Ionization Loss at Relativistic Velocities in Nuclear Emulsion

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The variation of grain density in the tracks of singly-charged relativistic particles traversing a nuclear emulsion has been investigated as a function of velocity in Ilford G.5 plates exposed to the cosmic radiation at 100 000 feet. Multiple scattering measurements and "blob counts" were made on long tracks of electrons, mesons, and protons with energies γ up to 3400 rest mass units. The blob density G_{pl} at the Fermi plateau of ionization was found to be 1.14 ± 0.03 times $G_{\min}$, in agreement with the result previously obtained by another method. Protons and electrons show the same value of G_{pl}. The data are compared with the theories of Halpern-Hall and of Sternheimer for AgBr, using the ionization potentials of Bakker and Segrè. Since only grains *along* the track were counted, the calculated energy loss is restricted to energy transfers less than an upper limit T_0. Calculations for two assumed values of T_0, 2 kev and 5 kev, fit the data equally well, yielding ratios $G_{\mathrm{pl}}/G_{\min}$ of 1.15 and 1.14, respectively. The data are also consistent with the slow rate of rise from minimum ($\gamma\sim4$) to plateau ($\gamma>100$), which is predicted by the theory.

INTRODUCTION

In our earlier work on the rate of ionization loss I of very fast charged particles ($\beta>0.95$) in nuclear emulsions,[1] we reported a relativistic rise of $\sim12_{-3}^{+4}$ percent above the ionization minimum to the Fermi "plateau." This result was obtained by comparing the Ag grain densities g of two groups of particle tracks, the thin shower tracks emerging from nuclear explosions and the "primary" tracks of high-energy stars with associated shower multiplicities $n_s\geq5$.

The former group originates primarily from pions and, from their known energy spectrum, their grain-density histogram was expected to have a peak at $g_{\min}$. Similarly, the latter group is attributable mainly to protons so energetic that their histogram should display a peak at or near the plateau value g_{pl}. The theoretically expected peaks were observed, and an experimental value of the ratio $I_{\mathrm{pl}}/I_{\min}$ for AgBr was deduced therefrom. Also, preliminary investigation of energetic electrons (total energy $\gamma>300$ rest masses, as determined from their multiple scattering) indicated that g_{pl} for electrons lies 16 ± 3 percent above $g_{\min}$ for mesons.

NEW MEASUREMENTS

We have improved and extended our previous work in the following ways:

(1) Measurements of multiple Coulomb scattering have been made on long tracks of many particles with velocities in the regions of minimum and plateau ionization. These measurements were especially desirable in view of the considerable uncertainty inherent in the estimates of the energies of the shower-producing primaries from their associated multiplicities.

(2) Measurements have also been made on tracks of particles with intermediate relativistic velocities in the region where the ionization increases appreciably with energy. It is important to provide experimental evidence on the variation of grain density in this region. Such evidence would show whether any ionization theory correctly describes this variation in nuclear emulsions. It is also essential for mass estimation (and hence identification) of particles in the energy interval $5<\gamma<100$.

(3) An experimental comparison between electrons and heavier particles in regards to their "restricted ionization loss" in AgBr grains at energies $\gamma>15$ rest masses, is possible from our new data. Contrary to the difference in *total* ionization between electrons on the one hand and mesons or protons on the other, there is no reason to expect a difference between the *grain densities* of these two groups of particles as a function of velocity.

(4) The tracks in our earlier study had been located on several plates of the same batch, exposed together and processed together in the same solutions. Nevertheless, even under these well-controlled conditions we found variations of a few percent in $g_{\min}$ among the several plates.[2] Therefore, we made the control more rigorous still by gathering most of the data presented here from a single plate 6×3 inches. The Ilford G.5 emulsion, 400 microns thick, had been exposed in the stratosphere for ~8 hours at an atmospheric depth of 11 g/cm². We took the precaution of looking for differences in $g_{\min}$ between various parts of this plate. Such differences, if any, were found to be well within experimental error. The plate selected was well suited for the work at hand; it had low background, high uniformity of grain density with depth, and a low level of distortion.

(5) Grain densities were replaced by "blob" densities. Even in very thin tracks, such as those with which we are dealing here, the grains of silver sometimes coalesce or overlap, so that two of them may be mistaken for one. This element of subjectivity can be removed by counting blobs instead of attempting to resolve grains. Thus we find that experienced observers agree within statistical errors in their blob

[1] M. Shapiro and B. Stiller, Phys. Rev. 87, 682 (1952).

[2] It is, of course, possible to normalize the level of grain density in one plate to that in another. This was done for the data from another plate in the same batch; see Fig. 2.

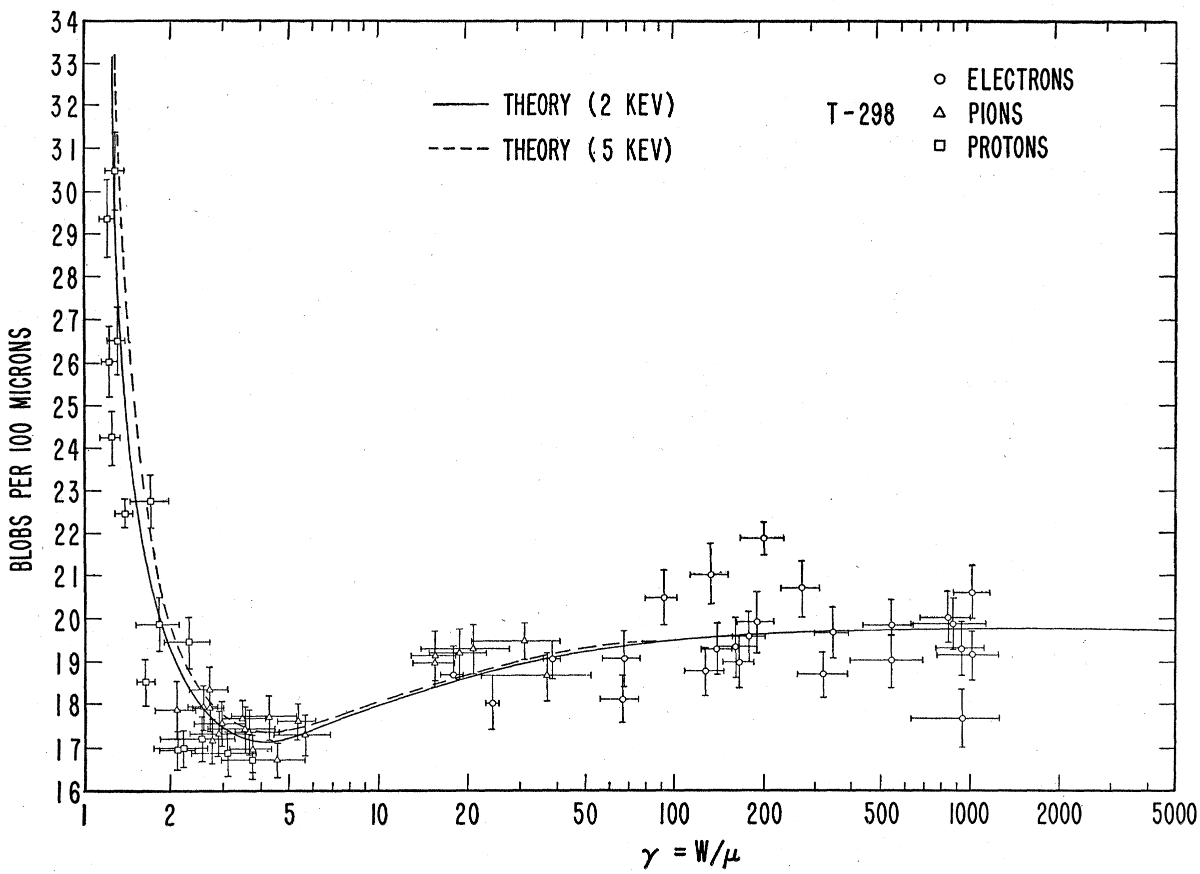

FIG. 1. Blob density is plotted as a function of γ, the total energy of the moving particle, deduced from its multiple Coulomb scattering. Measurements on 55 long tracks found in one 3 in.×6 in., Ilford G.5, 400-μ plate are shown.

counts, although not necessarily in their grain counts. For our thin tracks, a blob is simply a grain in most cases; in fact, the blob density G is only ~7 percent below the grain density g, and we find no variation in the ratio G/g at these low levels ($G<1.25\,G_{\min}$).

RESULTS

Figure 1 shows the blob density as a function of γ for tracks gathered from a single plate, No. T-298. Since the abscissa is a function of velocity only, it is instructive to plot together the various particles of unit charge. It will be noticed that, at energies $\gamma>100$, only electron data appear. In the interval $10<\gamma<100$, both electron and meson tracks appear. Finally, at lower velocities, mesons and protons, but no electrons, are represented.

Ideally, one would prefer to measure the rate of ionization loss over the whole range of velocities for a single type of particle. In practice, such procedure proves to be extremely difficult because of the precision required in this experiment, where the variation with which we are dealing is so small. Electrons in the region of the theoretical minimum have energies <3 Mev and mean scattering angles $>7°/(100\mu)^{1/2}$. Their strong scattering imposes several obstacles to accurate determinations of grain or blob densities: (a) such electrons seldom stay in the emulsion over a sufficient path length; (b) when they do, the inclusion of spurious background grains in a highly bent track is more probable than in a straight track; (c) even the measurement of true length of track is subject to appreciable error; and (d) there is danger of shifting unwittingly to a neighboring electron track which crosses the one in question. For these reasons, measurements of G on electron tracks have been made with adequate precision only at energies $\gamma>10$. In the vicinity of the ionization minimum ($2<\gamma<6$), it has been found exceedingly difficult to use electrons.[3]

Conversely, if one wants energy determinations from scattering (as well as G) at energies $\gamma>70$, one must perforce use electrons. Protons of $\gamma>10$ have mean angles $\bar{\alpha}$ of multiple scattering which are much too

[3] Recent investigation of electron tracks in emulsions established the existence of an ionization plateau at high energies. See D. R. Corson and M. R. Keck, Phys. Rev. **79**, 209 (1950); I. B. McDiarmid, Phys. Rev. **84**, 851 (1951); A. H. Morrish, Phil. Mag. **43**, 533 (1952). However, because their investigations were confined to electrons, their measurements were not extended down to the minimum of ionization and hence could not yield the ratio $I_{\mathrm{pl}}/I_{\min}$.

small and, hence, too close to the noise level to permit useful measurements. For pions the corresponding limit is $\gamma \sim 70$, whereas electron tracks lend themselves to reliable determinations of $\bar{\alpha}$ and G at energies up to several thousand rest masses. On the other hand, in the vicinity of $G_{\min}$, both $\bar{\alpha}$ and G can be measured for protons, and especially well for pions.

Fortunately, there also is an intermediate region $10<\gamma<40$ in which $\bar{\alpha}$ and G can be determined for both electrons and mesons. Thus, a direct comparison can be made between the two groups, and a useful link is provided between the regions of $G_{\min}$ and G_{pl} where, for our present purpose, only one or the other type of particle can be employed. Moreover, for both electrons and mesons, the rate of variation of I can be compared with theory in this intermediate region where experimental delineation of the ionization curve is particularly important.

The electron tracks were selected phenomenologically by their occurrence in electron pairs or in tridents. Long tracks (averaging $\sim$8000 microns) of mesons and protons occurring in the same 6×3 inch emulsion were obtained from shower stars. The multiple scattering of these tracks was measured by the coordinate method of Fowler[4] on a special Bausch and Lomb microscope stage which has a "noise" <0.15 micron. Grain and blob counts (>1000 blobs/track) were repeated by three observers for each track. The errors in both G and γ shown in Figs. 1 and 2 are standard errors. The curves shown in these figures are theoretical curves, which were calculated as described in the next section.

To arrive at a value of $G_{\text{pl}}/G_{\min}$ which might be compared with the theoretical $I_{\text{pl}}/I_{\min}$, we adopted the least-squares value $G_{\text{pl}}=19.70\pm0.40$ blobs/100μ for the electron tracks with $\gamma>100$. The least-squares slope is $0.15\pm5°$, i.e., the best-fitting line in this velocity region is a horizontal one. For $G_{\min}$ we averaged the G values of the meson tracks in the interval $3<\gamma<6$, which yielded the value 17.24 ± 0.35 blobs/100μ. (The theoretical minimum, in the theory described below, occurs at $\gamma=4.2$.) In this way we obtained the result

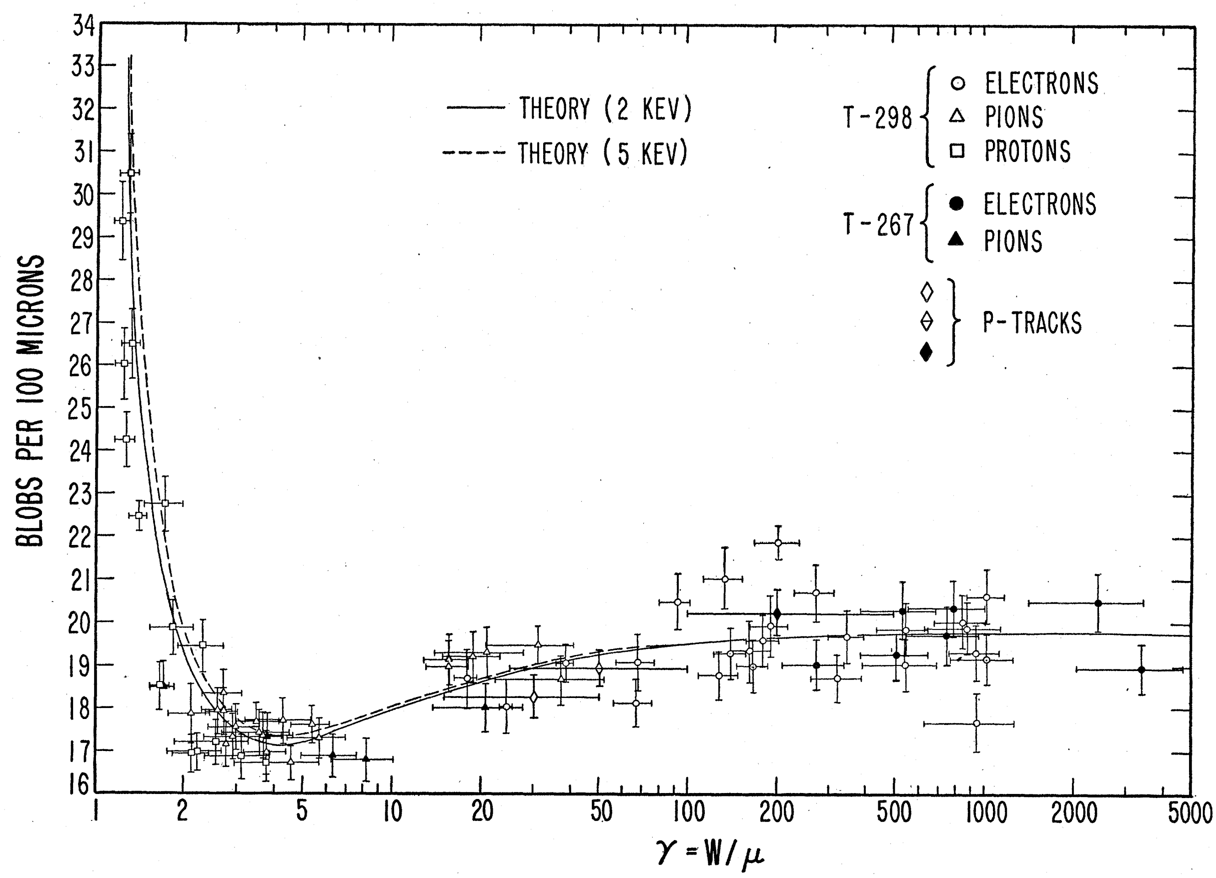

FIG. 2. Blob density is plotted as a function of γ, the total energy of the moving particle. Measurements on 85 tracks found in two 3 in.× 6 in., Ilford G.5, 400μ plates, of the same batch, exposure and processing are shown. The "p tracks" are tracks of shower-generating particles. The open diamond gives the average blob density of 4 tracks with $n_s=5$; the barred diamond shows the average of 6 tracks with $n_s=7$ or 8; and the black diamond, the average of 4 tracks with $n_s=9$ or 10.

[4] P. H. Fowler, Phil. Mag. 41, 169 (1950).

$G_{pl}/G_{min} = 1.143 \pm 0.03$. Since there is evidence that G is proportional to I at low-grain densities[4,5] this value applies also to the ratio I_{pl}/I_{min} in AgBr, where I is defined more precisely below.

Additional tracks were measured in T-267, a plate of the same batch and having the same history as T-298. G_{pl} was found to be 8 percent higher in the former than in the latter, but the ratio G_{pl}/G_{min} was unchanged. These data were normalized at plateau to those of Plate T-298 and the results for both plates are shown in Fig. 2.

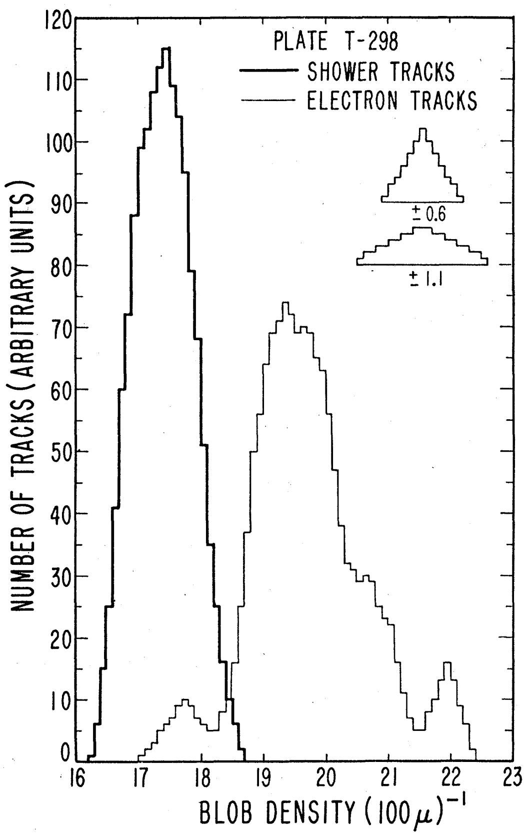

FIG. 3. Frequency of tracks as a function of blob density. Only data shown in Fig. 1 are plotted. Shower tracks with densities <18 blobs/100μ are included. The electron peak is 12.5 percent above the shower-track peak. Tracks are plotted as triangles of equal area but with a base proportional to the standard error in the blob density.

The three diamond-shaped points in Fig. 2 represent the tracks of singly-charged shower-generating particles which gave rise to shower multiplicities $n_s \geq 5$. We call these p tracks, where p denotes "primary," although it seems safe to assume that all or most of these are protons. These three points differ from the others in two respects: (a) each represents several (4 to 6) tracks rather than only one, and (b) no $\bar{\alpha}$ measurement could be made on these in view of their small $\bar{\alpha}$, as explained above. The lower limit $n_s = 5$ was chosen so as to increase the likelihood that the shower primaries had very high energies (tens of Bev or higher).

In order to see how closely G for protons agrees with G for electrons at or near the plateau, we have estimated the energy of the p particles in each n_s group from their multiplicities. This can only lead to a very rough estimate of the energy, since for either plural or multiple theories of meson production one can expect a considerable spread in energy for a given n_s. Moreover, neither of these theories alone seems likely to account for the generation of meson showers in the variety of nuclei present in emulsions. It is reasonable to suppose that, especially in the heavier nuclei such as Ag or Br, meson production is in general a pluro-multiple process. Therefore, to arrive at energy estimates, we have taken into account both Fermi's theory of multiple production and the Heitler-Janossy theory of plural production. Our assignments of energy ($\gamma = 30$ for $n_s = 5$; $\gamma = 50$ for $n_s = 6, 7$; and $\gamma = 200$ for $n_s = 9, 10$) represent values considerably smaller than the mean-value predictions of multiple theory, and somewhat larger than those of plural theory. The crudeness of these energy estimates is indicated by the very wide limits of error for these three points shown in Fig. 2. Thus, for the point at $\gamma = 200$, the limits are $100 < \gamma < 500$. Fortunately, the variation of I at these high velocities is so insensitive a function of the energy that it is useful to include these points despite the large uncertainty in energy. The G values, of course, must be determined rather precisely, and this was possible. It will be seen that the p track data agree satisfactorily with the electron and meson data.

If one wishes to evaluate the ratio G_{pl}/G_{min} alone, this can be done more quickly, without multiple scattering measurements, by the histogram method outlined in the Introduction, and described more fully in reference 1. In order to compare the present data directly with our earlier results, we have plotted in Fig. 3 the track frequency as a function of blob density for two groups of tracks from plate T-298, those around the minimum and those near the plateau. The former group consisted of star-shower tracks having less than 18 blobs per 100μ. The plateau group consisted exclusively of the fast electron tracks whose selection has been described above. From the locations of the peaks in this histogram, read at the centers of the respective half-maxima, we estimate that G_{pl} lies 12.5 percent above G_{min}.

A plate H-146, from another batch, having a completely different history, was scanned for similar shower-star and electron tracks and the data obtained were also plotted on a histogram, shown in Fig. 4. From this diagram, we estimate an increase of 13 percent between G_{min} and G_{pl}. Since G_{pl} in Fig. 4 is 8 percent higher than G_{pl} in Fig. 3, and since the height of G_{pl} above G_{min} is the same in the two plates, our data indi-

[5] Fowler's evidence (reference 4) is actually for g rather than G. However our results show that G is proportional to g for these thin tracks.

cate that the magnitude of the relativistic rise in ionization loss as measured by blob densities is not sensitive to the intensity of development, which largely determines the absolute value of G_{pl}.

It is noteworthy that, although the G_{pl} peak in Figs. 3 and 4 is obtained from electron tracks, the value of G_{pl}/G_{min} is the same as that in our earlier determination, which employed "*p* tracks" (mainly protons) for G_{pl}. This shows that, within our experimental error, plateau protons and plateau electrons produce the same blob densities.

The values of the relativistic rise deduced from Figs. 1 to 4 as well as the histograms in reference **1** agree within experimental error. This suggests that the faster histogram method, which omits scattering measurements, is adequate for some purposes, though it gives no information about the *rate* of rise of G between minimum and plateau. As to the magnitude of the rise, we attach more weight to the value 14±3 percent than to those deduced from the histograms, since the former is based on tracks for each of which the energy was measured. Thus the blob density, at minimum, is probably fixed more precisely.

COMPARISON WITH IONIZATION THEORY

The ionization theory of Bethe-Bloch-Williams, which predicted an indefinitely continued relativistic increase in I with velocity, was corrected by Fermi[6] to take account of the polarization of the medium.[7] He showed that the rate of ionization loss should saturate at sufficiently high velocities. Fermi's treatment, which employed a single dispersion frequency for the electrons in the medium, was modified by Wick,[8] Halpern and Hall,[9] Bohr,[10] and Sternheimer[11] who showed that it is necessary to construct a multifrequency theory.

We have calculated theoretical ionization curves for AgBr according to the Halpern-Hall and Sternheimer theories using the constants based on the recent experiments of Bakker and Segrè[12] on ionization potentials. For comparison with our experiment it is necessary to calculate not the *total* rate of ionization loss usually discussed in the theories but the limited rate of loss along a track, measured by its Ag grain density, which arises only from energy transfers to electrons up to a few kev.[13] Larger transfers give rise to delta rays consisting of 2 or more Ag grains and the grains lying off the track are ignored in ordinary grain counting. Accordingly, we computed the restricted rate of loss I as a function of γ, using 5 kev[14] for the "limiting effective energy transfer T_0" to the electrons of the medium.

The theoretical expression which we employed for the rate of energy loss is the one used by both Halpern-Hall and Sternheimer, *viz.*[15]

$$I \equiv -\frac{1}{\rho}\frac{dE}{dx} = \frac{2\pi ne^4}{\rho mv^2}\left[\ln\frac{mv^2T_0\gamma^2}{w^2} + \frac{1}{\gamma^2} - \delta\right].$$

Here I is the rate of loss per g cm^{-2}; E is the energy of the ionizing particle; ρ is the density of AgBr; n is the number of electrons per cm^3; m is the electron mass; $v=\beta c$ is the velocity of the particle; the upper limit T_0 (defined above) replaces T, the absolute maximum energy transfer; $\gamma=(1-\beta^2)^{-1/2}$; w is the average ionization potential of AgBr; and δ is the "density-effect correction," which is a function of the particle's velocity.

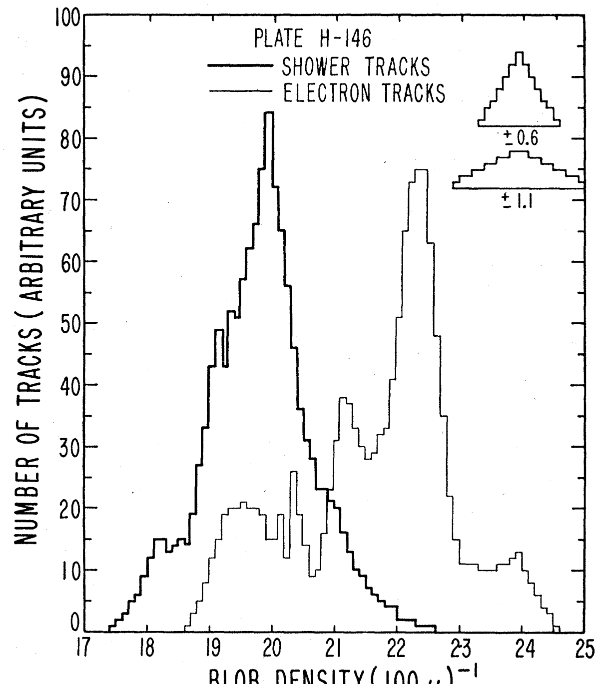

FIG. 4. Frequency of tracks as a function of blob density. This plate is of a different batch, exposure and processing than T-298 and T-267. The peak for electrons is 13 percent above the shower-track peak.

[6] E. Fermi, Phys. Rev. **56**, 1242 (1939); **57**, 485 (1940).

[7] The diminution of ionization loss by the shielding effect of electron displacement was suggested by W. F. G. Swann, J. Franklin Inst. **226**, 598 (1938).

[8] G. C. Wick, Ricerca sci. **11**, 274 (1940); **12**, 858 (1941); Nuovo cimento **1**, 302 (1943).

[9] O. Halpern and H. Hall, Phys. Rev. **57**, 459 (1940); **73**, 477 (1948).

[10] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **24**, No. 19 (1948).

[11] R. M. Sternheimer, thesis, University of Chicago, 1946 (unpublished); Phys. Rev. **88**, 851 (1952). We used the constants for AgBr rather than those for emulsion, since we are concerned with the rate of ionization loss manifested in the AgBr grains which were rendered developable.

[12] C. J. Bakker and E. Segrè, Phys. Rev. **81**, 489 (1951).

[13] H. Messel and D. M. Ritson [Phil. Mag. **41**, 169 (1950)] first called attention to the importance of this distinction.

[14] The range of a 5-kev electron in AgBr is approximately the same as the radius of an average undeveloped grain of AgBr in a G.5 emulsion.

[15] See Eq. (11) of R. M. Sternheimer, Phys. Rev. **88**, 851 (1952).

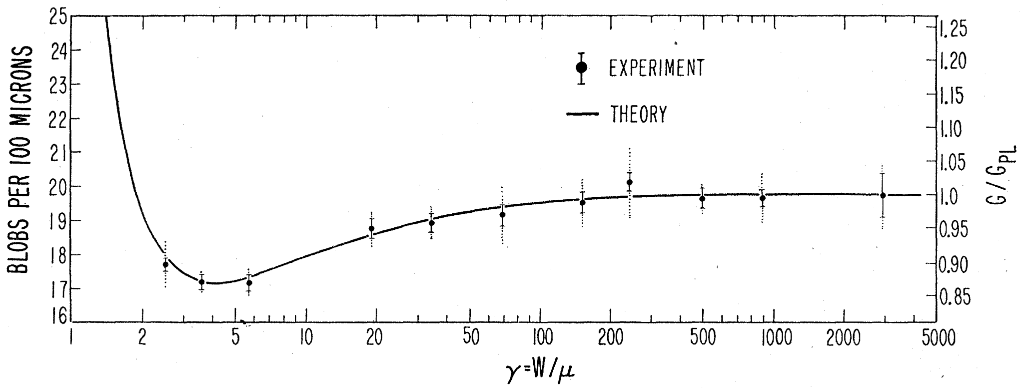

FIG. 5. Mean blob density $\bar{G}$, as given in Table I, plotted as a function of $\bar{\gamma}$. Tracks with $\gamma<2$ are omitted. The solid error line is based on the number of blobs contributing to the point; the dotted one represents the standard deviation of the group from their mean.

For computation we have used the convenient form[16]

$$I=\frac{A}{\beta^2}\left[B+\ln T_0+\frac{1}{\gamma^2}-\Delta\right],$$

where $A=2\pi ne^4/mc^2\rho$, $B=\ln[mc^2(10^6ev)/w^2]$,

$$\Delta=\delta-2\ln\beta\gamma,$$

and T_0 is in Mev. For AgBr, Sternheimer[17] gives $A=0.0668$ Mev/g cm^{-2} and $B=15.1$. (This corresponds to an average ionization potential $w=376$ ev.) Δ was computed according to his Eq. (10) and Table II.

The dashed curve in Figs. 1 and 2 shows the results, normalized to the experimental least-squares plateau value at a single point, $\gamma=300$. We have also computed I for another value of T_0, 2 kev. The results of this calculation are shown in the solid curve which lies slightly below the dashed one. It is evident that I is not sensitive to the choice of T_0, and that either of these theoretical curves fits our data satisfactorily. The calculated ratio $I_{\rm pl}/I_{\rm min}$ for the solid curve is 1.152, that for the dashed curve is 1.137. These may be compared with our experimental value, 1.143 ± 0.03. Our data as to the rate of rise toward the plateau support the theoretical prediction of a slow rise followed by saturation in the region $\gamma>100$.

The multiplicity of points and the overlapping of error lines in Figs. 1 and 2 make it difficult to use those diagrams in comparing theory with experiment. We have, therefore, constructed Fig. 5 in which the same data are grouped into energy intervals each containing about the same number of tracks. Table I gives the mean blob density, mean γ, and the total number of blobs counted for each group of tracks represented by a point in Fig. 5. Two types of standard error are shown for each point in this figure: the solid error line is based on the number of blobs contributing to the point; the dotted one represents the standard deviation of the group from their mean. The theoretical curve shown in the figure is the same as the solid curve in Figs. 1 and 2, i.e., it was calculated for $T_0=2$ kev. The tracks with $\gamma<2$ are omitted from this figure.

While our results show excellent agreement with the theories of Halpern-Hall and Sternheimer, they conflict with the theory of Huybrechts and Schönberg,[18] which departs more radically from Fermi's treatment. These authors predict an increase in I from minimum to plateau in AgBr of 3.8 percent, and they suggest a method whereby this theoretical value could be increased to 6 percent. Neither value seems capable of accounting for our results.[19]

As can be seen in Fig. 2, our G values show no significant differences between electrons and heavier particles. This is just what should be expected of the restricted ionization loss with which we are concerned. The predicted difference in total rate of ionization loss between the two groups of particles arises from the different upper limits to the possible energy transfers between the moving particle and the electrons of the surrounding medium. If one applies an upper limit of a few kilovolts in order to compute the restricted energy transfer to silver bromide grains *along* the track, then the difference vanishes. Our results bear this out in the plateau region, as well as in the interval $10<\gamma<100$.

The role of Čerenkov radiation in the energy-loss of relativistic particles was noted by Fermi[6] and further elucidated by A. Bohr.[10] The possibility that some of this radiation might escape from the AgBr crystals along the path of a particle in a nuclear emulsion and thus detract from the observable relativistic rise in grain density has been investigated by Messel and Ritson,[13] Schönberg,[19,20] Huybrechts,[18] Budini,[21] and Sternheimer.[22]

According to Sternheimer's analysis the escape of

[16] This is equivalent to Sternheimer's Eq. (13) [Phys. Rev. **88**, 851 (1952)], since Sternheimer's δ includes as its first term $2\ln(p/\mu c)=2\ln\beta\gamma$.

[17] Footnote 12 of reference 15.

[18] M. Huybrechts and M. Schönberg, Nuovo cimento **9**, 764 (1952).

[19] Schönberg's earlier theory disagrees even more sharply with our experimental results; see Nuovo cimento **8**, 159 (1951).

[20] M. Schönberg, Nuovo cimento **9**, 210 (1952); **9**, 372 (1952).

[21] P. Budini, Phys. Rev. **89**, 1147 (1953); Nuovo cimento **10**, 236 (1953).

[22] R. M. Sternheimer, Phys. Rev. **89**, 1148 (1953); a more detailed analysis is given in Phys. Rev. **91**, 256 (1953).

energy from the AgBr crystals in the form of Čerenkov radiation is so small, compared with the energy deposited in the grains, that the former may be neglected in comparing the theory with measurements on grain-density in emulsions. This conclusion is supported by the agreement between our experimental results and calculations based on Sternheimer's theory.

COMPARISON WITH OTHER EXPERIMENTS

In 1949 the Brussels group found no evidence in emulsions for the relativistic rise in g exceeding their experimental error.[23] In 1950 similar results appeared in publications from Bristol[4] and Rochester.[24] During the same year Pickup and Voyvodic[25] reported indications of a relativistic increase of about 10 percent to a plateau starting at $\gamma\sim20$. These were, according to the authors, preliminary results based on a few tracks. Voyvodic subsequently extended this work and found a rise of 8 or 9 percent.[26] Similar results were obtained by Daniel *et al.*[27] Our present value is 14 ± 3 percent.

As to the rate of rise of g toward saturation, Voyvodic found indications that saturation is completed more quickly than for the ionization loss as calculated by him. Morrish's results[3] on electrons and those of Daniel *et al.*,[27] on heavier particles similarly suggest that saturation in g already sets in at $\gamma\sim20$. Our data, on the other hand, are altogether consistent with the slow rate of rise calculated according to the Halpern-Hall-Sternheimer theory, and suggest that the polarization plateau is reached at values of $\gamma>100$ as predicted by this theory.

In view of the appreciable magnitude of the total rise, we believe that it should now be possible to discriminate between pions and protons of 3 to 6 Bev and perhaps higher. The upper energy limit previously assigned to such discrimination was 1 Bev. The identification of these very fast particles requires, of course, tracks of sufficient length for good measurement of $\bar{\alpha}$.

In analyzing previously available data on the relativistic rise, Brown[28] is concerned with the explanation of a "trough in the experimental grain density *vs* energy curve $\frac{1}{3}$ to $\frac{2}{3}$ as deep and 3 to 10 times as narrow as the corresponding trough in the theoretical curve. . . ." As we have seen, however, our data are completely consistent with the theoretical trough both in "depth" and "width." The considerable area of agreement provides at least empirical justification for comparison with the theory upon which our calculations are based.

Brown shows that a detailed theoretical treatment of grain densities should take into account the phenomenon of fluctuations in the energy loss. His discussion makes it plausible that *large* differences in the intensity of development, with large resultant differences in the absolute values of G_{pl} may noticeably affect the value of the ratio G_{pl}/G_{min}. However, our measurements indicate that *moderate* changes ($\sim$10 percent) in the level of G_{pl} affect this ratio by less than 3 percent, if at all.

TABLE I. Data for the groups of tracks plotted in Fig. 5.

γ interval	No. of tracks	No. of blobs	Blobs per 100μ	$\bar{\gamma}$
$2\leq\gamma<3$	11	11,319	17.7	2.5
$3\leq\gamma<4$	7	7,593	17.2	3.6
$4\leq\gamma<10$	6	7,588	17.2	5.7
$10\leq\gamma<25$	7	6,591	18.8	19
$25\leq\gamma<50$	7	7,779	18.9	34
$50\leq\gamma<100$	9	8,143	19.2	69
$100\leq\gamma<190$	6	5,622	19.5	150
$190\leq\gamma<340$	9	9,397	20.1	240
$340\leq\gamma<700$	5	5,088	19.6	500
$700\leq\gamma<1500$	8	7,575	19.6	900
$1500\leq\gamma<3000$	2	2,058	19.7	2900

CONCLUSIONS

We draw the following conclusions about the restricted rate of ionization loss I in AgBr of singly charged particles moving at relativistic velocities:

1. As the velocity increases, I passes through a minimum at $\gamma\sim4$ and then rises to a limiting value I_{pl} which lies 14 ± 3 percent above I_{min}.
2. At velocities corresponding to the interval $10<\gamma<100$, our data are consistent with the slow rate of rise predicted by theory.
3. I saturates at values of $\gamma>100$, and maintains the plateau value at least as far as $\gamma=3400$.
4. Electrons, mesons, and protons show a similar variation in I.

In all these respects our results are in agreement with the theoretical curves we computed according to Halpern and Hall and Sternheimer. We believe that these curves can be used, e.g., for velocity determinations based on G and for mass estimates based on both G and $\bar{\alpha}$. It should be emphasized, however, that in this region of extremely high velocities, such determinations are still subject to considerable errors because of the inherent insensitivity of G to the velocity.

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[23] G. P. S. Occhialini, Nuovo cimento 6, 413 (1949).
[24] Bradt, Kaplon, and Peters, Helv. Phys. Acta 23, 24 (1950), see footnote, p. 48.
[25] E. Pickup and L. Voyvodic, Phys. Rev. 80, 89 (1950).
[26] According to unpublished reports by L. Voyvodic and C. F. Powell at the Bristol Conference, December, 1951. Recently in a private communication, Dr. Voyvodic informed us that his data are consistent with a rise of 10 ± 5 percent.
[27] Daniel, Davies, Mulvey, and Perkins, Phil. Mag. 43, 753 (1952).
[28] L. M. Brown, Phys. Rev. 90, 95 (1953).