Returning now to the elastic cross section [Eq. (1)] and setting $A_p = A_n$, we obtain $\sigma_{\text{elastic}} \approx 0$ for destructive interference ($g_n = +g_p$) and $\sigma_{\text{elastic}} \approx 4A_p^2 |I(\mathbf{k}/2)|^2$ for constructive interference ($g_p = -g_n$).[6] $I(\mathbf{k}/2)$ is evaluated by Chew and Lewis[2] using Hulthén functions for the ground state of the deuteron. Using their results and previously measured cross sections for hydrogen,[4,7] we obtain the calculated result:

$$\frac{d\sigma_{\text{elastic}}}{d\Omega}(\pi/2, E_\gamma = 275 \text{ Mev}) \approx 5 \times 10^{-30} \frac{\text{cm}^2}{\text{sterad}}.$$

The experimental result is $d\sigma/d\Omega = 3.5_{-1}^{+2} \times 10^{-30}$ cm²/sterad for γ rays between 250–300 Mev. The error shown includes estimates of what we believe to be all the uncertainties involved in measuring the absolute cross section.

This result offers rather strong evidence that the neutron and proton are oppositely coupled to the $\pi^0 (g_p = -g_n)$ as required by the "symmetrical theory." The strength of this conclusion depends on the validity of the impulse approximation for this calculation. Chappalear and Brueckner[8] have recently attempted to evaluate the effect of multiple scattering. They find that the multiple scattering tends to reduce the difference between the two cases. However, the calculations are not yet complete and no quantitative information is yet available.

We are continuing this experiment in an attempt to measure the angular and energy dependence of this cross section.

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Radioactive Copper Nuclides Produced by Slow Negative Pions and Muons from Zinc*

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ISOTOPES of copper are made when negative pions and muons are absorbed by zinc nuclei and only neutrons are emitted. Such events correspond to the "0-prong" stars in photographic emulsions that are a good fraction (~25 percent) of all slow π^--meson interactions with complex nuclei,[1] and are the predominant mode of interaction of slow μ^- mesons with nuclei.[2] This letter reports the preliminary results of the use of radiochemical methods to get more detailed information on such events.

Zinc, in the form of kilograms of zinc chloride, was exposed to magnetically analyzed 122-Mev negative pion beam of the University of Chicago synchrocyclotron.[3] This beam is contaminated[4] with several percent of 139-Mev negative muons. The copper radioactivities formed when the beam is stopped completely in the target were determined radiochemically. Under our conditions of irradiation and chemistry, the five species listed in Table I were identified.

TABLE I. Production of copper nuclei from zinc by 122-Mev π^- beam.

		Relative yields[a]		
Half-life	Assignment	thick target	at pion range	at muon range
10 min	Cu^{58} or Cu^{59} and Cu^{62}	1.48	1.9	1.1
25 min	Cu^{60}	0.16	0.17	0.01
3.3 hr	Cu^{61}	0.62	0.80	0.2
12.8 hr	Cu^{64}	1.00	1.0	1.0
60 hr	Cu^{67}	0.22	0.10	0.6

[a] The observed activities with an end-window proportional counter were corrected for the background activity induced when the analyzing magnet was shut off and for the fraction of electron capture in the different nuclides.

The production of three of the copper nuclides (Cu^{61}, Cu^{64}, and Cu^{67}) in a 2-in. collimated beam of pions was studied as a function of depth of penetration into a 5-in.×5-in. target of zinc chloride. Figure 1 graphs the results. The production of all three nuclides

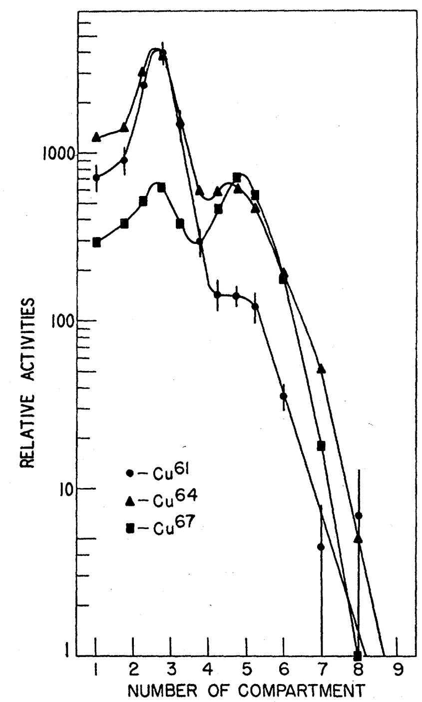

FIG. 1. Production of Cu^{61} (●), Cu^{64} (▲) and Cu^{67} (■) activities as a function of depth of penetration of a 122-Mev π^- beam (after 36.8 g/cm² of Cu initial absorber) into a $ZnCl_2$ target. Each compartment corresponds to approximately 6.2 g of $ZnCl_2$/cm².

is a maximum at the range of the pions (53 g/cm²). In addition, there is a definite second maximum in the production of Cu^{67} and Cu^{64} at approximately the range of the muons (70 g/cm²). The relative production of these three nuclides in the regions of the pion range and the muon range is also given in Table I.

On the assumption that all the stable zinc isotopes (Zn^{64}, Zn^{66}, and Zn^{68} compose 95 percent of the element) capture mesons equally and react in the same way, these data lead to the following conclusions:

1. The reaction $Zn(\mu^-; yn)Cu$ gives a different distribution of radioactive nuclides than does the corresponding reaction with pions. The data of column 5 of Table I, in particular, the very low yield of Cu^{60} and the relatively high yield of Cu^{67}, indicate a maximum probability for y a little greater than 1. This is in agreement with low neutron multiplicities observed[5] upon muon capture in elements near zinc.

2. The reaction $Zn(\pi^-; xn)Cu$ with slow pions has an apparent maximum probability at $x = 2-3$. A distribution with a maximum probability at $x = 5-6$ such as was found radiochemically in the reaction (π^-; p, xn) with bromine,[6] arsenic,[7] and iodine[8] and which is implied by the average neutron multiplicity upon π^- absorption in nuclei in this region of atomic number[9] is definitely excluded. The effect of the secondary neutrons born inside the target at the death of a π^- and contributing via (n; p, xn) reac-

tions has not been completely investigated yet, but targets varying in size by a factor of two give essentially the same results.

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Coupled-Field Green's Functions

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ALL reference to divergent quantities can be avoided in the equations for the Green's functions of quantum electrodynamics by extending the idea of the limiting process introduced by Dirac[1] and Heisenberg.[2] The Green's functions $G_+(x_1, x_2) \sim S_{F1}'$, $\mathcal{G}(x_1, x_2) \sim D_{F1}'$ are solutions of the equations

$$(i\nabla + e\mathbf{A} + \mu + \Sigma_c^*)G_+ = I, \quad (\Box + P_c)\mathcal{G} = I, \tag{1}$$

in which well-defined functions Σ_c^*, P_c replace the divergent functions of Schwinger's equations.[3] The definitions can be formulated without reference to the power series expansion, though the analysis of the latter was used in their derivation.

In order to illustrate the method, we consider in the lowest order approximation the contribution to Σ^* from a loop with an open end,

$$\Sigma^{*(2)}(x', x''; x_1) = -ie^2\gamma^\nu S_+(x'-x_1)\gamma_\nu D_+(x_1-x''). \tag{2a}$$

When we introduce $x = \frac{1}{2}(x'+x'')$, $\xi = x'-x''$, a straightforward calculation gives for the Fourier transform formed with $e^{ik(x_1-x)}$,

$$\Sigma^{*(2)}(k, \xi) = -ie^2(4\mu + \mathbf{k} + 2i\nabla_\xi) \times \int_0^1 du \exp[ik(u-\tfrac{1}{2})\xi]\Delta_+^0(\xi, \mu^2 u - k^2 u(1-u)), \tag{2b}$$

where $\Delta_+^0(\xi, \mu_1^2)$ is a Hankel function of order 0, with the Fourier transform $(\mu_1^2 - p^2)^{-2}$. If we disregard the infrared problem which should be dealt with separately, (2a, b) are well-defined functions with singularities at $\xi^2 = 0$. These singularities have to be compensated by a term $\Sigma_q^{*(2)}$ before closing the loop, and Σ_c^* is defined as

$$\Sigma_c^*(x, x_1) = \int d^4\xi\delta(\xi)\{\Sigma^*(x', x''; x_1) + \Sigma_q^*(x', x''; x_1)\}, \tag{3}$$

where $\int d^4\xi\delta(\xi)\cdots$ indicates a limiting process $\xi \to 0$ to be performed from a space-like direction ξ_μ. If we write $k^2 = \mu^2$ in (2b) and form the difference of the two expressions, $\{\Delta_+^0(\xi, \mu^2 u - k^2 u(1-u)) - \Delta_+^0(\xi, \mu^2 u^2)\}$ and its first derivatives are finite for $\xi \to 0$, the finite part resulting by a kind of self-regularization. In order to exhibit a factor $(\mu + \mathbf{k})^2$ in the finite part, a further finite contribution is to be compensated which is obtained on writing $\mathbf{k} \to -\mu$, after separation of a factor $(\mu + \mathbf{k})$. We write

$$\Sigma_q^*(k, \xi) = -[\Sigma^*(k, \xi)]_{k^2=\mu^2;\ \mathbf{k}\to-\mu}, \tag{4}$$

which compensates, in the order considered, singularities corresponding to the divergent term $A + (\mathbf{k}+\mu)B$ of the usual calculations and a term with $\boldsymbol{\xi}\Delta_+$ corresponding to ambiguities in the latter which have to be removed there by regularization prescriptions. Further analysis of the power series expansion reveals a one-to-one correspondence of the singularities to the divergent parts as defined by Dyson,[4] with additional terms corresponding to the ambiguities. All these singularities can be duly compensated and no divergent integrals or infinite constants appear in the calculations.

One can also define Σ_c^* without reference to the power series expansion by Eq. (3). $\Sigma^*(x', x''; x_1) = \Sigma^*(x', x''; x_1, x_1)$ is conveniently obtained from $\Sigma^*(x', x''; x_1', x_1'')$ which corresponds to two open ends of the self-energy loop and can be written with an abbreviated notation as

$$\Sigma^* = ie^2(\gamma^\nu + \Lambda_q^\nu)G_+(\gamma^\kappa + \Lambda_q^\kappa + \Lambda^{*\kappa})\mathcal{G}_{\nu\kappa}. \tag{5}$$

The second term of the first factor is to compensate the singularities related to the "b" divergences of Dyson;

$$\Lambda^{*\nu}(x', x''; y; x_1) = \delta\Sigma^*(x', x''; x_1)/\delta eA_\nu(y), \tag{6}$$

and Λ_q^ν is obtained with a minus sign from $(\Lambda^{*\nu})A_\nu = 0$ on writing $k_1 = 0$, $\mathbf{k} \to -\mu$ in its Fourier transform formed with $e^{ik_1(y-x_1)} \times e^{ik(x_1-x)}$. Σ_q^* in (3) is given as the sum of the expression defined by (4) for $A_\nu = 0$ and of the field-dependent term $-\int d^4ye \times A_\nu(y)\Lambda_q^\nu(x', x''; y; x_1)$.

$P_c^{\mu\nu}$ in (1) is defined by an equation analogous to (3). The first term is obtained from

$$P^{\mu\nu} = -ie^2 \operatorname{tr}(\gamma^\mu + \Lambda_q^\mu)G_+(\gamma^\nu + \Lambda^\nu)G_+, \tag{7}$$

where the detailed prescriptions for (7), which include the opening and closing of loops, will not be given here. $P^{\mu\nu}(x', x''; x_1)$ is the sum of a term $P_{[\xi]}^{\mu\nu}$ with direction-dependent ξ_μ factors (including singular contributions related to the known gauge-dependent ambiguities), and a term which in the limit $A_\nu \to 0$ has a Fourier transform $(k^\mu k^\nu - g^{\mu\nu}k^2)P_1(k, \xi)$. $P_q^{\mu\nu}$ compensates $P_{[\xi]}^{\mu\nu}$ and the contributions from $(k^\mu k^\nu - g^{\mu\nu}k^2)[P_1(k, \xi)]_{k^2=0}$.

Details will be published in due course. The quantized field equations which are still under investigation are to be defined by means of the limiting process and counter terms given by $P_q^{\mu\nu}$, Σ_q^*, Λ_q^ν.

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The Directional Correlation and Electric Quadrupole Interaction in the γ-γ Cascade of 48.7-min Cd^{111m}†

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THE correlation in the directions of emission of the two successive γ rays of the 48.7-min isomer Cd^{111m} has been measured using several sources differing in chemical and physical state. One purpose was to test further the hypothesis that such correlations can be disturbed through the medium of the electric quadrupole moment of the nucleus in its intermediate state.[1,2] Such effects have been observed by others[3-6] in the γ-γ correlation of the Cd^{111} following K capture in 2.8-day In^{111}. The decay schemes of these two nuclei have the same intermediate excited level, of half-life 8.5×10^{-8} sec. The use of the 48.7-min isomer allows full knowledge of the chemical and physical environments of the nucleus because no changes in electronic configuration and no important recoil energies are involved after the source has been prepared.

The directional correlations were measured using equipment and methods that have been previously described.[7] One of the counters was screened by a 0.9-mm thickness of lead and both were biased to exclude γ rays below about 0.075 Mev to eliminate coincidences due to either back-scattered γ rays or to x-rays. Samples of about 2 mg of Cd, enriched to 70.0 percent Cd^{110},[8] were bombarded in the Brookhaven reactor for 20 to 30 minute periods and data taken for about one half-life. Geometrical correction factors were determined using a collimated beam of γ rays (the 0.19-Mev γ rays of In^{114}) in a manner similar to that employed by Lawson and Frauenfelder.[9] Measurements near 180° had to be corrected by subtraction of coincidences resulting from a background of positron annihilation present in the sources.