Atmospheric Effects on Cosmic-Ray Intensity Near Sea Level*†

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(Received June 30, 1953)

The effects of atmospheric temperature and pressure on the μ-meson intensity are studied theoretically for locations near sea level. The analysis is based on a unidimensional equation for the vertical differential intensity of μ mesons, studied originally by Sands. The treatment is rigorous in the sense that it includes the continuous production as well as the ionization losses of μ mesons in the atmosphere. With the help of a newly derived range spectrum of μ mesons at production and the exact expression for the survival probability of μ mesons, a three-term regression formula for the relative changes of the μ-meson intensity is derived and discussed in detail. According to this formula, the relative intensity changes are correlated not only with the average production height and the ground pressure (a customarily employed two-term correlation) but also with the average tropospheric temperature. This additional correlation, resulting from the ionization losses of μ mesons in the air, seems to remove some apparent difficulties in the interpretation of experimental data. In particular, it seems to explain the discrepancies found in the decay coefficients determined from diurnal and seasonal observations, respectively.

I. INTRODUCTION

NUMEROUS experiments[1] of the last two decades have shown that the intensity of cosmic rays is influenced by the atmospheric conditions existing during the period of observation. It has been found from experiments on the ground that the variations of the cosmic-ray intensity are closely related to the changes of pressure and temperature of the atmosphere above the observer. While the pressure effect indicates the dependence of cosmic radiation on the amount of air traversed, the temperature effect has its origin in the instability of μ mesons[2] which form the preponderant part of the penetrating component of cosmic rays at sea level. The following simple argument shows qualitatively that an increase of the temperature causes a decrease of the μ-meson intensity at sea level. Each μ meson produced at a certain atmospheric depth (i.e., at a given pressure level) has to travel the distance from the production layer to the level of detection. An increase of the temperature will increase this distance and thus enhance the probability of decay in flight. If all mesons were produced at the same atmospheric depth, and if one could neglect their ionization losses, the fluctuations in the height H, of a single pressure level—the production level—would suffice to account for the temperature effect on the μ-meson intensity. One could then express the variations of the μ-meson intensity I with an equation of the following type:

$$\delta I/I = A_H'\delta H + A_P'\delta x_0, \tag{1}$$

where δH and δx_0 are the variations of the height of production and of the ground pressure, respectively. The coefficients A_H' and A_P' are often referred to as the "decay" and "absorption" coefficients, respectively.

Early investigations seemed to confirm the approximate validity of the above regression formula. Duperier,[3,4] in particular, found through the statistical analysis of the observed data, that the ground pressure and the height corresponding to the atmospheric depth of 100 g cm^{-2} were the controlling factors in the variations of cosmic-ray intensity at sea level. However, the more recent studies by the same author,[5] as well as those by Dolbear and Elliot,[6] have shown that the two-term Eq. (1) is inadequate to account fully for the variations of the cosmic-ray intensity at sea level. The inconsistencies have not been explained quantitatively in a satisfactory manner.

In this paper we attempt to give a more rigorous, quantitative treatment of the problem outlined above. In particular, the two following facts, neglected thus far, are taken into account in present calculations: (1) the fact that the μ mesons are produced *continuously* throughout the atmosphere, and (2) the fact that the μ mesons suffer *ionization losses* during their propagation through air. As we shall see in the following sections, the first fact will justify and clarify the notion of an average production layer for the bulk of μ mesons, and the second fact will introduce a third term into the regression formula discussed above. This additional term will turn out to be primarily controlled by the temperature changes in the *lower* atmosphere, where the ionization losses are relatively more important than in the upper atmosphere.

All considerations in this paper apply only to the *vertical* intensities (both differential and integral) of μ mesons measured at locations near sea level.

* Part of a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the Massachusetts Institute of Technology.
† This work was supported in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] For a review, see, e.g., H. Elliot, *Progress in Cosmic-Ray Physics* (North-Holland Publishing Company, Amsterdam, 1952), Chap. VIII.
[2] P. M. S. Blackett, Phys. Rev. **54**, 973 (1938).
[3] A. Duperier, Terrestial Magnetism and Atm. Elec. **49**, 1 (1944).
[4] A. Duperier, Proc. Phys. Soc. (London) **61**, 34 (1948).
[5] A. Duperier, Proc. Phys. Soc. (London) **62**, 684 (1949).
[6] D. W. N. Dolbear and H. Elliot, J. Atmos. Terr. Phys. **1**, 215 (1951).

II. SURVIVAL PROBABILITY OF μ MESONS

In order to see the manner in which the temperature of the atmosphere enters into the expression of the μ-meson intensity, we must first consider the expression for the survival probability of μ mesons. For a μ meson produced at the elevation z and traveling vertically toward the earth with the velocity $c\beta$, the survival probability is given by[7]

$$w=\exp\left[-\int_{z_0}^{z}\frac{(1-\beta^2)^{\frac{1}{2}}}{c\tau\beta}dz'\right], \tag{2}$$

where τ is the mean life of the μ meson at rest, and z_0 the elevation of the point of observation. The factor $(1-\beta^2)^{\frac{1}{2}}$ accounts for the relativistic time dilatation. It is convenient to express the distance dz' in terms of the increment of atmospheric depth dx'. Since $dx'=\rho(x')dz'$, one may write for Eq. (2)

$$w(x,R)=\exp\left[-\frac{\mu}{\tau}\int_{x}^{x_0}\frac{dx'}{\rho(x')p(R')}\right], \tag{2'}$$

where now x and x_0 denote the atmospheric depths at the points of production and observation, respectively; μ is the rest mass of the μ meson, and $p(R')$ is the momentum at the depth x' expressed as a function of the corresponding residual range $R'=R+x_0-x'$. R is the residual range of the μ meson at the point of observation. It is useful to approximate $p(R')$ by the following analytical formula:

$$\frac{\mu c}{p(R)}=\frac{B}{b+R'}-\kappa, \tag{3}$$

where $B=53.5$ g cm^{-2} and $b=56$ g cm^{-2} if R is measured in g cm^{-2}; $\kappa=\text{constant}=2.07\times10^{-3}$. With the numerical values of the constants quoted above, Eq. (3) is applicable for all μ mesons with residual ranges R', between 30 g cm^{-2} and 6000 g cm^{-2} air equivalent. In this region, it reproduces the theoretical curve[8] within an accuracy of one percent. Sinceal most all μ mesons recorded at sea level have residual ranges from 30 to 6000 g cm^{-2}, Eq. (3) may be considered as sufficient for the purposes of this paper.

The evaluation of Eq. (2′) also requires the knowledge of the vertical behavior of the air density $\rho(x')$ or of the atmospheric temperature T. These two quantities are related by the following equation:

$$x'/\rho(x')=\mathcal{R}T(x')/Mg, \tag{4}$$

where $\mathcal{R}$ is the universal gas constant, M is the effective molecular weight of air, and g is the acceleration of gravity. If the temperature $T(x')$ is measured in absolute units, $\mathcal{R}/Mg=2.87\times10^3$ cm/°C. It is more appropriate to discuss the survival probability w in terms of $T(x')$ rather than in terms of $\rho(x')$, because $T(x')$ is a quantity that can be measured directly by means of radio sondes.

By combining Eqs. (2′), (3), and (4) one may write for the survival probability

$$w(x,R)=\exp\left[-\frac{\mathcal{R}}{c\tau Mg}\int_{x}^{x_0}\frac{T(x')}{x'}\left(\frac{B}{x_e-x'}-\kappa\right)dx'\right], \tag{5}$$

where $x_e=x_0+R+b$. Since $1/[x'(x_e-x')]=1/[x_e(x_e-x')]+1/(x_ex')$ one may also express Eq. (5) as follows:

$$w(x,R)=\exp[\alpha_H(R)H(x)+\alpha_K(R)K(x,R)], \tag{5'}$$

where

$$\alpha_H(R)=-\frac{1}{c\tau}\left(\frac{B}{x_e}-\kappa\right)=-\frac{\mu}{\tau p(x_0+R)}, \tag{6}$$

$$\alpha_K(R)=-\mathcal{R}B/Mgc\tau x_e^2, \tag{6'}$$

and

$$H(x)=\frac{\mathcal{R}}{Mg}\int_{x}^{x_0}\frac{T(x')}{x'}dx', \tag{7}$$

$$K(x,R)=x_e\int_{x}^{x_0}\frac{T(x')}{x_e-x'}dx'. \tag{7'}$$

The representation of $w(x,R)$ given by Eq. (5′) has some advantages over those given by Eqs. (2′) or (5). The two terms in the exponent of Eq. (5′) have a direct physical significance. The function $H(x)$, defined by Eq. (7), represents simply the distance from the pressure level x_0 (the ground) to the pressure level x (the production level). The first term in Eq. (5′), $\exp[\alpha_H H(x)]$, is the main term; it represents the survival probability for a μ meson produced at x and traveling the distance $H(x)$ with a *fixed* momentum $p(x_0+R)$ [see Eq. (6)]. Since, for most mesons, x is considerably smaller than x_0 the fixed momentum $p(x_0+R)$ will not differ significantly from the actual momentum at production $p(x_0+R-x)$, so that

$$\exp[\alpha_H H(x)]\approx\exp\left[-\frac{\mu H(x)}{\tau p(x_0+R-x)}\right].$$

One recognizes in the right-hand side expression a formula often quoted in the interature; it represents the survival probability of μ mesons if one neglects their *ionization losses* in the air. The second term in Eq. (5′), $\exp(\alpha_K K)$, may be regarded as a corrective term accounting for the *ionization losses* of μ mesons in the air. The function $K(x,R)$, defined by Eq. (7′), will, in general, depend not only on $T(x)$ but also on the residual range of the μ mesons R. However, for sufficiently large R (say $R>1000$ g cm^{-2}), $K(x,R)$ may be written roughly as

$$K(x,R)\approx\int_{x}^{x_0}T(x')dx'; \tag{8}$$

[7] See, e.g., B. Rossi, *High Energy Particles* (Prentice-Hall, Inc., New York, 1952), p. 157.
[8] See reference 7, pp. 40–41.

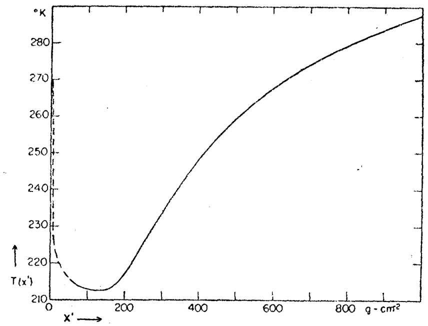

FIG. 1. Vertical distribution of the annual mean of the temperature at 40°N geographic latitude plotted as a function of the atmospheric depth. The curve was computed from the graph given by B. Haurwitz and J. M. Austin in *Climatology* (McGraw-Hill Book Company, Inc., New York and London, 1944), p. 37.

i.e., in the case of fast mesons, the function $K(x, R)$ is closely related to the temperature averaged over the region between the level of production and the ground.

For the numerical evaluation of Eq. (5′) one needs the function $T(x')$ which will, in general, be different for different seasons, different geographical locations, etc. We have based our calculations of $w(x, R)$ on the annual average of $T(x')$ for 40°N geographical latitude (see Fig. 1). Figure 2 shows graphically the behavior of the negative logarithm of this "average" survival probability *versus* the atmospheric depth x for various residual ranges R and for $x_0=1000$ g cm^{-2}.

In order to study the deviations from the average survival probability, $w_{\text{Av}}(x, R)$, which are due to the departure of the temperature from its annual mean, one may make use of the first-order perturbation with respect to $T(x')$. From Eq. (5′), one then gets

$$\delta_T w(x, R)=w_{\text{Av}}(x, R)[\alpha_H \delta H(x)+\alpha_K \delta K(x, R)], \quad (9)$$

where

$$\delta H(x)=\frac{\mathcal{R}}{Mg}\int_x^{x_0}\frac{\delta T(x')}{x'}dx', \quad (10)$$

and

$$\delta K(x, R)=x_e\int_x^{x_0}\frac{\delta T(x')}{x_e-x'}dx'. \quad (10')$$

As we shall see in the next section, the above Eqs. (9), (10), and (10′) will be of primary importance for the evaluation of the temperature coefficient of the differential intensity of μ mesons.

III. TEMPERATURE EFFECT ON THE COSMIC-RAY INTENSITY AT SEA LEVEL

We now turn to our main problem: the evaluation of the variations of the cosmic-ray intensity caused by changes of atmospheric temperature. As we have mentioned in the introduction, we shall limit ourselves to the case of the *vertical* intensities of the hard component at sea level. Thus we may follow the treatment given by Sands[9] and assume that differential intensity may be represented by

$$i_v(R)=\int_0^{x_0}G(R_s)e^{-x/L}w(x, R)dx, \quad (11)$$

where L is the absorption mean free path of the meson-producing radiation and $G(R_s)$ is the range spectrum of μ mesons at production. $G(R_s)$ will be assumed to be a function of the residual range $R_s=R+x_0-x$ *only*. We shall use, for the purpose of this paper, the following approximate expression for this function:

$$G(R_s)=\frac{7.31\times10^4}{(520+R_s)^{3.58}}(\text{g}^{-2}\text{ cm}^2\text{ sec}^{-1}\text{ sterad}^{-1}), \quad (12)$$

where R_s is measured in g cm^{-2}. This empirical formula for the production spectrum of μ mesons has been derived on the basis of more recent experimental data,[10]

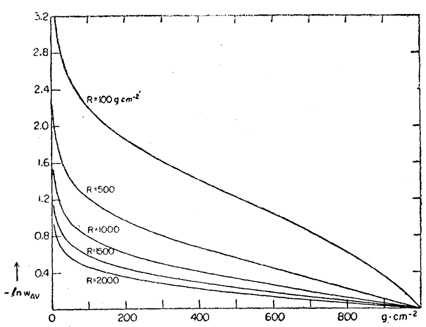

FIG. 2. Negative natural logarithm of the survival probability of μ mesons produced at the atmospheric depth x and reaching sea level with various residual ranges between 100 and 2000 g cm^{-2}. The curves correspond to the annual mean of the vertical temperature distribution at 40°N geographic latitude.

and is believed to be slightly better than the expression given by Sands. It is compatible with measurements at 50° geomagnetic latitude for the residual ranges at production extending from $R_s=100$ g cm^{-2} to $R_s=7000$ g cm^{-2}.

Let us discuss first the case where the ground pressure x_0 is kept *constant*. Then the variation of $i_v(R)$ due to the temperature changes $\delta T(x')$ is given, according to Eqs. (11) and (9), by

$$\delta_T i_v(R)=\int_0^{x_0}[\alpha_H\delta H(x)+\alpha_K\delta K(x, R)]\Phi(x, R)dx, \quad (13)$$

where we have put for short

$$\Phi(x, R)=G(R_s)e^{-x/L}w_{\text{Av}}(x, R). \quad (14)$$

[9] M. Sands, Technical Report No. 28, Laboratory for Nuclear Science, Massachusetts Institute of Technology, 1949 (unpublished); Phys. Rev. 77, 180 (1950).

[10] For a detailed discussion on this subject, the reader is referred to S. Olbert, thesis, Massachusetts Institute of Technology, June, 1953 (unpublished).

In order to evaluate Eq. (13) we shall make use of the mean value theorem which states that a definite integral of a product of two regular functions, say $f(z)$ and $g(z)$, may be written as follows:

$$\int_a^b g(z)f(z)dz=g(\xi)\int_a^b f(z)dz, \tag{15}$$

where $a<\xi<b$. One can estimate the actual value of ξ rather accurately if the following two conditions are satisfied: (1) $g(z)$ is a slowly varying function in the interval (a, b), and (2) $f(z)$ displays a sharp maximum in (a, b). One then finds, by expanding $g(z)$ into a Taylor series at ξ and neglecting the terms of second order or higher, that

$$\xi\int_a^b f(z)dz=\int_a^b zf(z)dz. \tag{16}$$

The error that one makes in Eq. (15) by applying the above estimate of ξ is of the order of

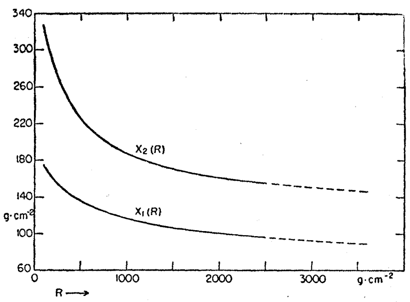

FIG. 3. Characteristic atmospheric depths $x_1(R)$ and $x_2(R)$ defined by Eqs. (21) and (22) and entering into the regression formula for the temperature effect on the differential μ-meson intensity near sea level.

$$\frac{1}{2g(\xi)}\left[\frac{d^2g(z)}{dz^2}\right]_{z=\xi}(\eta^2-\xi^2), \tag{17}$$

where

$$\eta^2\int_a^b f(z)dz=\int_a^b z^2f(z)dz. \tag{18}$$

Evidently, the function $\Phi(x, R)$, defined by Eq. (14), satisfies the condition (2) very well for $R>100$ g cm^{-2}; furthermore, according to Eq. (10′), the function $\delta K(x, R)$ satisfies the condition (1). Hence, we may apply the method outlined above directly to the second term in Eq. (13). We cannot apply it directly to the first term, because the function $H(x)$ displays a logarithmic divergence at $x=0$. However, if we consider separately the functions $\delta H(x)/\ln(x_0/x)$ and $\Phi(x, R)\ln(x_0/x)$, rather than the functions $\delta H(x)$ and $\Phi(x, R)$, we readily verify that these two functions do satisfy conditions (1) and (2), respectively. Consequently, we may write Eq. (13) as follows:

$$\delta_T i_v(R)=\frac{\alpha_H\delta H(x_1)}{\ln(x_0/x_1)}\int_0^{x_0}\ln(x_0/x)\Phi(x, R)dx+\alpha_K\delta K(x_2, R)\int_0^{x_0}\Phi(x, R)dx \tag{19}$$

or

$$\delta_T i_v/i_v=\gamma\alpha_H\delta H(x_1)+\alpha_K\delta K(x_2, R), \tag{20}$$

where, according to Eq. (16),

$$x_1=\int_0^{x_0}x\ln(x_0/x)\Phi(x, R)dx\Big/\int_0^{x_0}\ln(x_0/x)\Phi(x, R)dx, \tag{21}$$

$$x_2=\int_0^{x_0}x\Phi(x, R)dx\Big/\int_0^{x_0}\Phi(x, R)dx, \tag{22}$$

and

$$\gamma=\left[\int_0^{x_0}\ln(x_0/x)\Phi(x, R)dx\right]\Big/\left[\ln(x_0/x_1)\int_0^{x_0}\Phi(x, R)dx\right]. \tag{23}$$

The three quantities, x_1, x_2, and γ, according to their definitions, are functions of the residual range R of μ mesons at x_0. Figures 3 and 4 show these quantities plotted *versus* $R(100<R<4000$ g cm$^{-2})$ for $x_0=1000$ g cm^{-2}. Note that the factor γ is practically one for all $R>500$ g cm^{-2}; note further that the atmospheric depths x_1 and x_2 are slowly decreasing functions of R. We shall discuss the physical significance of these results in Sec. V.

In order to give Eq. (20) a simpler physical interpretation, it is useful to approximate the function $\delta K(x, R)$ by the following expression:

$$\delta K(x_2, R)=\frac{x_e}{x_0-x_2}\left(\ln\frac{x_e-x_2}{x_e-x_0}\right)\int_{x_2}^{x_0}\delta T(x')dx';$$

one may readily verify that the above formula is a good approximation to Eq. (10′) if $\delta T(x')$ does not

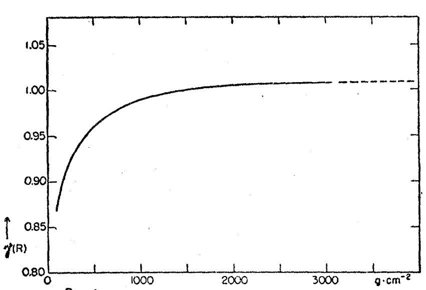

FIG. 4. Factor $\gamma(R)$ defined by Eq. (23) and entering into the regression formula for the temperature effect on the differential μ-meson intensity near sea level.

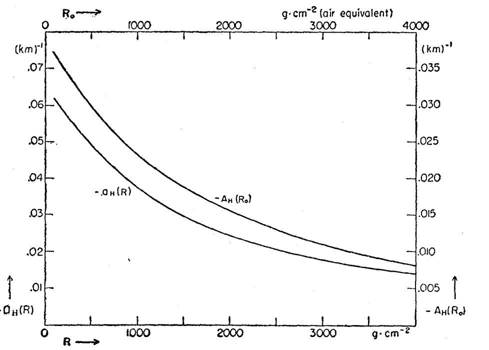

FIG. 5. The coefficients a_H and A_H correlating the differential and integral intensity changes with the variations of the mean production height; a_H and A_H are plotted *versus* the residual range R and the cut-off thickness R_0, respectively.

vary too strongly with x' in the region between x_0 and x_2. In this case Eq. (20) simplifies to

$$\delta_T i_v/i_v = a_H(R)\delta H(x_1) + a_K(R)[\delta T(x_2)]_{\mathrm{Av}}, \quad (24)$$

where

$$a_H(R) = \alpha_H(R)\gamma(R)$$

$$a_K(R) = \frac{\mathcal{R}B}{Mgc\tau x_e}\ln\left(\frac{x_e - x_2}{x_e - x_0}\right)$$

$$[\delta T(x_2)]_{\mathrm{Av}} = \frac{1}{x_0 - x_2}\int_{x_2}^{x_0} \delta T(x')dx'. \quad (25)$$

Equation (24), although not so accurate as Eq. (20), gives us an insight into the problem in question: The temperature effect on the differential intensity i_v at sea level can be described by means of a two-term regression formula. The variations of i_v are correlated with the variations of the height of x_1, and the variations of the temperature averaged between x_2 and x_0. Figures 5 and 6 show the coefficients a_H and a_K plotted *versus* the residual ranges R.

We conclude this section with a few remarks concerning the temperature effect for the *integral* intensity near sea level, I_v. Evidently one obtains an expression applicable to this effect by integrating Eq. (20) or Eq. (24) over all residual ranges R above a certain cut-off value, R_0, determined by the experimental arrangement, *viz.*:

$$\delta_T I_v(R_0) = \int_{R0}^{\infty} a_H(R) i_v(R)\delta H dR + \int_{R0}^{\infty} a_K(R) i_v(R)(\delta T)_{\mathrm{Av}} dR. \quad (26)$$

Since the functions δH and $(\delta T)_{\mathrm{Av}}$ vary slowly with respect to R, Eq. (26) may be evaluated by means of the mean value theorem. Thus, according to Eqs. (15) and (16),

$$\delta_T I_v/I_v = A_H(R_0)\delta H(\bar{x}_1) + A_K(R_0)[\delta T(\bar{x}_2)]_{\mathrm{Av}}, \quad (26')$$

where

$$A_{H,K}(R_0) = \frac{1}{I_v(R_0)}\int_{R0}^{\infty} a_{H,K}(R) i_v(R) dR;$$

$$\bar{x}_1 = x_1(R_1), \quad \bar{x}_2 = x_2(R_2);$$

$$R_{1,2}\int_0^{\infty} a_{H,K} i_v dR = \int_{R0}^{\infty} R a_{H,K} i_v dR. \quad (27)$$

Figures 5 and 6 show the coefficients A_H and A_K plotted against the cut-off range R_0. These curves are convenient for comparative purposes with experimental observations. We shall return to them in Sec. V.

The numerical values of $\bar{x}_1$ and $\bar{x}_2$ for the case of $I_v(R_0)$ may be estimated directly from Fig. 3 where one has to take those values of x_1 and x_2 which correspond to the average residual ranges, R_1 and R_2, respectively. One finds that $\bar{x}_1$ will lie somewhere between 115 and 110 g cm^{-2}, and $\bar{x}_2$ between 190 and 160 g cm^{-2}, depending on the experimental value of R_0.

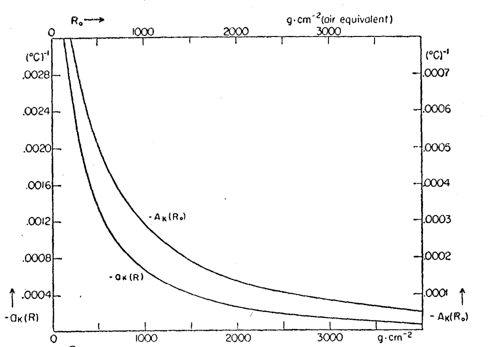

FIG. 6. The coefficients a_K and A_K correlating the differential and integral intensity changes with the variations of the mean temperature of the troposphere; a_K and A_K are plotted *versus* the residual range R and the cut-off thickness R_0, respectively.

IV. PRESSURE EFFECT ON THE COSMIC-RAY INTENSITY AT SEA LEVEL

To complete the discussion of the atmospheric effects on the cosmic-ray intensity, it remains to evaluate the effect of the atmospheric pressure. This evaluation may be carried out by a method analogous to that discussed in the preceding section. If the temperature overhead is kept constant, the partial variation of the differential intensity $i_v(R)$, due to the changes of the ground pressure x_0, is given by

$$\delta_P i_v = (\partial i_v/\partial x_0)\delta x_0. \quad (28)$$

Assuming that i_v may be represented by Eq. (11), one finds by differentiation

$$\delta_P i_v = \left\{ G(R)e^{-x_0/L} + \int_0^{x_0} \partial/\partial x_0[\ln G(R_s) + \ln w(x, R)]\Phi dx \right\}\delta x_0. \quad (28')$$

The first term in Eq. (28′) is negligible at sea level. The two last terms may be evaluated by means of the mean value theorem discussed previously. By substituting Eqs. (12) and (5) for $G(R_s)$ and $w(x, R)$, respectively, one gets

$$\delta_P i_v / i_v = a_P(R)\delta x_0, \tag{29}$$

where the "pressure" coefficient a_P is given by

$$a_P(R) = \frac{3.58}{x_0 + R - x_2 + 520} + \gamma \frac{\partial \alpha_H}{\partial x_0} H(x_1) + \alpha_H \frac{\mathcal{R}T}{Mgx_0} + \frac{\partial}{\partial x_0}[\alpha_K K(x_2, R)]. \tag{30}$$

The evaluation of Eq. (30) with help of Eqs. (6) and (7′) shows that the first term above is predominant. We have computed the pressure coefficient a_P for $100 < R < 4000$ g cm^{-2} under the assumption that the temperature, $T(x)$, is given by Fig. 1. The result is presented in Fig. 7, where a_P is plotted *versus* R.

For the convenience of the reader we have also computed the pressure coefficient of the integral intensity, i.e., the quantity

$$A_P(R_0) = \frac{1}{I_v(R_0)} \int_{R_0}^{\infty} a_P(R) i_v(R) dR. \tag{31}$$

The behavior of A_P, as a function of R_0, is shown in Fig. 7.

Before concluding this section, we should like to point out that the coefficient a_P is directly related to the measurements on the altitude dependence of the differential intensity i_v; according to its definition, a_P simply represents the slope of the intensity-depth curve. Therefore, referring to Eq. (30), the observed values of a_P may be alternately used as part of the data needed for the determination of the range spectrum at production $G(R_s)$.

V. SUMMARY—COMPARISON WITH EXPERIMENTS

(a) Atmospheric Effect on the Differential Intensity

According to the results obtained in Secs. III and IV, the total variation of the differential intensity near sea level may be represented by a three-term regression formula, *viz.*:

$$\delta i_v / i_v = a_H \delta H(x_1) + a_K[\delta T(x_2)]_{\text{Av}} + a_P \delta x_0; \tag{32}$$

i.e., the fractional changes in i_v are correlated with the following variables of the atmosphere:

(1) the departures, $\delta H(x_1)$, from the mean height of the pressure level $x_1(R)$;

(2) the departures from the mean temperature overhead averaged between the pressure level $x_2(R)$ and the sea level pressure, x_0, *viz.*:

$$[\delta T(x_2)]_{\text{Av}} = \frac{1}{x_0 - x_2} \int_{x_2}^{x_0} \delta T(x') dx';$$

(3) the departures from the mean pressure at sea level, δx_0. The numerical values of the two characteristic pressure levels, x_1 and x_2, are to be taken from Fig. 3. For the numerical values of the three coefficients, a_H, a_K, and a_P, one is referred to Figs. 5, 6, and 7.

(b) Atmospheric Effect on the Integral Intensity

Similarly to case (a), the total variation of the integral intensity near sea level is given by

$$\delta I_v / I_v = A_H \delta H(\bar{x}_1) + A_K[\delta T(\bar{x}_2)]_{\text{Av}} + A_P \delta x_0. \tag{33}$$

Here the coefficients A_H, A_K, and A_P are constant quantities for a given experimental arrangement. Their numerical values will be, however, different from case to case, depending on the amount of shielding material above the detector. Taking 400 g cm^{-2} air equivalent of shielding material as a typical case, we find from Figs. 5, 6, and 7

$$\begin{aligned} A_H &= -3.15 \text{ percent per km}, \\ A_K &= -0.059 \text{ percent per °C}, \\ A_P &= -1.79 \text{ percent per cm Hg}. \end{aligned} \tag{34}$$

In order to determine the above coefficients experimentally, one has to correlate the observed changes in I_v with the three atmospheric variables $\delta H(\bar{x}_1)$, $[\delta T(\bar{x}_2)]_{\text{Av}}$, and δx_0. The characteristic pressure levels, $\bar{x}_1$ and $\bar{x}_2$, have now practically fixed values: $\bar{x}_1 \simeq 115$ g cm^{-2}, and $\bar{x}_2 \simeq 190$ g cm^{-2}, if the amount of shielding above the detector is of the order of a few hundred grams per square centimeter. It is worth mentioning that the pressure level $\bar{x}_2$ nearly coincides with that of the tropopause at moderate latitudes. This implies that the vertical average of the temperature changes above the observer $(\delta T)_{\text{Av}}$, extends only over the region of the

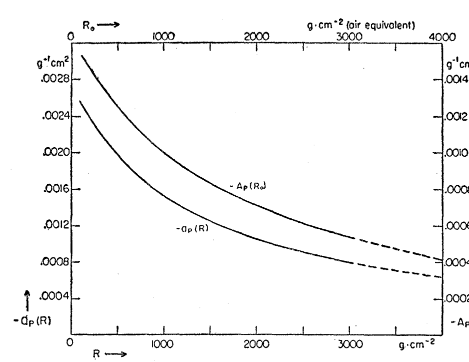

FIG. 7. The coefficients a_P and A_P correlating the differential and integral intensity changes with the variations of the sea-level pressure; a_P and A_P are plotted *versus* the residual range R and the cut-off thickness R_0, respectively.

troposphere, and does not include the temperature of the stratosphere.

(c) Comparison with Experiments

As we have mentioned in the Introduction, it has been customary to correlate the observed variations of the μ-meson intensity to only two atmospheric variables, δH and δx_0. Lack of knowledge of the third variable $(\delta T)_{\text{Av}}$, makes an exact comparison of our results with those found experimentally impossible. However, the following semiquantitative considerations appear to indicate substantial agreement between observations and theory.

In the first place, our value of 115 g cm^{-2} for the "mean pressure level" of μ-meson production is in agreement with the observations. For example, A Duperier obtained the best correlation between changes in cosmic-ray intensity and changes in the height of a given pressure level by choosing a value of this pressure level in the proximity of 100 g cm^{-2}.

In the second place, if one writes Eq. (33) in the form

$$\delta I_v/I_v=\left(A_H+A_K\frac{(\delta T)_{\text{Av}}}{\delta H}\right)\delta H+A_P\delta x_0, \tag{35}$$

one sees that the so-called "decay" coefficient of Eq. (1), A_H', is related to our coefficients A_H and A_K by the equation

$$A_H'=A_H+A_K[(\delta T)_{\text{Av}}/\delta H]. \tag{36}$$

The ratio $(\delta T)_{\text{Av}}/\delta H$ will depend, of course, on the geographic location, the season during which the experiment was performed, etc. Meteorological observations indicate that, for moderate latitudes, the above ratio will have on the average the following approximate values:

for diurnal departures of $(T)_{\text{Av}}$ and H from their means:

$$[(\delta T)_{\text{Av}}/\delta H]_{\text{diurnal}}\simeq 50°\text{C/km},$$

for seasonal departures of $(T)_{\text{Av}}$ and H from their means:

$$[(\delta T)_{\text{Av}}/\delta H]_{\text{seasonal}}\simeq 20°\text{C/km}.$$

It follows that the decay coefficient A_H' will be different depending on whether it is inferred from the diurnal or the seasonal changes of the intensity. According to Eqs. (34) and (36), one would expect for A_H' in these two cases:

$$[A_H']_{\text{diurnal}}\simeq -6.1\% \text{ per km},$$

$$[A_H']_{\text{seasonal}}\simeq -4.3\% \text{ per km}.$$

Indeed, the diurnal and seasonal decay coefficients estimated above are in substantial agreement with those observed experimentally. For instance, Dolbear and Elliot[6] deduced for A_H' the values -5.7 percent km^{-1} and -3.6 percent km^{-1} from the daily and monthly correlations, respectively.[11]

Regarding the pressure effect on the μ-meson intensity, we shall limit ourselves to the remark that the values of the pressure coefficient A_P, given in Fig. 7, seem to be in essential agreement with those observed experimentally. The agreement may be considered as a partial check for the consistency of the assumptions made about the production spectrum of μ mesons [see Eqs. (11) and (12)].

In conclusion we would like to discuss briefly the problem of the so-called "positive temperature effect." As we have mentioned in the Introduction, recent observations by Duperier have shown that the two-term regression formula given by Eq. (1) was inadequate to account fully for the observed variation of the cosmic-ray intensity. The analysis of the correlation coefficients indicated that there must be an additional atmospheric variable which, together with H and x_0, plays a controlling role in the intensity variations. Duperier assumed this variable to be the temperature of the pressure layer between 100 and 200 mb (lower stratosphere). Hence, he replaced Eq. (1) by the following:

$$\delta I/I=A_H\delta H+A_T\delta T+A_P\delta x_0, \tag{37}$$

where δT is the deviation from the mean of the temperature of the 100–200 mb pressure layer, and the other symbols have the same meaning as before. The temperature coefficient A_T, deduced from the observational data, turned out to be positive ($A_T=+0.12$ percent per °C). Duperier attempted to interpret this positive temperature effect as due to the competing processes of nuclear capture and decay of π mesons. However, a quantitative estimate of the effect of these processes, based on more recent data for the mean life and the cross section for nuclear capture of π mesons, has shown that the observed value of A_T is much too high to be ascribed exclusively to the finite life span of π mesons.

It is interesting to compare Duperier's regression formula, Eq. (37), with that given by Eq. (33). Since the coefficients A_H and A_P determined by Duperier are roughly in numerical agreement with those calculated in this paper, the ratio $(\delta T)_{\text{Av}}/\delta T$ should be equal to A_T/A_K. The value of our coefficient A_K is about -0.06 percent per °C while the experimental value of A_T is 0.12 percent per °C from which it follows that $(\delta T)_{\text{Av}}/\delta T\simeq -2$. The negative sign of the ratio $(\delta T)_{\text{Av}}/\delta T$ is not in contradiction with the general behavior of the free atmosphere. For it is well known that the warming of the troposphere [$(\delta T)_{\text{Av}}$ positive] is, as a rule, ac-

[11] In the experimental arrangement used by Dolbear and Elliot the threefold counter coincidence set was inclined at 45° to the vertical. Therefore the results of these authors are to be corrected before being compared with the results corresponding to the vertical intensity. It can be shown, however, that this correction is not crucial, and can be neglected in our semiquantitative discussion.

companied by a cooling of the lower stratosphere (δT negative) and vice versa. The magnitude of the above ratio is probably too large. However, a knowledge of the actual values of $(\delta T)_{Av}$ for the period and location of Duperier's experiment is needed to check our results quantitatively.

In view of the above discussion we conclude that the additional term, $A_K(\delta T)_{Av}$, in the regression formula may possibly remove the apparent variability of Elliot's decay coefficient as well as the anomalous value of Duperier's coefficient for the positive temperature effect. An experimental verification of this conclusion would be desirable. Unfortunately, because of the strong correlation between δH and $(\delta T)_{Av}$, it will be very difficult to separate experimentally the effects caused by δH from those caused by $(\delta T)_{Av}$.

The author would like to express his gratitude to Professor Bruno Rossi for his interest and assistance in the preparation of this paper, and to Professor H. Elliot and Professor A. Duperier for their criticisms. The author is also grateful to Professor J. M. Austin of the Meteorology Department at Massachusetts Institute of Technology for valuable discussions concerning meteorological data.

PHYSICAL REVIEW VOLUME 92, NUMBER 2 OCTOBER 15, 1953

Multiple Cores in Air Showers*

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(Received July 6, 1953)

Multiple cores in cosmic-ray air showers are difficult to observe because of the overlapping of the cores. After transition to equilibrium with water the cores are much smaller and readily identified. Decoherence measurements of the coincidences between pulses in ionization chambers have been made for depths in water from 0 to 3 meters and with separation of the chambers up to 6 meters. The decoherence measurements are consistent with an average air shower at 9000 feet which has about 20 cores within a distance of about 5 meters from the shower's center.

INTRODUCTION

SEVERAL experiments have been performed to study the structure of the extensive showers in cosmic rays. Calculations[1] based on the hypothesis of the cascade origin of these showers have been shown to be in agreement with experiments performed with Geiger counters[2] and ionization chambers.[3] It has been proposed by Lewis, Oppenheimer, and Wouthuysen[4] that the originating particles are produced in nucleon-nucleon collisions with high multiplicity. Hence, the resulting showers should have multiple cores corresponding to the multiplicity of the events. However, experiments[2,3] failed to detect any multiplicity in the cores of the extensive showers. These negative results have been explained by Blatt.[5] He pointed out that statistical fluctuations in experimental data are responsible for this apparent agreement between Molière[1] distribution and the experimental data.

Fretter and Ise[6] have reported another type of experiment to detect the presence of multiple cores, using water as an absorber and Geiger tubes as detectors. The experiments were carried out by Barrett,[7] and results were reported to agree with the single core distribution.

In the research reported here, another experimental arrangement was applied to study further the structure of the shower cores. Water was used as an absorber and fast ionization chambers as detecting instruments. The reason for these two choices can be explained as follows:

In water, the characteristic quantities for electron showers are quite different from those in air. Table I gives numerical values of the characteristic quantities for electron showers in both materials.

The unit r_1 was introduced by Euler and Wergeland[8] as a convenient unit for studying the spatial distribu-

TABLE I. Characteristic quantities for electron showers in air and water.

	Critical energy E_J $\times 10^9$ ev	Radiation length X_0 cm	Lateral unit r_1 cm
Air	11.3	33 000	5950
Water	11.3	43	7.8

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
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