companied by a cooling of the lower stratosphere (δT negative) and vice versa. The magnitude of the above ratio is probably too large. However, a knowledge of the actual values of $(\delta T)_{\rm Av}$ for the period and location of Duperier's experiment is needed to check our results quantitatively.

In view of the above discussion we conclude that the additional term, $A_K(\delta T)_{\rm Av}$, in the regression formula may possibly remove the apparent variability of Elliot's decay coefficient as well as the anomalous value of Duperier's coefficient for the positive temperature effect. An experimental verification of this conclusion would be desirable. Unfortunately, because of the strong correlation between δH and $(\delta T)_{\rm Av}$, it will be very difficult to separate experimentally the effects caused by δH from those caused by $(\delta T)_{\rm Av}$.

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Multiple Cores in Air Showers*

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Multiple cores in cosmic-ray air showers are difficult to observe because of the overlapping of the cores. After transition to equilibrium with water the cores are much smaller and readily identified. Decoherence measurements of the coincidences between pulses in ionization chambers have been made for depths in water from 0 to 3 meters and with separation of the chambers up to 6 meters. The decoherence measurements are consistent with an average air shower at 9000 feet which has about 20 cores within a distance of about 5 meters from the shower's center.

INTRODUCTION

SEVERAL experiments have been performed to study the structure of the extensive showers in cosmic rays. Calculations[1] based on the hypothesis of the cascade origin of these showers have been shown to be in agreement with experiments performed with Geiger counters[2] and ionization chambers.[3] It has been proposed by Lewis, Oppenheimer, and Wouthuysen[4] that the originating particles are produced in nucleon-nucleon collisions with high multiplicity. Hence, the resulting showers should have multiple cores corresponding to the multiplicity of the events. However, experiments[2,3] failed to detect any multiplicity in the cores of the extensive showers. These negative results have been explained by Blatt.[5] He pointed out that statistical fluctuations in experimental data are responsible for this apparent agreement between Molière[1] distribution and the experimental data.

Fretter and Ise[6] have reported another type of experiment to detect the presence of multiple cores, using water as an absorber and Geiger tubes as detectors. The experiments were carried out by Barrett,[7] and results were reported to agree with the single core distribution.

In the research reported here, another experimental arrangement was applied to study further the structure of the shower cores. Water was used as an absorber and fast ionization chambers as detecting instruments. The reason for these two choices can be explained as follows:

In water, the characteristic quantities for electron showers are quite different from those in air. Table I gives numerical values of the characteristic quantities for electron showers in both materials.

The unit r_1 was introduced by Euler and Wergeland[8] as a convenient unit for studying the spatial distribu-

TABLE I. Characteristic quantities for electron showers in air and water.

	Critical energy E_J $\times 10^9$ ev	Radiation length X_0 cm	Lateral unit r_1 cm
Air	11.3	33 000	5950
Water	11.3	43	7.8

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TABLE II. Experimental results at Lake Sabrina (elevation 2765 meters).

Depth under water, D in cm	Separation of the chambers, S in cm	Twofold coincidence counting rates counts/hr
0	87	0.870 ± 0.1
91	87	1.024 ± 0.15
185	87	0.469 ± 0.046
	345	0.294 ± 0.039
	440	0.175 ± 0.033
	645	0.046 ± 0.034
274	87	0.277 ± 0.043
	158	0.196 ± 0.04
	345	0.162 ± 0.037
	440	0.075 ± 0.039

tion in the showers, and can be defined by: $r_1 = E_s X_0 / E_j$, where X_0 is the radiation length, E_j the critical energy, and E_s is a constant ($E_s = 21$ Mev). The value of r_1 for any material can then be shown to be of the order of magnitude of the lateral dimension of the showers in equilibirum with this material. This lateral dimension being sometimes expressed as a half radius, meaning the radius enclosing half the total number of particles in the shower at any depth.

From the value of r_1 (the lateral unit of length), it is clear that showers which extend in air to distances of several meters, after reaching equilibrium in water, will shrink to a few centimeters. X_0 the radiation length, being only 43 cm in water, indicates that the transition effect will have disappeared at a depth of two meters or so. The multiplicity of cores which escapes detection in air due to overlapping, is more readily revealed if the showers are studied under water.

Fast ionization chambers were chosen as the detecting instrument in the experiment because of their proportionality. The pulse output of a chamber is proportional in height to the amount of ionization in the chamber. The instrument can then be applied to record events larger than any arbitrary value when used in conjunction with a discriminating circuit.

EXPERIMENTAL ARRANGEMENT

The apparatus for the experiment consisted of two spherical ionization chambers, the necessary amplifiers, registering circuits, and power supplies. The equipment was carried on a raft and floated on Lake Sabrina (elevation 2765 meters) near Bishop, California. The chambers were then lowered in water by ropes. The depth D of the chambers below the surface of the lake as well as the separation S between the chambers were changed throughout the experiment and the twofold coincidence counting rate was recorded.

Each ionization chamber consisted[9] of two spun-aluminum hemispheres, 13.25 in. in diameter and $\frac{1}{8}$ in. thick, welded together, enclosing a small, hollow aluminum sphere 3 in. in diameter serving as a collecting electrode. The ionization chambers were fitted to steel boxes with watertight covers leaving the top hemispheres exposed. The boxes housed the lower halves of the chambers, the high voltage circuits and the preamplifiers. Power and signals were conducted to and from the chambers by cables which entered the boxes through watertight glands. The chambers were filled with pure argon gas at a pressure of 2.85 atmospheres. A high degree of purity of the gas was achieved by having a calcium gas purifier[10] connected permanently to each chamber.

Signals from the chambers after going through the preamplifiers are transmitted through long cables to the linear amplifier.[11] Each linear amplifier has a rise time of 0.7 μsec and a clipping time of 50 μsec. The overall gain of amplification is 8×10^5. The output from the amplifiers is then fed into a discriminator circuit, a twofold coincidence circuit, and registering circuits. The registering circuits recorded the individual channels' counts as well as the twofold coincidences on separate registers. The resolving time of the coincidence circuit was measured to be 5 μsec.

For calibration, each chamber was provided with a weak polonium source placed on the inside of the outer wall. A bias curve was obtained for each chamber and the discriminator was set during the experiment to correspond to $\frac{1}{2}$ the energy of a polonium α particle. This energy corresponds to 20 shower particles in the chamber or a density of 230 particles per square meter.

RESULTS AND ANALYSIS

The experimental data are given in Table II. Counting rates were corrected for accidentals and the errors indicated are standard deviations due to statistical fluctuations. Other errors were too small, compared to the statistical fluctuations, to be of significance.

Calculations were carried out to determine the shape of the twofold decoherence curve expected for this experimental arrangement assuming the Molière distribution is valid and a single core. If the apparatus is biased to detect densities above a certain value ρ, the twofold coincidence counting rate $W(\rho,S)$ for any separation of chambers, S, can be calculated. An expression was given by Blatt[5] for $W(\rho,S)$ as a function of the density distribution function $f(r)$ and the integral number spectrum of showers. This expression in conjunction with the Ise and Fretter[12] discussion of shower spectra and the Molière[1] distribution function can be reduced to

$$W(\rho,S) = CS^{-4}, \tag{1}$$

where C is a proportionality constant. Equation (1) is

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shown in Fig. 1 by the dotted curve 4. It was normalized at the experimental values at the smallest separation.

It is obvious that the calculated curve does not fit the experimental data. This is expected, in view of the argument presented in connection with Table I. It actually means that the values of S used in the experiment are so large compared to the dimensions of showers in water, that the counting rate function drops very rapidly.

As the idea of single cores was found inconsistent with the experimental data, an assumption was made that showers have multiple cores. This assumption is also supported by meson production theories and theories on the origin of showers, as was pointed out in the Introduction. An attempt was then made to obtain information about the distribution of these cores.

Let the cores of the shower be distributed around the axis, according to a function $\phi(r)$. In other words, let $\phi(r)$ be the probability of finding a core at a distance r in the horizontal plane from the axis of the shower. If ν is the total number of cores within the individual shower, and σ is the density of cores at a distance r, then

$$\sigma=\nu\cdot\phi(r).$$

An expression can then be set up for the twofold coincidence counting rate W as a function of the abovementioned variables. A linear function was assumed for $\phi(r)$, decreasing from a value corresponding to density of cores of σ_0 at the center, to the value of 0 at a limiting distance r_0. Calculations were carried out for an average shower, hence no assumption as to the form of dependence of the shower frequency on ν was necessary. By choosing arbitrary values for the multiplicity ν and the limiting radius r_0 it was possible to calculate graphically the value of the counting rate W as a function of the separation S. The results of the calculations are shown in Fig. 1. Curve 1 was calculated for $r_0=7$ meters and $\nu=30$, curve 2 for $r_0=7$ meters, $\nu=35$, and curve 3 for $r_0=5$ meters and ν ranging from 10 to 15.

The fairly good fit of curve 3 to the experimental points suggests that the assumptions introduced in the analysis are consistent with the experimental data. It was noticed that the shape of the curve is sensitive to changes in r_0 but rather insensitive to changes in ν. The values of r_0 indicate that the cores of the shower extend to a distance of the order of 5 meters from the axis. No definite value for ν can be evaluated. However, upper and lower limits can be estimated as 30 and 10, respectively.

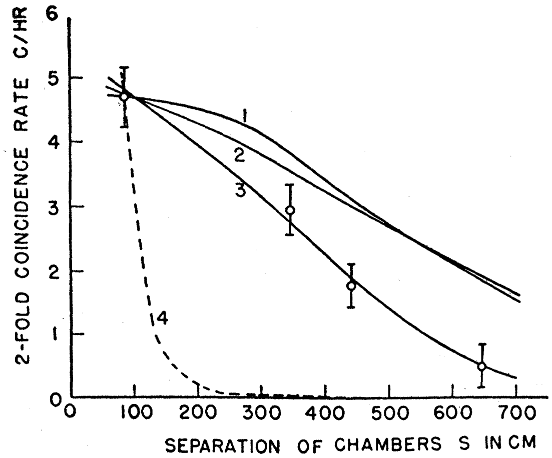

FIG. 1. Calculated decoherence curves assuming multiple cores (curves 1, 2, and 3) and for single core (curve 4). Experimental points taken at a depth of 185 cm are also shown.

CONCLUSIONS

The conclusions that may be drawn from the experiments and calculations are somewhat qualitative. The decoherence curve calculated from the Molière[1] distribution was found to be inconsistent with the experimental results. The observed decoherence can be satisfactorily explained by the presence of multiple cores in extensive showers. It is reasonable to assume that the density of cores in the average extensive shower decreases linearly with the distance from the center of the shower to a value of zero at a distance r_0.

A density of cores σ_0 of about 1 per square meter at the center and a limiting radius of $r_0=5$ meters appear to fit the observed data. This gives a total number of cores in the average shower of about 20.

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