}, and the former is obtained for C^{13} from the yield of the (d, p) stripping reaction on C^{12}],

(6) the magnetic moment of C^{13}.

Of these data, the predicted values of (1), (2), and (3) are not sensitive to the mode of coupling in the shell model, but only to the potential well of the model. These six independent data can all be fitted with a certain choice of square well (depth 35 Mev, radius 4.05×10^{-13} cm). The predictions for the other data (4), (5), and (6) are all sensitive to the mode of coupling in the shell model, some of them being extremely so. Analysis shows that all the five data of (4), (5), and (6) are fitted extremely well by an intermediate coupling model with a value of the usual parameter a/K of about 4.5 (a being the strength of the spin-orbit coupling and K the exchange integral).[5]

Consequently every observable feature of the first four states of C^{13} and N^{13} is consistent with an intermediate coupling model of the nucleus. This appears to give by far the most consistent support for the model found hitherto. A complete account of this work will appear in a series of articles in the *Proceedings of the Physical Society (London)*,[6] and as part of a review article on nuclear reactions.[7]

[1] F. Azjenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).
[2] G. Racah, Phys. Rev. **63**, 367 (1943); H. A. Jahn and H. van Wieringen, Proc. Roy. Soc. (London) **A214**, 502 (1951); B. H. Flowers and A. R. Edmonds, Proc. Roy. Soc. (London) **A214**, 515 (1952).
[3] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[4] R. G. Thomas, Phys. Rev. **88**, 1109 (1952).
[5] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).
[6] A. M. Lane, Proc. Phys. Soc. (London) (to be published); A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) (to be published).
[7] R. G. Thomas and A. M. Lane, Revs. Modern Phys. (to be published).

Periodic Ellipse of the Strong-Focusing Equation*

B. C. CARLSON
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received September 11, 1953)

THE equation of free betatron oscillations in the strong-focusing synchrotron is a Hill's equation of the form[1]

$$d^2x/d\theta^2 + n(\theta)x = 0, \tag{1}$$

where $n(\theta)$ can be taken (by including a scale factor in θ) to alternate between 1 in converging sectors and -1 in diverging