Multiple Production of Pions in Nucleon-Nucleon Collisions at Cosmotron Energies*

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The statistical theory of multiple pion production is applied in some detail to the discussion of nucleon-nucleon collisions for primary energies of 1.75 Bev and 2.2 Bev. Probabilities are given for single and multiple productions of pions and nucleons with different charges.

THE availability of high-energy nucleons from the Brookhaven cosmotron makes it now possible to compare the results of the statistical theory[1] of multiple pion production with experiment.[2] In Table I of A, a tentative estimate of the relative probabilities that in a nucleon-nucleon collision various numbers n of pions are emitted together with two nucleons was given. According to formula (22) of A, these probabilities for for bombarding energies of a few Bev should be proportional to

$$f_n(w)=\left\{\frac{251}{w}(w-2)^3\right\}^n \Big/ \left(\frac{3}{2}\times\frac{5}{2}\times\cdots\times\frac{6n+1}{2}\right). \quad (1)$$

In this formula w is the total energy of the two colliding nucleons in the center-of-mass system including their rest energy. The nucleon rest energy is taken as unit of energy. A number of crude simplifying approximations have been introduced in A in deriving the preceding formula. One of them was to neglect the effects of the different possible charges of the nucleons and of the pions. We propose to improve the earlier results by a consideration of this factor. This will be done for low multiplicity production up to a maximum number of pions $n=3$. In doing this we shall make use of the conservation of isotopic spin as a limitation to the possible types of transitions.

The fundamental hypothesis of the statistical calculation of high-energy nuclear events is that in a collision process, all possible final states are formed with a probability proportional to the statistical weight of the final state. In listing all the possible final states, however, one should exclude all those that cannot be reached from the ground state because of conservation theorems. In addition to the classical conservation theorems of energy, momentum, and angular momentum, one should include in the present discussion also the conservation of isotopic spin and, of course, of charge. To be sure, the conservation of isotopic spin is not exact. It is believed, however, that only weak transitions are possible between states of different isotopic spin. Therefore, the statistical equilibrium postulated in A will normally not have time to be established except for states of equal isotopic spin.

In a collision of two nucleons, the initial state may have either isotopic spin $T=1$ or $T=0$. In computing the final states, only those with isotopic spin 1 or 0 shall have to be counted. For each final state characterized, for example, by the momenta of its particles, there are a number of different charge possibilities. Let p_n be the number of such possibilities for states of isotopic spin 1 with the given total charge, and q_n the similar number for isotopic spin 0. In Table I, we list the numbers p_n and q_n for states of two nucleons and n pions.

For example, in the collision of two high-energy protons, the isotopic spin of the initial state is $T=1$. A final state will be formed abundantly only when its isotopic spin is also 1 and we may assume that the probability of its formation will be proportional to $f_n(w)$ given by Eq. (1). In computing the relative probabilities for the formation of n pions, we shall take into account, however, that there are p_n states of isotopic spin 1. Therefore, the probabilities to form n pions will be proportional to p_nf_n and be given by

$$P_n=p_nf_n/\sum p_nf_n. \quad (2)$$

If the two colliding nucleons are a neutron and a proton, the initial state is a mixture of 50 percent isotopic spin 1 and 50 percent isotopic spin 0. If the initial state has $T=1$, the probability to form n pions will again be given by Eq. (2). For $T=0$, the probability will be given by a similar expression with p_n replaced by q_n:

$$Q_n=q_nf_n/\sum q_nf_n. \quad (3)$$

The resultant probability will be, therefore, the arithmetic average of Eqs. (2) and (3).

In discussing the comparison of these figures with experiment, it is important to give not only the number of pions that accompany the two nucleons in the final state, but also their charges. In order to do this, we must subdivide the numbers p_n and q_n of states with n pions into numbers of states corresponding to the

TABLE I. Number of states of isotopic spin 1 and 0 for a system of two nucleons and n pions.

n	0	1	2	3
p_n	1	2	4	9
q_n	1	1	2	4

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[1] E. Fermi, Progr. Theoret Phys. (Japan) 5, 570 (1950), quoted as A; Phys. Rev. 81, 683 (1951).
[2] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. (to be published).

different possible charges of the particles emitted. For example, according to Table I there are two states of isotopic spin 1 containing two nucleons and one pion. If we are discussing the collision of two protons (total charge = 2) the two states can be written as follows:

$$(-1/\sqrt{2})(pp0)+(1/2)(pn+)+(1/2)(np+) \quad (4)$$

and

$$(1/\sqrt{2})(pn+)-(1/\sqrt{2})(np+), \quad (5)$$

where, for example, $(pn+)$ means a state in which the first nucleon is a proton, the second a neutron, and the pion is a positive pion. If the final state were Eq. (4), the probability would be $\frac{1}{2}$ that in the final state there is a proton, a neutron, and a positive pion and $\frac{1}{2}$ that there are two protons and a neutral pion. If the final state were Eq. (5), the only possible final state would be a neutron, a proton, and a positive pion. In this case, therefore, the number $P_2=2$ is divided in a part 3/2 for the formation of a neutron, a proton and positive pion, and a part 1/2 corresponding to the formation of two protons and a neutral pion. The weights computed in this manner for the various cases are listed in Tables II and III.

TABLE II. Weights for different final states in a collision of two protons.

State	Weight	Number of pions	Number of prongs	Probabilities for primaries of energy 1.75 Bev	2.2 Bev
pp	1	0	2	8.6	4.0
$pp0$	$\frac{1}{2}$	1	2	14.1	11.3
$pn+$	$\frac{3}{2}$	1	2	42.3	32.7
$pp+-$	1.2	2	4	9.1	12.2
$pp00$	0.4	2	2	3.0	4.0
$pn+0$	1.8	2	2	13.6	18.2
$nn++$	0.6	2	2	4.6	6.0
$pp+-0$	154/60	3	4	1.4	3.2
$pp000$	18/60	3	2	0.2	0.4
$pn++-$	175/60	3	4	1.5	3.7
$pn+00$	121/60	3	2	1.0	2.5
$nn++0$	72/60	3	2	0.6	1.5

Table II illustrates the case of a collision between two protons. The first column gives the different types of particles that may appear in the final state compatible with charge conservation. The second column gives the weight of the state, the third is the number n of pions emitted and the fourth column is the number of charged particles emitted. The fifth and sixth columns will be discussed later.

Table III gives similar data for a neutron-proton collision. In this case two isotopic spins, $T=1$ and $T=0$, are possible, and therefore, two weights are given for each case in columns 2 and 3. Columns 4 and 5 give respectively the number of pions and the number of charged particles emitted. Again, the last two columns will be discussed later.

In order to show the use of the tables, we consider first the collisions of 1.75-Bev and 2.2-Bev protons against a proton at rest. One finds in this case $w=2.781$ for 1.75 Bev and $w=2.948$ for 2.2 Bev. The corresponding values of f_n are

$$f_0=1;\ f_1=3.28;\ f_2=0.88;\ f_3=0.062; \quad \text{for 1.75 Bev.}$$
$$f_0=1;\ f_1=5.53;\ f_2=2.49;\ f_3=0.30; \quad \text{for 2.2 Bev.}$$

The probability that the collision gives rise to an event of the type listed in the first column of Table II is given by the product of the weight listed in column 2 of the same table times the appropriate f_n. The probabilities calculated in this manner and normalized to 1 are listed in columns 5 and 6.

TABLE III. Weights for different final states in a collision of a proton and a neutron for the cases $T=1$ and $T=0$.

Final state	$T=1$	$T=0$	Number of pions	Number of prongs	Probabilities for primaries of energy 1.75 Bev	2.2 Bev
pn	1	1	0	1	12.3	6.0
$pp-$	$\frac{1}{2}$	$\frac{1}{3}$	1	3	15.7	12.9
$pn0$	1	$\frac{1}{3}$	1	1	22.8	18.5
$nn+$	$\frac{1}{2}$	$\frac{1}{3}$	1	1	15.7	12.9
$pp-0$	0.8	$\frac{1}{3}$	2	3	5.3	7.3
$pn00$	0.6	$\frac{1}{3}$	2	1	4.6	6.3
$pn+-$	1.8	1	2	3	13.8	18.9
$nn+0$	0.8	$\frac{1}{3}$	2	1	5.3	7.3
$pp-00$	0.9	0.4	3	3	0.4	1.0
$pp--+$	1.2	0.6	3	5	0.6	1.5
$pn000$	0.6	0.2	3	1	0.2	0.6
$pn+-0$	4.2	1.8	3	3	2.0	4.6
$nn+00$	0.9	0.4	3	1	0.4	1.0
$nn++-$	1.2	0.6	3	3	0.6	1.5

A similar calculation can be carried out for a collision of 1.75-Bev and 2.2-Bev neutrons with a proton. In this case, one should first compute in a similar manner the probabilities corresponding to isotopic spin 1 and 0, and then take the average of the results. These averages are given in columns 6 and 7 of Table III. For example, one can see from Table III that the probabilities of having a star with 1, 3, or 5 prongs in a neutron-proton collision of 1.75 Bev are 61.3, 37.8, 0.6 percent, and for a 2.2-Bev neutron are 52.6, 46.2, 1.5 percent. In particular, notice the very low probability of 5-pronged stars at these energies.

In statistics of 3-pronged stars, one will expect the probabilities of events in which a single negative pion, or at least a negative and a neutral pion, or a positive and a negative pion are produced should be 42, 15, 43 percent at 1.75 Bev and 28, 18, 54 percent at 2.2 Bev.

It should be stressed that all these figures are at best indicative of orders of magnitude. Obviously, a statistical theory of the type under discussion can, at best, yield qualitative results. In addition, in deriving formula (1) in A, many features like statistical correlation of the various particles and conservation of angular momentum have been neglected and it is to be expected that some sizeable error may be introduced thereby in the results.