Theory of High-Energy Deuteron Pickup*

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THE standard stationary state Green's function technique employed by Gerjuoy[1] for the (d,p) reaction can be used to determine the deuteron pickup[2,3] scattering amplitude. We will consider exactly the idealized (p,d) process where a proton impinges on a nucleus which is considered as a fixed center of force. Physically, this corresponds to the production of high-energy deuterons by high-energy protons where we neglect the energy exchanged with the residual nucleus. Spin and Coulomb effects are neglected also.

The Schrödinger equation for this problem is

$$(T_{\mathrm{R}}+T_{\mathrm{r}}+V_{NP}+V_N+V_P-E)\Psi=0 \tag{1}$$

where the potentials are defined as in reference 1, and for convenience, the kinetic energies T_{R} and T_{r} are expressed in terms of $\mathbf{R}=\frac{1}{2}(\mathbf{r}_p+\mathbf{r}_n)$ and $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$. The solution of Eq. (1) at infinity is of the form

$$\Psi=\psi_0+\Phi, \tag{2}$$

where Φ is the outgoing part, and

$$\psi_0=e^{i\mathbf{k}\cdot\mathbf{r}_p}\varphi(\mathbf{r}_n,\lambda_0), \tag{3}$$

where $\varphi(\mathbf{r}_n,\lambda_0)$ is the initial bound state wave function of the picked-up neutron which has the energy λ_0, so that

$$(T_{\mathrm{R}}+T_{\mathrm{r}}+V_N-E)\psi_0=0. \tag{4}$$

For this problem, the scattering amplitude $A(\mathbf{K})$ can be determined by

$$A(\mathbf{K})\frac{e^{i\mathbf{K}\cdot\mathbf{R}}}{R}=\lim_{R\to\infty}\int d\mathbf{r}w^*(\mathbf{r},\epsilon_f)\Psi, \tag{5}$$

where $w(\mathbf{r},\epsilon_f)$ is the final bound state wave function of the deuteron.

In the symbolic notation,[1] we can write

$$\Psi=\psi_D-G(V_N+V_P)\Psi, \tag{6}$$

where

$$(T_{\mathrm{R}}+T_{\mathrm{r}}+V_{NP}-E)\psi_D=0, \tag{7}$$

$$(T_{\mathrm{R}}+T_{\mathrm{r}}+V_{NP}-E)G=\delta(\mathbf{r}-\mathbf{r}')\delta(\mathbf{R}-\mathbf{R}'). \tag{8}$$

The outgoing Green's function can be written in the form

$$G=\Sigma_\epsilon g(\mathbf{R}-\mathbf{R}',E-\epsilon)w(\mathbf{r},\epsilon)w^*(\mathbf{r}',\epsilon), \tag{9}$$

where

$$g(\mathbf{R}-\mathbf{R}',E-\epsilon)=\frac{1}{4\pi}\frac{4M}{\hbar^2}\frac{\exp[i(4M/\hbar^2)^{\frac{1}{2}}(E-\epsilon)^{\frac{1}{2}}|\mathbf{R}-\mathbf{R}'|]}{|\mathbf{R}-\mathbf{R}'|}. \tag{10}$$

M is the mass of a nucleon, and Σ_ϵ is the summation over the complete set of discrete and continuum deuteron wave functions which satisfy

$$(T_{\mathrm{r}}+V_{NP}-\epsilon)w(\mathbf{r},\epsilon)=0. \tag{11}$$

From Eqs. (2) and (6), the difference between ψ_0 and ψ_D is outgoing. Then, if we rearrange Eq. (4) in the form

$$(T_{\mathrm{R}}+T_{\mathrm{r}}+V_{NP}-E)\psi_0=(V_{NP}-V_N)\psi_0 \tag{12}$$

we may write

$$\psi_0=\psi_D+G(V_{NP}-V_N)\psi_0. \tag{13}$$

If we use Eq. (13), we can eliminate ψ_D from Eq. (6), so that

$$\Psi=\psi_0-G(V_{NP}-V_N)\psi_0-G(V_N+V_P)\Psi. \tag{14}$$

Substituting the Ψ of Eq. (14) into Eq. (5), we have for the scattering amplitude, $A(\mathbf{K})=A_1(\mathbf{K})+A_2(\mathbf{K})$,

$$A_1(\mathbf{K})=-\frac{1}{4\pi}\frac{4M}{\hbar^2}\int d\mathbf{R}d\mathbf{r}e^{-i\mathbf{K}\cdot\mathbf{R}}w^*(\mathbf{r},\epsilon_f)(V_{NP}-V_N)\psi_0, \tag{15}$$

$$A_2(\mathbf{K})=-\frac{1}{4\pi}\frac{4M}{\hbar^2}\int d\mathbf{R}d\mathbf{r}e^{-i\mathbf{K}\cdot\mathbf{R}}w^*(\mathbf{r},\epsilon_f)(V_N+V_P)\Psi. \tag{16}$$

The ψ_0 term in Eq. (14) does not contribute to $A(\mathbf{K})$ since the bound state wave function of the neutron vanishes as $R\to\infty$. Likewise, since conservation of energy requires that

$$\frac{\hbar^2K^2}{4M}+\epsilon_f=E=\frac{\hbar^2k^2}{2M}+\lambda_0, \tag{17}$$

it can be readily shown from Eqs. (4) and (11) via integration by parts that

$$A_1(\mathbf{K})=0.^4 \tag{18}$$

Thus, $A(\mathbf{K})=A_2(\mathbf{K})$ describes the deuteron pickup scattering amplitude. Unlike the time-dependent theory, the explicit appearance of V_N instead of V_{NP} in $A_2(\mathbf{K})$ is not too surprising since the spherically scattered deuteron waves are already associated with the $-G(V_N+V_P)\Psi$ term in Eq. (6). If we make a Born approximation in $A_2(\mathbf{K})$ in which Ψ is replaced by ψ_0, we may then replace V_N by V_{NP} as a consequence of Eq. (18). With $V_P=0$, the scattering amplitude then takes the same form that is usually used, as in reference 2.

I would like to thank Dr. Walter Selove for his discussion of the deuteron pickup experiments being performed at Harvard. Scattering amplitudes are now being evaluated in conjunction with these experiments.

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[2] G. F. Chew and M. L. Goldberger, Phys. Rev. 77, 899 (1950).
[3] M. Gell-Mann and M. L. Goldberger, Phys. Rev. 91, 398 (1953).
[4] Compare with Eq. (4.28) in reference 3, which also can yield this result.

Meson Production by 500-Mev Negative Pions*

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THE first event of meson production in meson interactions was found by the Bristol Group in nuclear emulsions,[1] exposed to cosmic radiation. Recently one event with two emitted mesons has been found by Fry[2] in the 220-Mev negative meson beam of the Chicago cyclotron. Among 460 meson interactions initiated by 500-Mev negative pions from the Brookhaven cosmotron, six events have been found in which two mesons leave the nucleus. These events represent production of mesons. It is highly probable that the created meson is produced directly in collisions of mesons on nucleons inside the nucleus, rather than indirectly by a nucleon recoiling in a meson scattering process.

Because of the large error in the energy determination of the emitted mesons and the uncertainty concerning the size of the nucleon aggregates involved, one cannot decide in each individual case which mechanism of production, by a single meson-nucleon

TABLE I. Characteristics of events with meson production. w is the energy of the emitted meson and θ is the space angle it makes with the direction of the incident meson.

No. of events	Emitted mesons w_1 in Mev	θ_1	w_2 in Mev	θ_2	Other particles emitted in interaction
1	2.5 ± 0.1	124°	80^{+30}_{-20}	95°	2 black tracks 1 recoil
2	50^{+15}_{-10}	32°	140^{+40}_{-25}	94°	1 black prong 1 recoil 1 slow electron
3	85^{+50}_{-40}	102°	42^{+16}_{-9}	130°	5 black prongs
4	80 ± 20	72°	90^{+30}_{-20}	154°	4 black prongs
5	25 ± 3	65°	29^{+5}_{-4}	137°	2 black prongs
6	3 ± 0.3	12°	85^{+30}_{-20}	115°	1 black prong
7	17 ± 3	7°	130 ± 40	102°	0

interaction, or by an interaction of a recoiling nucleon with another nucleon, occurred. Because of the low energy of the recoiling nucleons and therefore the small cross section (about 3×10^{-27} cm^2),[3] the latter process is improbable. Only 80 events where a backward scattered meson is observed, in the 460 analyzed, can lead to recoil nucleons more energetic than 250 Mev. These recoil nucleons would be expected to produce at most 0.2 meson, not six.

Table I describes the characteristics of the events, listing kinetic energy and angle of emission (laboratory system) of the two mesons and the number of additional particles emitted in the disintegration. The relatively great number of prongs in events 1, 3, and 4 suggest that these are processes occurring in light nuclei. Specifically, event 1 can be explained on energy grounds as having taken place in a carbon nucleus, disintegrating into two protons of 4.7 and 6.5 Mev (range determination) and a $_4Be^9$ nucleus of 2–3 Mev.

Figure 1 is a photomicrograph of event 1; *i* is the incoming meson and *a* and *b* the emitted mesons.

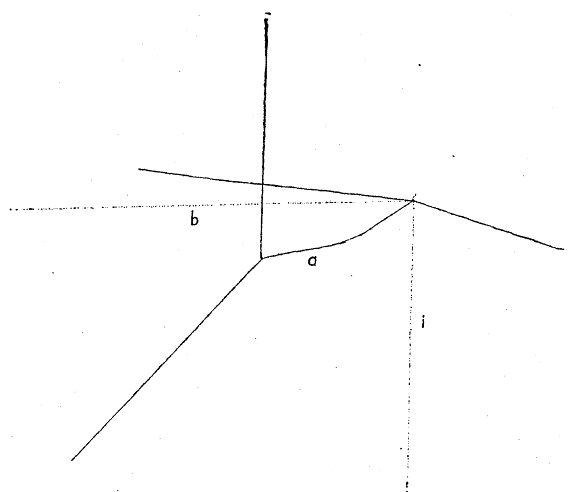

FIG. 1. Meson production, event 1.

Event 7 (Fig. 2) has been found after scanning more than the 460 events referred to. Here only the incoming meson *i* and the emitted mesons *a* and *b* can be seen. This event can, on the basis of energy and momentum considerations (within the limit of experimental errors), be explained as meson production in a hydrogen nucleus of the emulsion.

Thus far no positive mesons coming out of stars could be identified. However, in adjacent emulsions several π-μ-e decays have been observed, supporting the conclusions on meson production.

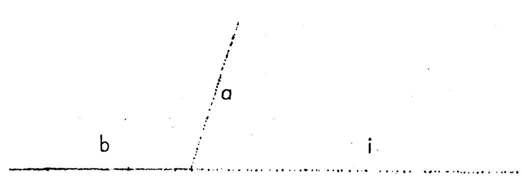

FIG. 2. Meson production, event 7.

In addition to the 6 events mentioned above, 8 more have been found which are consistent with interactions with meson production. However, the tracks are too short for unmistakable identification.

Therefore the percentage of events with two charged mesons is 1.3 percent to 3 percent, considering the 460 interactions among which the events have been found. This percentage has to be increased in order to take into account events where one or both mesons have been absorbed. We have found that with 500-Mev incident mesons, charged mesons leave the nucleus in only 38 percent of the interactions (55 percent if neutral mesons are considered). Since the energy of the emitted mesons is generally smaller in two-meson events, still fewer of these mesons escape. It is estimated (based on experiments with incident mesons of lower energy) that the number of 2-meson events reported here constitute at the most 25 percent of all events in which a charged meson—observed or absorbed—is produced. The above figure would then represent 5.2 percent to 12 percent of all interactions.

From these experiments nothing can be learned about the production of π^0 mesons, and therefore the actual cross section of meson-meson production at 500-Mev meson energy cannot be compared with theoretical data.

Recently Yang and Fermi[4] independently calculated the probabilities for meson-meson production for various meson energies available from the cosmotron. The calculations are based on the Fermi theory of meson production and on considerations concerning isotopic spin conservation; the calculations apply for meson-nucleon collisions and assume an interaction radius $r=\hbar/\mu c$.

Assuming (1) the validity of these calculations for meson-nucleon collisions inside the nucleus, (2) the equality of number of protons and neutrons for emulsion nuclei, and (3) the equality of π^--p and π^--n cross sections,[5] the fraction of events leading to two charged mesons is 16 percent or 18 percent (the second number includes events with 2 charged and 1 neutral meson). This figure is about twice the value calculated from experimental data, but considering the meager statistics and the simplified assumptions, the disagreement is probably not too serious

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Operation of a Glaser Bubble Chamber with Liquid Hydrogen

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GLASER has recently demonstrated that ionizing radiation can trigger violent boiling in highly superheated ether and pentane.[1] He has further shown that it is possible to photograph tracks of bubbles which mark the paths of ionizing particles passing through the liquid.

A bubble chamber using liquid hydrogen should prove useful for studying nuclear events involving protons. For example, it would provide a hydrogen target of greater density and purity than can be achieved in a cloud chamber.

In this letter we report preliminary experiments which strongly suggest that a useful liquid hydrogen bubble chamber can be