absorption simply $\hbar\omega$, since there is but a single level. These results are in disagreement with those of Bethe and Levinger only because they neglect correlations in the gound state wave function of the nucleus. Such correlations are of great importance in our model, and seem to be necessary in order to explain the experimentally observed variation of the resonance energy with atomic number.

Details will be published in a forthcoming paper.

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Nuclear Radii*

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We would like to point out that nuclear radii as predicted from isotope shift and high-energy electron scattering are in excellent agreement. Recent experiments on μ-mesonic x-rays corroborate these results.[1,2] These radii are considerably smaller than those usually quoted. However, they are in excellent agreement with those obtained from the most recent semi-empirical mass formula and may be reconciled with radii as obtained from the Coulomb energy difference in light nuclei.

Electron scattering experiments have been performed by Lyman, Hanson, and Scott[3] with electrons at 15.7 Mev, by Hammer, Raka, and Pidd[4] at 33 and 43 Mev, and by Hofstadter *et al.*[5] at 116 Mev, for a variety of elements. We shall analyze the first set of experiments. The second set gives similar results as far as nuclear radii are concerned. The third set does not show Ramsauer minima, again indicating a small radius. For the lower-energy experiments, according to theory[6] only one phase shift, η_0' is required. We have, therefore, evaluated the phase shift required to match the experimental data at each scattering. The resulting values should be constant. There are, however, a number of difficulties. For very light elements, and for small angles for all elements, the effect of nuclear size is small and would require experiments of great accuracy. For this reason the aluminum data are not useful for the present purpose. For large angles and for heavy elements the scattering is very small, again making the experiments difficult. Moreover, the theoretical uncertainties are greatest at large angles. The most consistent results are obtained for copper and silver, less consistent results for gold. (See Table I.)

Table I. Values of the phase shift η_0'.

ϑ	Copper	Silver	Gold
90°	0.0054	0.0213	0.141
120°	0.0052	0.0228	0.112
150°	0.005	0.0213	0.120

Employing the theoretical results,[6,7] we find that for a homogeneous charge distribution the copper and silver radii are $1.0\times10^{-13}A^{\frac{1}{3}}$ cm and $1.1\times10^{-13}A^{\frac{1}{3}}$ cm, respectively. The nuclear radius for gold is not well determined; but if an average phase shift of $\eta_0'=0.120$ is taken, the nuclear radius is $1.2\times10^{-13}A^{\frac{1}{3}}$ cm. These results are in agreement with those of Raka *et al.*, who obtain a radius of $(1.1\pm0.075)\times10^{-13}A^{\frac{1}{3}}$ cm for Sn and $(1.03)\times10^{-13}A^{\frac{1}{3}}$ cm for W.

The isotope-shift data in Fig. 1 has been summarized by Brix and Kopfermann.[8] We have replotted the data taking the nuclear radius as $1.1\times10^{-13}A^{\frac{1}{3}}$ cm. The agreement with the data is very much better than that obtained with $1.5\times10^{-13}A^{\frac{1}{3}}$ cm. It clearly would be of interest to do electron scattering experiments with the rare earths Ce, Sm, Eu, as well as with Rb, Xe, Ba, which show large deviations from the average line. It should be noted that the tacit assumption is made here that the isotope shift is a pure volume effect.

It is of interest to show that these radii may not be reconciled with the larger Coulomb energy radii of $1.47\times10^{-13}A^{\frac{1}{3}}$ cm commonly quoted,[3] for we shall show that no positive definite charge distribution exists which will give a smaller nuclear radius for scattering and a larger one for the nuclear Coulomb energy. The effective nuclear Coulomb radius R_N is defined by

$$\frac{1}{R_N}=\frac{5}{6Z^2e^2}\int\int\frac{\rho(\mathbf{r})\rho(\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|}\frac{d\mathbf{r}}{4\pi}\frac{d\mathbf{r}_0}{4\pi}. \tag{1}$$

The scattering and isotope shift depend primarily upon the volume integral of the perturbing potential,[6]

$$\int\frac{\rho(\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|}d\mathbf{r}_0-\frac{Ze}{r}.$$

It may be shown that this difference is proportional to

$$\int\rho(\mathbf{r})r^2d\mathbf{r}.$$

Hence, the scattering radius R_s is defined by

$$R_s^2=\frac{5}{3Ze}\int\rho(\mathbf{r})r^2(d\mathbf{r}/4\pi). \tag{2}$$

We now ask for what charge distribution the ratio R_s/R_N is stationary:

$$\frac{\delta(R_s/R_N)}{\delta\rho}=0,\quad \delta\int\rho d\mathbf{r}=0;$$

or

$$\frac{1}{R_s}\frac{\delta R_s}{\delta\rho}=\frac{1}{R_N}\frac{\delta R_N}{\delta\rho};\quad \delta\int\rho d\mathbf{r}=0. \tag{3}$$

From Eq. (2) we find

$$\frac{\delta R_s}{\delta\rho}=\frac{5}{6Ze}\frac{r^2}{R_s}, \tag{4}$$

while from Eq. (1) it follows that

$$\frac{\delta R_N}{\delta\rho}=-\frac{5}{3}\frac{R_N^2}{Z^2e^2}V, \tag{5}$$

where

$$V(\mathbf{r})=\int\frac{\rho(\mathbf{r}_0)}{|\mathbf{r}-\mathbf{r}_0|}d\mathbf{r}_0.$$

Inserting (4) and (5) into (3), we find that $V(\mathbf{r})\sim A-Br^2$, where A and B are constants. For positive definite charge density the constants correspond to a homogeneous charge distribution. We determine that the homogeneous charge distribution corresponds to a minimum for the ratio (R_s/R_N) by evaluating the ratio for an actual example. It is, of course, possible to obtain $R_s<R_N$ by relaxing the positive definite charge density condition and thus permitting regions of negative charge within the nucleus, as might be possible in a meson theory of the nucleus.

We turn now to other evidence for nuclear radii. Here it is interesting to note that the most recent determination of the semi-empirical mass formula by Green and Engler[9] give the Coulomb energy term as $0.750(Z^2/A^{\frac{1}{3}})$mMU. This corresponds to the relation $R_N=(1.23\times10^{-13})A^{\frac{1}{3}}$ cm, in agreement with our determination. The second source of evidence is obtained from mirror nuclei. These are light and, as Wigner has pointed out, correlation effects are important. In particular, the exchange Coulomb energy has the effect of reducing the Coulomb energy and therefore increasing the effective radius. Both Elton[10] and Cooper and Henley[1] have pointed out that the nucleon involved in the β transition

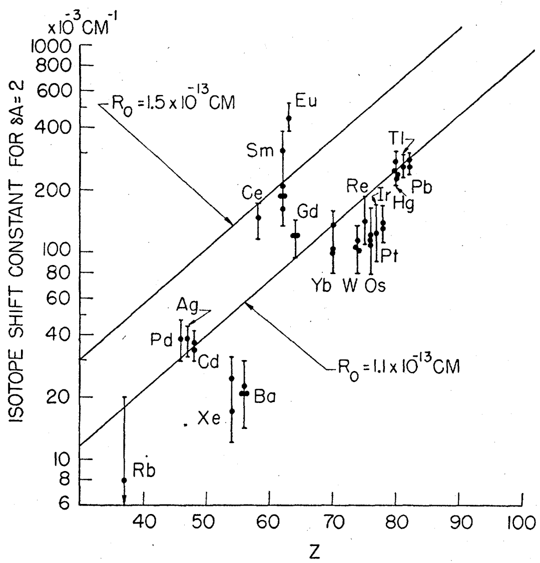

FIG. 1. Isotope shift constant as a function of atomic number.

between mirror nuclei is near the nuclear surface and will have a smaller than average Coulomb interaction. Cooper and Henley have tested these suggestions by a model and have shown it to be plausible that both of these effects are sufficient to reconcile the values of R_s and R_N. Finally, we have the evidence from nuclear reactions. It is perhaps not too surprising that these radii are large because the strong absorption properties of nuclear matter would tend to weight the surface regions more heavily. Actually, this may be seen by defining a radius in terms of an effective range theory.[11] Of course it no longer is possible to take the nucleus as having a uniform distribution, but rather a tail of some extension must be assumed.

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Charged Particles of Mass Intermediate between Proton and Deuteron

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OBSERVATIONS of singly charged unstable particles with mass intermediate between those of proton and deuteron have been described by Leighton[1] and by Levi-Setti.[2] Two examples of such heavy particles have been found in this laboratory. One was observed as a moderately fast product of a fundamental nucleon-nucleon collision.[3] The other came to rest, and apparently decayed with the emission of light meson, probably a pion. The mass estimates are in fair agreement, although different methods of mass determinations were appropriate for the two cases: measurements of ionization and multiple scattering for the fast particle, scattering and residual range for the slow one. Both events occurred in Ilford G.5 emulsions exposed in combination with C and Pb absorbers at an atmospheric depth of $\sim$11 g/cm^2 and geomagnetic latitude 56°N. Table I summarizes the track data.

The track of our first example originates in a "fundamental" collision; i.e., there are no tracks attributable to evaporation particles or to a recoil nucleus. The star comprises a total of 5 tracks, all of grain density less than 2.5 times the "plateau" value, and one of these tracks is very probably due to a charged primary particle. The nature of the generating interaction suggests that the incident particle had at least several Bev of energy. With respect to the forward direction of the assumed primary, the particle of interest was emitted at an angle of $\sim$120° in the laboratory system, and it left the emulsion at a distance of 18.6 mm from its origin. This considerable length of track, and the fact that its grain density lies well above the insensitive region of the ionization minimum, permit the mass determination, $2560\pm500m_e$. For velocity calibration, 34 tracks of protons and pions in the same emulsions, having comparable lengths and grain densities, were measured.

The primary track of our second example exhibits the increase in scattering and ionization characteristic of a charged particle coming to rest. The "constant saggita" method of multiple scattering measurement, which utilizes the range-energy relation for known particles, was applied to this track and to those of 18 calibration tracks of stopped protons. It can be shown that when the scattering cell size s is varied with residual range R according to the relation $s\sim R^{0.385}$, then the mean saggitas (second differences $\bar{D}$ and $\bar{D}_p$, respectively) for a singly charged particle of mass m and for the proton mass m_p are related by $m/m_p=(\bar{D}_p/\bar{D})^{2.37}$. Thus, using the $\bar{D}$ values in Table I, the mass $(2860\pm850)m_e$ is obtained. The sensitivity of mass to $\bar{D}$ leads to considerable error; nevertheless this method seems the best available for particles arrested in emulsion. Application of the alternative constant-cell method of scattering yields a mass value $2940m_e$ which agrees with the one above within experimental error. In arriving at the $\bar{D}$ values, cutoff was applied for large single scatters and correction was made for spurious scattering noise. Omitting the data for the last 800 microns and the last 200 microns of range, respectively, led to values $m=1.42m_p$ and $m=1.70m_p$. We have provisionally adopted the mean value $(1.56\pm0.45)m_p$.

As seen from Table I, the secondary of the slow heavy particle is very probably a pion. It is possible definitely to rule out a proton secondary. A similar example, involving a $\pi^{\pm}$ secondary, has been reported by Peters.[4]

Using the velocity of our meson secondary, we have computed the Q value and primary mass for each of two assumed decay schemes,[5]

$$Y^{\pm}\rightarrow \text{neutron}+\pi^{\pm}+Q, \tag{1}$$

$$Y^{\pm}\rightarrow V_1{}^0+\pi^{\pm}+Q'. \tag{2}$$

TABLE I. Summary of track data.

Fast particle	
Track length	18.6 mm
Multiple scattering (600μ cells)	$(0.068°\pm0.012°)/(100\mu)^{\frac{1}{2}}$
Grain density/plateau density	2.25 ± 0.07
$p\beta$	390 ± 70 Mev/c
Mass (in m_e)	2560 ± 500
Stopped particle	
(a) Primary	
Track length	3.7 mm
Scattering: mean saggita	0.34μ
Mean saggita, protons	0.41μ
Mass (in m_e)	2860 ± 850
(b) Secondary	
Track length	2.2 mm
Multiple scattering	$(0.17°\pm0.04°)/(100\mu)^{\frac{1}{2}}$
Grain density/plateau density	1.15 ± 0.07
$p\beta$	150 ± 35 Mev/c
Mass (in m_e)	330 ± 90